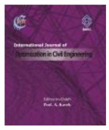
INTERNATIONAL JOURNAL OF OPTIMIZATION IN CIVIL ENGINEERING
Int. J. Optim. Civil Eng., 2026; 16(1):31-47



A HYBRID DIFFERENTIAL EVOLUTION AND SWARM ALGORITHM FOR ENGINEERING OPTIMIZATION

M. Shahrouzi*, †, Y. Naserifar, A.M. Taghavi, and S.-Sh. Emamzadeh
Civil Engineering Department, Faculty of Engineering, Kharazmi University, Tehran, Iran

ABSTRACT

Although metaheuristic algorithms are popular tools for global optimization, none of them is reported as the best for all problems. Hybridization is an advanced solution to overcome the shortcomings of individual methods by using the power points of the others. Here, a popular swarm intelligent algorithm with high explorative capability is combined with an exploitative operator of differential evolution and some dynamic parameter variation, as well as a greedy operator to enhance the search refinement. The proposed method is evaluated on a variety of engineering and constrained engineering problems, including the optimal design of Belleville Spring, pressure vessel, car side impact problem, and Morrow point dam. According to the results, considerable improvement is observed with respect to the standard particle swarm optimizer as well as competitive performance with a number of metaheuristic algorithms.

Keywords: Hybrid metaheuristic algorithms; differential evolution; particle swarm optimizer; mechanical engineering benchmarks; concrete dam; shape optimization.

Received: 20 November 2025; Accepted: 8 January 2026

[DOI: 10.22068/ijoce.2026.16.1.661]

1. INTRODUCTION

Metaheuristic algorithms are popular solutions for many real-world problems that have implicit or non-differentiable functions [1–3]. They utilize stochastic search aiming to overpass local optima. But the search refinement of any such individual method may be challenged due to its tuning or setup of operators for balancing of exploration and exploitation for every new case. In the other hand, no-free-lunch theory states that no single algorithm deserves as the best in all optimization scenarios.

Problem-specific approaches rise the need for enhancement of a single algorithm to overcome its case-dependent shortcomings. One of the solutions, is proper parameter

*Corresponding author: Faculty of Engineering, Kharazmi University, Tehran & Karaj, Iran
†E-mail address: shahruzi@khu.ac.ir (M. Shahrouzi)

tuning [4–7]. Generally some recommended values for parameters of an algorithm are derived from experimental results on specific set of benchmarks [8,9]. However, they may not be optimum for every new case. The matter brings about challenges in the real-world problems when the true optimum is not yet known or due to excessive computational costs [10,11]. It is difficult to make a reasonable relation between each set of parameter values with the nature of the problem in hand. In this regard, the use of artificial intelligence may be concerned; however, it is still a state of the art [12].

An alternative solution is hybridizing some components of various algorithms in a new framework to take benefit of every other method or overcome its specific shortcomings [11,13]. It is generally based on theoretical or analytical knowledge about pros-cons of different operators and their effect on the overall search process.

Investigators prefer to hybridize the algorithms from different categories so that variety of their features allows to achieve better performance when combined in a unified framework [14,15]. Some frameworks combine algorithms completely; while the other offers combining some of their operators. The latter usually brings about lower computational burden. The present work, considers hybridization of a popular algorithm in the category of swarm intelligence with the other in the category of evolutionary computation. They are hybridized so that no new parameter is added, in order to prevent from any extra tuning effort. For some parameters a dynamic control is utilized to achieve better balance of exploration and exploitation during the search. Besides, a greedy approach is embedded to improve the algorithm's efficiency.

[DOI: 10.22068/ijoce.2026.16.1.661]

2. SOLUTION ALGORITHMS

2.1 Differential Evolution

Differential Evolution, DE, is a population-based heuristic algorithm aiming to overpass local optima by sampling the cost function over the space of alternative solutions [16]. DE deals with some guiding vectors to make a walk in such a design space. Particularly, it utilizes a self-organizing operator that perturbs an existing vector of a population due to the difference vector between other two vectors that are randomly picked up. Such a walk from a current position X_i is represented in the form of:

$$X_i^{trial} = X_i + c_d(X_j - X_k) \tag{1}$$

X_i stands for the i^{th} design vector in the population and the direction of walk will be parallel to the difference vector from the position of X_k toward X_j; scaled by the factor c_d.

As X_i , X_j and X_k are picked up from the current population, no extra information outside it is required. By such an operator (called DE mutation), a trial solution X_i^{trial} is generated. During a greedy technique, X_i is replaced with X_i^{trial} only if it has less cost than X_i . Although DE borrows some other operators like crossover, it is the mutation operator that majorly distinguishes DE from many other evolutionary algorithms. Unlike genetic mutations that act on genotypic space after consequent encoding, the DE mutation utilizes vector-sum walks in continuous phenotypic space.

2.2 Particle Swarm Optimizer

Particle Swarm Optimization, PSO [17] is a pioneering swarm intelligent algorithm already applied to wide range of engineering problems. It applies perturbation on a current design vector X_i by summation of three other vectors:

$$X_i^t = X_i^{t-1} + c_i V_i^{t-1} + rand \times c_c (X_{Pbest,i}^{t-1} - X_i^{t-1}) + rand \times c_s (X_{Gbest}^{t-1} - X_i^{t-1}) \quad (2)$$

The first term of such a vector-sum perturbation, addresses the direction of previous velocity vector; so it is called the inertial term. The second (cognitive) term is directed from the current position toward its own experience; denoted by $X_{Pbest,i}^{t-1}$. The social term targets the third part of the velocity vector at the best experience of all particles in the swarm; that is referred to as X_{Gbest}^{t-1}. The function $rand$ generates random scale factors in the range [0, 1]; i.e. c_i, c_c and c_s as the corresponding inertial, cognitive and social factors. These form three specific control parameters of PSO in addition to the population size N_P and the number of iterations $t_{\max}$.

2.3 The proposed hybrid algorithm

Despite DE, no greedy selection is applied in the standard form of PSO. In the other hand, PSO keeps a level of diversity during the search that provides explorative capability. However, such a non-decreasing diversity level is a drawback for its search refinement. DE utilizes its operator to construct the new solution based on the existing individuals in the population. As the search progresses, the population of search agents gets closer to the best-so-far position reserved by greedy selection. Therefore, DE is expected to provide improved search refinement at the final iterations. The matter rises the idea to combine DE mutation and greedy selection with PSO.

Here, a hybrid method is developed to take advantageous of such different operators in PSO and DE. It is a called HPSODE and represented via the following steps:

Step 1. Randomly distribute N_P particles in the design space within X^L as the vector of lower bounds and X^U as the upper bounds on the design variables.

Step 2. Evaluate each particle in such an initial population and set the iteration number as $t=1$. For every i^{th} particle, set $X_{Pbest,i}^1 = X_i^1$.

Step 4. While $t < t_{\max}$ repeat the main loop (perform steps 5 and 6):

Step 5. Update the global best vector as X_{Gbest}^t and increase the iteration number t by 1.

[DOI: 10.22068/ijoce.2026.16.1.661]

Step 6. For every i^{th} particle do:

- Generate a trial position X_i^{trial} by cognitive and social terms of PSO:

$$X_i^t = X_i^{t-1} + rand \times c_c (X_{Pbest,i}^{t-1} - X_i^{t-1}) + rand \times c_s (X_{Gbest}^{t-1} - X_i^{t-1}) \tag{3}$$

- Greedy selection: evaluate the trail position and replace X_i^{t-1} with the trial vector X_i^{trail} when it is fitter than X_i^{t-1}.
- Generate another trial position X_i^{trial} by combination of DE mutation and an inertial term similar to PSO:

$$X_i^{trail} = X_i^t + c_i V_i^{t-1} + rand \times c_d (X_j^{t-1} - X_k^{t-1}) \tag{4}$$

where X_j^{t-1} and X_j^{t-1} are two randomly chosen particles in the current swarm population and update the inertial factor c_i at the current iteration t as:

$$c_i = (1 - \frac{t-1}{t_{\max} - 1})(\frac{c_c + c_s + c_d}{3}) \tag{5}$$

- Greedy selection: evaluate the trail position and replace X_i^{t-1} with the trial vector X_i^{trail} when it is fitter than X_i^{t-1}.
- Update $X_{Pbest,i}^t$ as the best of $X_{Pbest,i}^{t-1}$ and X_i^t .

Step 7. Update the global best vector X_{Gbest}^t as the fittest particle over $X_{Pbest,i}^t$, $i = 1,...,N_P$ and announce it as the optimum solution.

3. PERFORMANCE EVALUATION

In order to evaluate performance improvement of PSO via the proposed hybridization, a variety of engineering examples are studied. The specific parameters of PSO and HPSODE are set as 1, 2 and 2 for the inertial, cognitive and social factors, respectively. The population size and total number of function evaluations (NFE_{max}) are set depending on each problem.

In the best run of the winner among HPSODE and PSO, the achieved designs and the convergence curves are compared and the number of analyses corresponding to the last improvement of the cost function, is reported as NFE_{LI}. For each example, the statistical results are derived over a set of independent trial runs; provided that the fair comparison conditions [18] are satisfied.

[DOI: 10.22068/ijoce.2026.16.1.661]

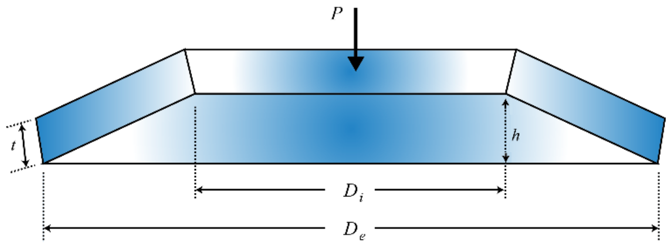


Figure 1: The Belleville Spring and its design parameters

3.1 Belleville spring design

This problem was introduced by Coello [19] to design a Belleville spring for minimum weight. Since then, it has been solved by several methods including *Genetic Algorithm* [19], *Teaching and Learning based optimization* (TLBO)[20], *Min Blast Algorithm* (MBA) [21] and *Particle Swarm Optimizer with a Diversity Classifier* [22]. It is subjected to constraints on the compressive stress, the deflection, height to deflection, height to maximum height, outer diameter, inner diameter, and slope. The design variables include the external diameter D_e and the internal diameter D_i of the spring, its thickness t and height h, as depicted in Fig.1. The problem is formulated as follows.

$$Min.f(X) = 0.07075\pi\left(D_e^2 - D_i^2\right)t \qquad X =< D_e, D_i, h, t > \tag{6}$$

Subjected to the following behavior constraints:

$$g_1(X) = \frac{4E\delta_{max}}{(1-\mu^2)\alpha D_e^2}\left[\beta\left(h - \frac{\delta_{max}}{2}\right) + \gamma t\right] - S \le 0 \tag{7}$$

$$g_2(X) = P_{max} - \frac{4E\delta}{(1-\mu^2)\alpha D_e^2}\left[\left(h - \frac{\delta_{max}}{2}\right)(h - \delta_{max})t + t^3\right] \le 0 \tag{8}$$

$$g_3(X) = \delta_{max} - \delta_l \le 0 \tag{9}$$

$$g_4(X) = h + t - H \le 0 \tag{10}$$

$$g_5(X) = D_e - D_{max} \le 0 \tag{11}$$

$$g_6(X) = D_i - D_e \le 0 \tag{12}$$

$$g_7(X) = \left(\frac{h}{D_e - D_i}\right) - 0.3 \le 0 \tag{13}$$

[DOI: 10.22068/ijoce.2026.16.1.661]

where $P_{max} = 5400\ lb$, $\delta_{max} = 0.2\ in$, $S = 200ksi$, $E = 30000\ ksi$, $\mu = 0.3$, $H = 2\ in$, $D_{max} = 12.01\ in$, $K = \frac{D_e}{D_i}$, $a = \frac{h}{t}$, $\delta_l = q(a)a$. The function $q(a)$ is given by Table 1, while the factors α, β and γ are defined by:

$$\alpha = \left(\frac{6}{\pi \ln(K)}\right)\left(\frac{K-1}{K}\right)^2, \beta = \left(\frac{6}{\pi \ln(K)}\right)\left(\frac{K-1}{\ln(K)}-1\right), \gamma = \left(\frac{6}{\pi \ln(K)}\right)\left(\frac{K-1}{2}\right) \qquad (14)$$

The lower and upper bounds on the design vector are given b X^L and X^U , respectively.

$$X^L =< 5.00, 5.00, 0.20, 0.20 >, \ X^U =< 15.00, 15.00, 0.25, 0.25 > \qquad (15)$$

Table 1: Definition of $q(a)$ for the Belleville spring design

a	$q(a)$	a	$q(a)$	a	$q(a)$
≤ 1.4	1.00	1.9	0.63	2.4	0.53
1.5	0.85	2.0	0.60	2.5	0.52
1.6	0.77	2.1	0.58	2.6	0.51
1.7	0.71	2.2	0.56	2.7	0.51
1.8	0.66	2.3	0.55	≥ 2.8	0.50

Table 2: Results comparison for the Belleville spring design

Var.	HPSODE	PSO	Gene AS I [23]	TLBO[20]	MBA[21]
x_1	12.01000	11.45447	11.627	12.01000	12.01000
x_2	10.03046	9.28284	9.534	10.03047	10.03047
x_3	0.204143	0.20703	0.205	0.204143	0.204143
x_4	0.20000	0.20202	0.201	0.20000	0.20000
Best Cost	1.97968	2.07233	2.01807	1.97968	1.97968
Mean Cost	2.02707	2.19907	-	1.97969	1.98469
SD	0.0530	0.0517	-	0.4500	0.0078
NFEmax (**NFELI)**	10000 (9949)	10000 (8395)	-	150000	15000 (10600)
Infeasibe	No	No	Yes	Yes	Yes

In the present work, the problem is solved using 40 particles and the results are compared in Table 2. It can be noticed that HPSODE has successfully captured the cost of 1.97968 within 10000 function calls. Such a result is competitive with the best of previous works by TLBO[20] and MBA [21], that have consumed more computational effort. At the same time, PSO has stopped in a local optimum after NFE_{LI} of 8395 with a higher cost of 2.07233. In addition, such a superiority of HPSODE over PSO stays reliable for the mean results of

[DOI: 10.22068/ijoce.2026.16.1.661]

Table 2. It can be noticed in Fig.2 that how better is the proposed algorithm with respect to PSO, regarding both the convergence rate and quality.

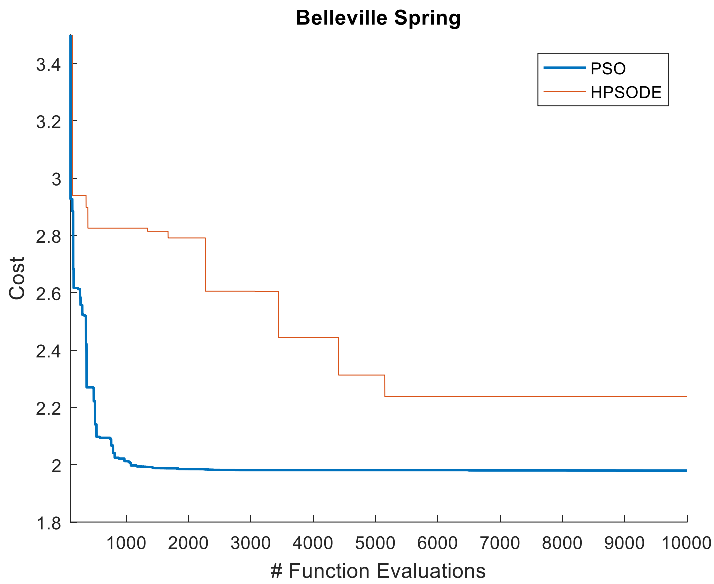


Figure 2: Convergence comparison of PSO and HPSODE in the Belleville spring design

3.2 Car side-impact problem

As a practical example, a car exposed to side-impact due to European Enhanced Vehicle-Safety Committee (EEVC) procedures, is considered here. Fig.3 demonstrates schematic of the side-impact problem. The design variables include thicknesses of B-Pillar inner, B-Pillar reinforcement, floor side inner, cross members, door beam, door belt-line reinforcement and roof rail, material properties of inner B-Pillar and floor side (x_8, x_9), the barrier height and the hitting position (x_{10}, x_{11}). It can be formulated as the following optimization problem to minimize the car weight.

$$Min.\, f(x) = 1.98 + 4.90x_1 + 6.67x_2 + 6.98x_3 + 4.01x_4 + 1.78x_5 + 2.73x_7 \quad (16)$$

Subjected to behavior constraints of:

$$g_1(x) = 1.16 - 0.3717x_2x_4 - 0.00931x_2x_{10} - 0.484x_3x_9 + 0.01343x_6x_{10} - 1 \le 0 \quad (17)$$

$$g_2(x) = 0.261 - 0.0159x_1x_2 - 0.188x_1x_8 - 0.019x_2x_7 + 0.0144x_3x_5 + 0.0008757x_5x_{10} + 0.080405x_6x_9 + 0.00139x_8x_{11} + 0.00001575x_{10}x_{11} - 0.32 \le 0 \quad (18)$$

$$g_3(x) = 0.214 + 0.00817x_5 - 0.131x_1x_8 - 0.0704x_1x_9 + 0.03099x_2x_6 - 0.018x_2x_7 + 0.0208x_3x_8 + 0.121x_3x_9 - 0.00364x_5x_6 + 0.0007715x_5x_{10} - 0.0005354x_6x_{10} + 0.00121x_8x_{11} - 0.32 \le 0 \quad (19)$$

[DOI: 10.22068/ijoce.2026.16.1.661]

$$g_4(x) = 0.074 - 0.061x_2 - 0.163x_3x_8 + 0.001232x_3x_{10} - 0.166x_7x_9 + 0.227x_2^2 - 0.32 \le 0 \quad (20)$$

$$g_5(x) = 28.98 + 3.818x_3 - 4.2x_1x_2 + 0.0207x_5x_{10} + 6.63x_6x_9 - 7.7x_7x_8 + 0.32x_9x_{10} - 32 \le 0 \quad (21)$$

$$g_6(x) = 33.86 + 2.95x_3 + 0.1792x_{10} - 5.057x_1x_2 - 11.0x_2x_8 - 0.0215x_5x_{10} - 9.98x_7x_8 + 22.0x_8x_9 - 32 \le 0 \quad (22)$$

$$g_7(x) = 46.36 - 9.9x_2 - 12.9x_1x_8 + 0.1107x_3x_{10} - 32 \le 0 \quad (23)$$

$$g_8(x) = 4.72 - 0.5x_4 - 0.19x_2x_3 - 0.0122x_4x_{10} + 0.009325x_6x_{10} + 0.000191x_{11}^2 - 4 \le 0 \quad (24)$$

$$g_9(x) = 10.58 - 0.674x_1x_2 - 1.95x_2x_8 + 0.02054x_3x_{10} - 0.0198x_4x_{10} + 0.028x_6x_{10} - 9.9 \le 0 \quad (25)$$

$$g_{10}(x) = 16.45 - 0.489x_3x_7 - 0.843x_5x_6 + 0.0432x_9x_{10} - 0.0556x_9x_{11} - 0.000786x_{11}^2 - 15.7 \le 0 \quad (26)$$

In this problem, two design variables x_8, x_9 are discrete, while the others are continuous. They are bounded by:

$$0.5 \le x_1, x_2, x_3, x_4, x_5, x_6, x_7 \le 1.5, x_8, x_9 \in \{0.192, 0.345\}, -30 \le x_{10}, x_{11} \le 30 \quad (27)$$

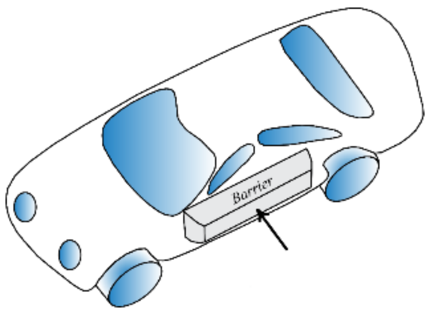


Figure 3: Schematic of the car side-impact

This problem is solved with a population of 50 particles. Fig. 4 shows that the standard PSO has been trapped in local optimum in early stages of the search; that is given in Table 4 as 2756 function calls out of 40000. It is while the proposed HPSODE has improved its search up to NFE_{LI} of 39939; achieving a cost of 22.844 that is much lower than 23.946 by PSO. According to Table 3, the proposed hybrid algorithm has also been successful in revealing better mean results over 20 trial runs. The best cost of HPSODE is quite competitive to the optimal cost of 22.843 as reported in previous works [24–26], provided that the present design is completely feasible.

[DOI: 10.22068/ijoce.2026.16.1.661]

Table 3: Results of optimal design for the car side impact problem

Var.	HPSODE	PSO	HGSO[24]	WWO[24]	MGGOA[24]
x_1	0.50000	0.51636	0.66947	0.58231	0.50000
x_2	1.11128	1.22862	1.16209	1.14379	1.11641
x_3	0.50000	0.55651	0.55690	0.52386	0.50000
x_4	1.31113	1.22308	1.20816	1.28768	1.30217
x_5	0.50000	0.50155	0.88163	0.56545	0.50000
x_6	1.50000	1.37451	1.04022	1.16623	1.50000
x_7	0.50000	0.57126	0.52033	0.52124	0.50000
x_8	0.34500	0.31533	0.34500	0.34500	0.34500
x_9	0.25583	0.31878	0.19200	0.19200	0.19200
x_{10}	-20.46989	-2.14095	0.46490	-7.7083	-19.552
x_{11}	-0.37526	7.35980	0.73053	-0.32270	0.27941
Best Cost	22.844	23.946	24.7113	23.7121	22.8432
Mean Cost	23.327	24.865	25.4618	23.7571	23.0823
SD	0.536	0.467	0.34610	0.34622	0.25023
NFE_{max} (NFE_{LI})	40000 (39939)	40000 (2756)	40000	40000	20000
Infeasible	No	No	No	No	Slight

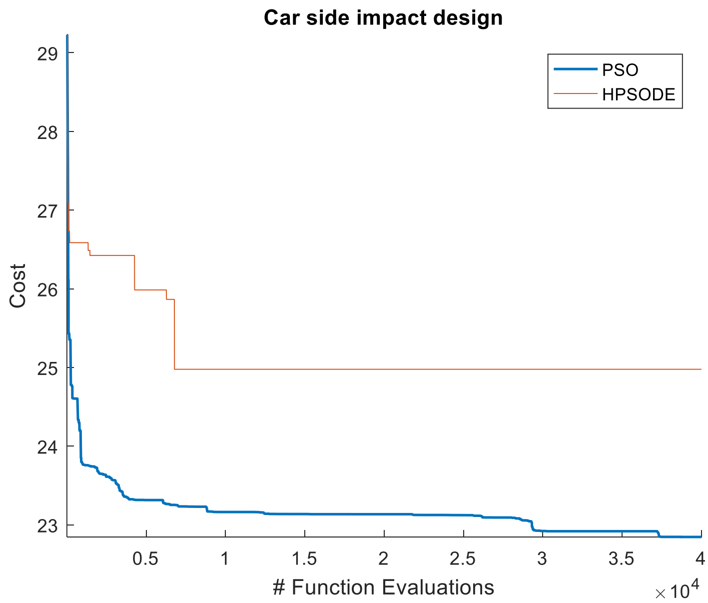


Figure 4: Convergence comparison of PSO and HPSODE in the car side impact problem

[DOI: 10.22068/ijoce.2026.16.1.661]

3.3 The pressure vessel design

As a well-studied engineering benchmark [27], design of the pressure vessel of Fig. 5 is considered due to ASME regulations to resist a pressure of 3000 psi. The problem variables are thickness of the cylinder shell: T_s, the head thickness: T_h, the inner radius: R and net length of the cylindrical part: L. The optimization problem is formulated as follows:

$$\min f(X) = 0.6224 x_1 x_3 x_4 + 1.7781 x_3^2 x_2 + 3.1661 x_1^2 x_4 + 19.84 x_3 x_1^2 \quad (28)$$

Subject to:

$$g_1(x) = 0.0193 x_3 - x_1 \le 0 \quad (29)$$

$$g_2(x) = 0.00954 x_3 - x_2 \le 0 \quad (30)$$

$$g_3(x) = 1296000 - \pi x_3^2 x_4 - \frac{4}{3}\pi x_3^3 \le 0 \quad (31)$$

$$g_4(x) = x_4 - 240 \le 0 \quad (32)$$

The variables are limited to the following bounds:

$$0.0625 \le x_1, x_2 \le 6.1875 \;\;,\;\; 10 \le x_3 \le 200 \;\;,\;\; 10 \le x_4 \le 200 \quad (33)$$

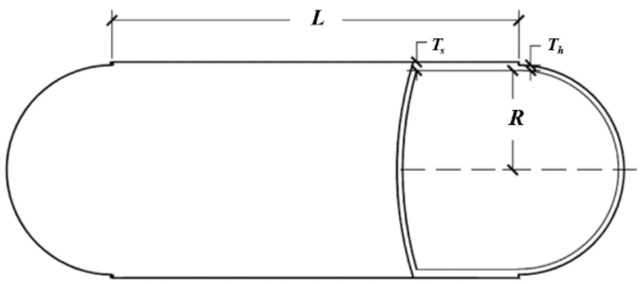


Figure 5: The pressure vessel design parameters [18]

Restricting the first two variables to be integer multipliers of 0.0625 inch, makes the problem of mixed discrete-continuous type; for which, the global optimum has been already reported [27] with the cost of 6059.7143. Table 4 gives statistical results of the present works and those reported by *Co-evolutionary Particle Swarm Optimization* (Co-PSO) [28], Cuckoo Search (CS)[29], and *Escaping Bird Search* (EBS) [18]. Comparison of PSOE and PSO convergence can be observed in Fig. 6. Among these methods, CS, EBS and HPSODE have captured the global optimum within a total of 20000, 12000 and 7000 analyses, respectively. It is while the hybrid method of Co-PSO has achieved a greater cost of 6061.077, even by consuming 200000 function calls. Despite the proposed HPSODE, the standard PSO has not been capable of continuing cost reduction after 5456 calls; that means a premature convergence. Besides, the mean and standard deviation of HPSODE have considerably been better than PSO.

Table 4: Results comparison for pressure vessel design

Var.	HPSODE	PSO	Co-PSO[28]	CS[29]	EBS[18]
x_1	0.8125	0.8750	0.8125	0.8125	0.8125
x_2	0.4375	0.6250	0.4375	0.4375	0.4375
x_3	42.0985	0.4246	42.09126	42.0985	42.0985
x_4	176.6366	1.9429	176.7465	176.6366	176.6366
Best Cost	6059.7143	7612.9785	6061.077	6059.714	6059.714
Mean Cost	6249.2896	8676.7452	6147.133	6447.736	6238.624
SD	250.6327	992.6354	86.5	502.6	238.7
NFE_{max} (NFE_{LI})	7000 (6204)	7000 (5456)	200000	20000	12000
Infeasible	No	No	No	Slight	No

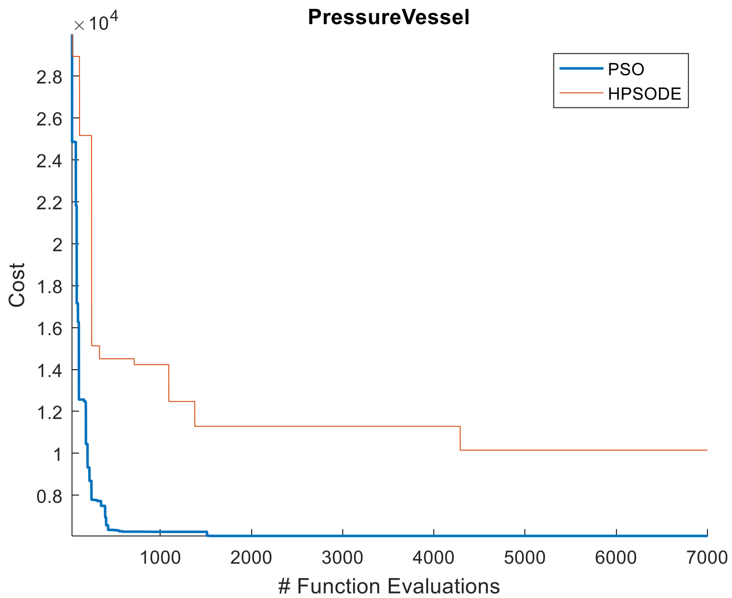


Figure 6: Convergence comparison of PSO and HPSODE in the pressure vessel design problem

3.4 Morrow point dam design

As a practical large-scale problem, optimal design of curved concrete dam is studied in this example. The dam resistance and cost is majorly affected by the shape of the upstream and downstream archs in the vertical and horizontal profiles [30]. The Morrow point dam is considered here, as a popular case study [31–33]. Table 5 reports the material properties for the finite element system of dam-water-rock.

[DOI: 10.22068/ijoce.2026.16.1.661]

Table 5: Material properties for the Morrow point dam design problem[11]

Parameter	Value (Unit)
Dam-body Concrete	
Mass Density	2483 (Kg/m^3)
Elasticity modulus	27580 (MPa)
Poisson's Ratio	0.20
Uniaxial compressive strength	30 (MPa)
Uniaxial tensile strength	1.5 (MPa)
Water	
Mass Density	1000 (Kg/m^3)
Velocity of Pressure Waves	1440 (m/s)
Wave reflection coefficient	0.90
Foundation rock Properties	
Mass Density	2483 (Kg/m^3)
Elasticity modulus	27580 (MPa)
Poisson's Ratio	0.25

The objective is to minimize the total volume of the dam body, V, by altering its shape design. It is penalized using the factor k_p via the following cost fucntion to satisfy the problem constraints.

$$Minimize\ Cost(X) = V(X) \times [1 + k_p \sum_i \max(0, g_i(X))] \quad (34)$$

The shaping design variables are demonstrated in Fig. 7. The dam curves are determined by interpolation using 6 control points. The corresponding parameters; i.e the thickness values, the upstream and downstrean radii, are included in the following design vector X. the other variables include the overhang and undercut slopes S_i , the height factor β and the roation angle of the dam central axis in the plan view as φ .

$$X = \left\{t_{c,1}, ..., t_{c,6}, r_{d,1}, ..., r_{d,6}, r_{u,1}, ..., r_{u,6}, S_1, S_2, \beta, \phi\right\}^T \quad (35)$$

Description of each constraint $g_i(X)$, is given in the previous studies [11,15]. They are simple bounds on the variables as well as the other stress, stability and geometry constraints. The latter set includes the overhang and undercut slopes, upstream and downstream radii and dam thickness among its height.

Such a large-scale example is solved using 10000 finite element analyses at each of the 50 indpendent run s. The best feasible design of this problem; i.e. the volume of $216314{,}93\ m^3$ belongs to the proposed HPSODE in comparison with the current PSO and the previous works [11,15]. Ta ble 6 reports such a best design by HPSODE and that of the PSO in the same run; in agreem ent with the convergence curves of Fig. 8. It can be noted that the best cost obtained by PSO (in another run) i.e. $237287\ m^3$ was yet worse than that reported for HPSODE in Table 6.

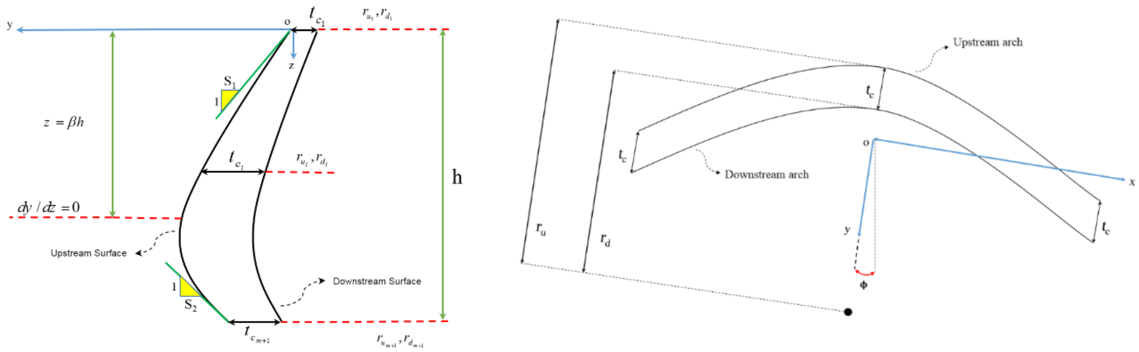
[DOI: 10.22068/ijoce.2026.16.1.661]

Table 6: Comparison of shape designs for Morrow Point dam by different methods

Design Var.	Lower bound	Upper bound	HPSODE	PSO	MPSO[11]	PSOTCHR[15]
$t_{c,1}$	7	10	7.00	7.00	7.02	7.00
$t_{c,2}$	8	15	9.80	10.18	9.86	9.81
$t_{c,3}$	12	20	12.00	13.32	12.02	12.00
$t_{c,4}$	15	25	15.00	15.03	15.03	15.00
$t_{c,5}$	17	30	17.00	17.09	17.02	17.00
$t_{c,6}$	20	35	20.00	21.28	20.10	20.00
$r_{d,1}$	115	156	129.00	127.42	134.41	130.90
$r_{d,2}$	99	133	116.10	117.54	120.97	117.81
$r_{d,3}$	82	111	103.61	102.15	108.98	100.90
$r_{d,4}$	65	88	88.00	88.00	87.86	87.98
$r_{d,5}$	48	65	65.00	65.00	64.86	65.00
$r_{d,6}$	31	42	42.00	42.00	41.61	42.00
$r_{u,1}$	115	156	122.06	122.66	123.54	121.98
$r_{u,2}$	99	133	115.96	116.43	117.36	115.88
$r_{u,3}$	82	111	103.61	101.25	108.19	100.90
$r_{u,4}$	65	88	87.99	84.09	87.74	87.98
$r_{u,5}$	48	65	65.00	60.28	64.79	65.00
$r_{u,6}$	31	42	33.12	41.99	41.42	33.12
s_1	0.09	0.36	0.36	0.34	0.36	0.36
s_2	0.09	0.36	0.16	0.09	0.09	0.10
β	0.50	0.90	0.90	0.51	0.73	0.84
$\varphi(^o)$	-1.00	1.00	1.00	-0.42	0.96	0.96
Best Cost			216314.93	255622.51	233065.38	217353.88
Mean Cost			235451.00	260815.65	261359.51	237778.04
SD			20315.6	16302.78	36246.3	9582.4
NFEmax (NFE$_{LI}$)			10000 (9539)	10000 (5993)	10000	10000
Infeasible			No	No	No	No

[DOI: 10.22068/ijoce.2026.16.1.661]

According to Fig. 8, considerable convergence improvement of the algorithm is observed due to the proposed hybridization in HPSODE; that continues its search refinement up to 9539 structural analyses. At the same run, the standard PSO has stopped cost reduction at NFE_{LI} of just 5993; with a premature convergence to the volume of $255622{,}51\ m^3$. In view of the mean results in Table 6, HPSODE has been superior to the others; however, the least standard deviation belongs to PSOTCHR [15].



(a) (b)

Figure 7. Shaping parameters of the dam model: (a) central section of the dam body, (b) rotation angle in the plan view [15]

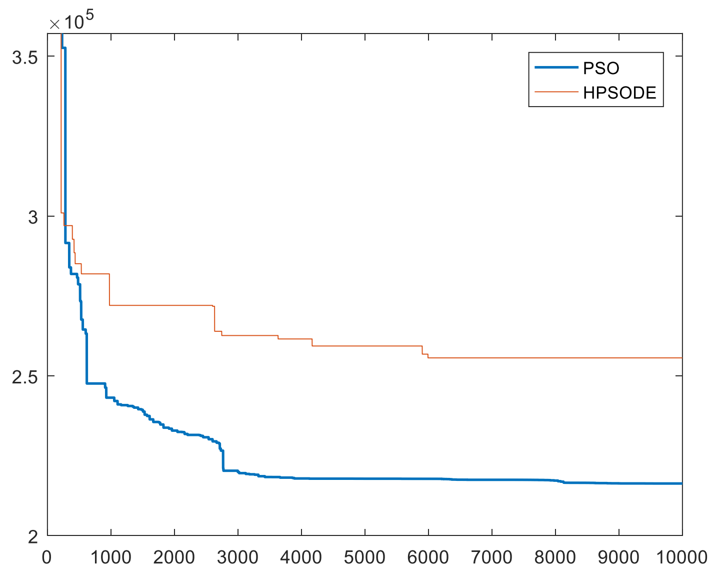


Figure 8: Convergence comparison of PSO and HPSODE in the dam shape optimization

4. CONCLUSION

Two metaheuristic algorithms in distinct categories of evolutionary and swarm optimization were combined to develop a new hybrid method. Applying greedy selection and dynamic

[DOI: 10.22068/ijoce.2026.16.1.661]

control of the inertial parameter finalized the proposed HPSODE without any extra parameters.

In order to evaluate performance enhancement of HPSODE with respect to PSO, they were examined on various mechanical and structural engineering problems. In this set the dam shape design and the car side impact are examples of practical highly constrained problems with numerous design variables. The latter and the pressure vessel problem are of mixed discrete-continuous variables.

It was observed that traditional PSO has not certain convergence and it may be trapped in local optima even in early stages of the search. In the other hand, the proposed hybrid algorithm could compensate such a shortcoming by more stable convergence toward global optimum. HPSODE continued its cost improvement up to more than 90% of the prescribed number of analyses in different treated problems. Higher difference was observed in the mixed discrete-continuous or large-scale problems. The proposed method could improve the quality of the best cost by up to 18% in the pressure vessel design and 15% in the dam shape optimization. In the treated set of problems, the corresponding mean results were improved up to 28% with respect to PSO. Additionally, in solving the previously addressed engineering benchmarks, HPSODE showed competitive performance in the final quality or the required effort to seek the global optimum. In conclusion, the proposed method offers as a successive and cost-efficient hybridization approach for improvement of such a popular swarm intelligent algorithm in engineering problems.

[DOI: 10.22068/ijoce.2026.16.1.661]

REFERENCES

1. Kaveh A. *Advances in Metaheuristic Algorithms for Optimal Design of Structures*. 3rd ed. Switzerland: Springer International Publishing; 2023.
2. Kaveh A, Mahdavi VR. Multi-objective colliding bodies optimization algorithm for design of trusses. *J Comput Des Eng*. 2019;**6**:49–59.
3. Rastegar Moghaddam M, Khanzadi M, Kaveh A. Multi-objective Billiards-Inspired Optimization Algorithm for Construction Management Problems. *Iran J Sci Technol - Trans Civ Eng*. 2021;**45**:2177–200.
4. Del Ser J, Osaba E, Molina D, Yang XS, Salcedo-Sanz S, Camacho D, et al. Bio-inspired computation: Where we stand and what's next. *Swarm Evol Comput*. 2019;**48**:220–50.
5. Yang XS. Nature-inspired optimization algorithms: Challenges and open problems. *J Comput Sci*. 2020;**46**:1–15.
6. Shahrouzi M. A new hybrid genetic and swarm optimization for earthquake accelerogram scaling. *Int J Optim Civ Eng*. 2011;**1**:127–40.
7. Taghavi AM, Shahrouzi M. Optimal design of spatial structures by a novel meta-heuristic algorithm: Sound energy optimizer. *Structures*. 2024;**70**:107570.
8. Shahrouzi M, Rafiee-Alavijeh F, Aghabaglou M. Configuration design of structures under dynamic constraints by a hybrid bat algorithm and teaching–learning based optimization. *Int J Dyn Control*. 2019;**7**:419–29.
9. Gandomi A, Yang X-S, Talatahari S, Alavi A. *Metaheuristic Applications in Structures and Infrastructures*. London, UK: Elsevier Inc.; 2013.

10. Shahrouzi M, Fahimi-Farzam M, Gholizadeh J. Optimization of multiple tuned mass damper inerter by escaping bird search for seismic control of buildings. *Int J Dyn Control*. 2025;**13**:362.
11. Shahrouzi M, Emamzadeh S, Naserifar Y. A hybrid Simulated Annealing, Particle Swarm and Ant Colony Optimization for Design of Double-Curved Dams. *Int J Optim Civ Eng*. 2023;**13**:413–38.
12. Kaveh A. *Applications of Artificial Neural Networks and Machine Learning in Civil Engineering*. vol. 1168. Cham: Springer Nature Switzerland; 2024.
13. Kaveh A, Shahrouzi M. A hybrid ant strategy and genetic algorithm to tune the population size for efficient structural optimization. *Eng Comput*. 2007;**24**:237–54.
14. Shahrouzi M, Sabzi AH. Damage detection of truss structures by hybrid immune system and teaching–learning-based optimization. *Asian J Civ Eng*. 2018;**19**:811–25.
15. Shahrouzi M, Naserifar Y. An Efficient Hybrid Particle Swarm and Teaching-Learning-Based Optimization for Arch-Dam Shape Design. *Struct Eng Int*. 2023;**33**:640–58.
16. Storn R, Price K. Differential Evolution – A Simple and Efficient Heuristic for global Optimization over Continuous Spaces. *J Glob Optim*. 1997;**11**:341–59.
17. Kennedy J, Eberhart R. Particle swarm optimization. Proc. ICNN'95 - Int. Conf. Neural Networks, IEEE; n.d., p. 1942–8.
18. Shahrouzi M, Kaveh A. An efficient derivative-free optimization algorithm inspired by avian life-saving manoeuvres. *J Comput Sci*. 2022;**57**:101483.
19. Coello CAC. Treating constraints as objectives for single- objective evolutionary optimization. *Eng Optim*. 2000;**32**:275–308.
20. Rao R V., Savsani VJ, Vakharia DP. Teaching-learning-based optimization: A novel method for constrained mechanical design optimization problems. *CAD Comput Aided Des*. 2011;**43**:303–15.
21. Sadollah A, Bahreininejad A, Eskandar H, Hamdi M. Mine blast algorithm for optimization of truss structures with discrete variables. *Comput Struct*. 2012;**102–103**:49–63.
22. Kim T, Cho M, Shin S. Constrained Mixed-Variable Design Optimization Based on Particle Swarm Optimizer with a Diversity Classifier for Cyclically Neighboring Subpopulations. *Mathematics*. 2020;**8**:1–29.
23. Deb K, Goyal M. Optimizing engineering designs using a combined genetic search. Proc. Sixth Int. Conf. Generic Algorithms, San Mateo: 1995, p. 521–8.
24. Gürses D. Enhanced Greylag Goose optimizer for solving constrained engineering design problems. *Mater Test*. 2025;**67**:900–9.
25. Yildiz AR, Abderazek H, Mirjalili S. A Comparative Study of Recent Non-traditional Methods for Mechanical Design Optimization. *Arch Comput Methods Eng*. 2020;**27**:1031–48.
26. Abderazek H, Yildiz AR, Sait SM. Mechanical engineering design optimisation using novel adaptive differential evolution algorithm. *Int J Veh*. 2019;**80**.
27. Yang XS, Huyck C, Karamanoglu M, Khan N. True global optimality of the pressure vessel design problem: a benchmark for bio-inspired optimisation algorithms. *Int J Bio-Inspired Comput*. 2013;**5**:329–335.
28. He Q, Wang L. An effective co-evolutionary particle swarm optimization for

[DOI: 10.22068/ijoce.2026.16.1.661]

constrained engineering design problems. *Eng Appl Artif Intell*. 2007;**20**:89–99.

29. Gandomi AH, Yang XS, Alavi AH. Cuckoo search algorithm: A metaheuristic approach to solve structural optimization problems. *Eng Comput*. 2013;**29**:17–35.
30. Mostafaei H, Sohrabi Gilani M, Ghaemian M. Stability analysis of arch dam abutments due to seismic loading. *Sci Iran*. 2017;**24**:467–75.
31. Duron ZH, Hall JF. Experimental and finite element studies of the forced vibration response of morrow point dam. *Earthq Eng Struct Dyn*. 1988;**16**:1021–39.
32. Salajegheh J, Salajegheh E, Seyedpoor SM, Gholizadeh S. Arch dam optimization considering fluid-structure interaction with frequency constraints using artificial intelligence methods. 14th World Conf. Earthq. Eng., Beijing: 2008, p. 1–10.
33. Talatahari S, Aalami MT, Parsiavash R. Optimum Design of Double Curvature Arch Dams Using A Quick Hybrid Charged System Search Algorithm. *Int J Optim Civ Eng*. 2016;**6**:227–43.