ENHANCED ANT COLONY OPTIMIZATION WITH A DIRECT CONSTRAINTS HANDLING STRATEGY FOR OPTIMAL DESIGN OF REINFORCED CONCRETE FRAMES

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ABSTRACT

In this paper an enhanced ant colony optimization algorithm with a direct constraints handling strategy is proposed for the optimization of reinforced concrete frames. The construction cost of reinforced concrete frames is considered as the objective function, which should be minimized subject to geometrical and behavioral strength constraints. For this purpose, a new probabilistic function is added to the ant colony optimization algorithm to directly satisfy the geometrical constraints. Furthermore, the position of an ant in each iteration is updated if a better solution is found in terms of objective value and behavioral strength constraints satisfaction. Five benchmark design examples of planar reinforced concrete frames are presented to illustrate the efficiency of the proposed algorithm.

Keywords: optimal design; ant colony optimization; constraint handling strategy; reinforced concrete frame.

1. INTRODUCTION

Optimal design of reinforced concrete (RC) frames is a complex optimization problem, due to the large number of variables affecting the design process, the different nature of the variables and the various reinforcement details available for beams and columns. For RC frames, three different cost components including concrete, steel and formwork should be considered and in this case a combination of design variables must be determined in such a way that the total cost is minimum [1-2]. Optimization of RC frames has attracted a great deal of attention in recent years [3]. Kaveh and Sabzi [4] conducted a comprehensive literature review which shows that metaheuristic algorithms have been widely used for the

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optimization of RC frames. Kaveh and Sabzi [4-5] used heuristic big bang-big crunch (HBB-BC) and heuristic particle swarm ant colony optimization (HPSACO) algorithms for the optimization of planar RC frames. Gharehbaghi and Fadaee [6] utilized particle swarm optimization (PSO) to optimize planar RC frames under earthquake loading. Gholizadeh and Aligholizadeh [7] employed bat algorithm (BA) for the optimization of RC frames. Esfandiary *et al.* [8] used a combination of PSO and multi-criterion decision-making strategy for design optimization of RC frames. Kaveh *et al.* [9] employed three metaheuristics for the optimal design of planar RC frames considering CO_2 emissions. Kaveh *et al.* [10] utilized enhanced colliding bodies optimization (ECBO) to deal with cost optimization problem of RC frames using an automated member grouping strategy.

One of the popular metaheuristic algorithms is ant colony optimization (ACO) developed by Dorigo [11]. This technique is based on swarm intelligence inspired by the behavior of real ants. Ants secrete pheromones to mark the direction of their movement, from nest to a food source, so that next ants can find shorter paths due to these trails [12]. In the past, some researchers has compared the performance of ACO with other metaheuristic algorithms in various fields of civil engineering, which shows its promising performance [13-14]. Penalty function methods are the most popular approach for handling constraints in solving constrained optimization problems due to their ease of implementation and simplicity. However, the major difficulty with the penalty function approach is to find appropriate penalty parameters required to effectively guide the optimization algorithm in the constrained design space towards the global optimum. Deb [15] showed that by using a direct constraints handling strategy, rather than penalty function approaches, it is possible to provide a search direction towards the feasible region. In this paper, an enhanced ant colony optimization (EACO) with the direct constraints handling strategy is proposed to tackle the cost optimization problem of RC frames.

In order to illustrate the efficiency of the proposed EACO algorithm, five benchmark design examples including 4-, 6-, 9-, 12- and 20-story planar RC frames are illustrated and the performance of EACO is compared with ACO and other algorithms in literature. The obtained numerical results demonstrate the superiority of the proposed EACO algorithm over the other algorithms.

2. OPTIMIZATION OF RC FRAMES

For beams and columns of RC frames a semi-infinite set of width, depth and steel reinforcement arrangements can be considered. This means that the computational effort of the optimization of RC frames increases as the dimensions of the design space increase. In order to address this issue, a countable number of cross-sections can be considered during the optimization process by constructing data sets in a practical range [4-5] and [16] according to the provisions of the ACI 318-08 code [17]. In this paper, the section databases provided in [4-5] and [16] are used for beams and columns.

In the size optimization of RC frames, the objective function is the total cost of the frame. The total cost of a RC frame includes the cost of concrete, steel reinforcement and framework of all beams and columns. In this case, the objective function for RC frames optimization can be stated as follows:

$$F = F_b + F_c \tag{1}$$

$$F_b = \sum_{i=1}^{nb} \left(C_C b_{b,i} h_{b,i} + C_S A_{S,b,i} + C_F \left(b_{b,i} + 2h_{b,i} \right) \right) L_i \tag{2}$$

$$F_c = \sum_{j=1}^{nc} \left(C_C b_{c,j} h_{c,j} + C_S A_{S,c,j} + 2C_F \left(b_{c,j} + h_{c,j} \right) \right) H_j \tag{3}$$

where F is the objective function; nb is the number of beams; $b_{b,i}$, $h_{b,i}$, L_i, and $A_{S,b,i}$ are the ith beam width, depth, length, and reinforcing bars area, respectively; nc is the number of columns; $b_{c,j}$, $h_{c,j}$, H_j, and $A_{S,c,j}$ are the jth column width, depth, length and area of the reinforcing bars, respectively; C_C, C_S, and C_F are the unit cost of concrete, steel, and the framework, respectively.

As the geometrical requirements, in each structural joint, the dimensions of the upper column (including width and height of the cross section $i.e.$, b_U^C, h_U^C) should not be larger than those of the bottom one (b_B^C, h_B^C), and also the number of reinforcing bars in the upper column (n_U) should not be greater than that of the bottom column (n_B). Also, the width of a beam (b^B) should not be greater than that of the bottom column.

During the optimization process of RC frames, structural analysis is performed to determine internal forces of the elements for the following load cases according to ACI 318-08 code [17].

$$\text{Load Case } 1 = 1.2D + 1.6L \tag{4}$$

$$\text{Load Case } 2 = 1.2D + 1.0L + 1.4E \tag{5}$$

$$\text{Load Case } 3 = 1.2D + 1.0L - 1.4E \tag{6}$$

$$\text{Load Case } 4 = 0.9D + 1.4E \tag{7}$$

$$\text{Load Case } 5 = 0.9D - 1.4E \tag{8}$$

where D, L and E are dead, live and earthquake loads, respectively.

According to the ACI 318-08 [17], for the design of RC beams, the applied moment in center, left and right joints of the beam should not exceed the capacity as follows:

$$M_u \le \phi M_n^+ \tag{9}$$

$$|M_{ul}^-| \le |\phi M_n^-| \tag{10}$$

$$|M_{ur}^-| \le |\phi M_n^-| \tag{11}$$

where M_u, M_{ul}^-, $and\ M_{ur}^-$ are the external moments applied in center, left and right joints of the beams, respectively; $M_n^+\ and\ M_n^-$ are positive and negative nominal moments; and $\phi = 0.9$ is the nominal strength reduction coefficient.

To evaluate the strength of RC columns subject to bending moment and axial force, the simplified P-M interaction diagram [17], shown in Fig. 1, is used in the current paper. For a designed column, the corresponding pair (M_u, P_u) under the applied loads should not fall outside the interaction diagram. Therefore, if the following conditions are met for a column,

it can be considered suitable and safe.

$$\sqrt{P_u^2 + M_u^2} \leq \sqrt{(\phi P_n)^2 + (\phi M_n)^2} \tag{12}$$

where P_u and M_u are externally applied axial force and moment, respectively; and P_n and M_n are nominal axial and flexural strengths, respectively.
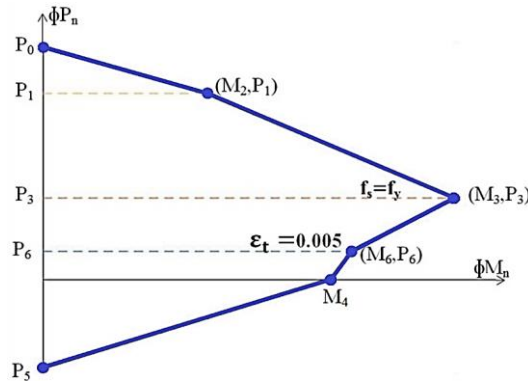

Figure 1. A simplified linear P-M interaction diagram [17]

Total cost optimization problem of RC frames subject to geometrical and strength constraints can be formulated as follows:

$$\text{Minimize: } F = F_b + F_c \tag{13}$$

Subject to:

Geometrical Constraints:
$$\begin{cases} g_1 = \dfrac{b_U^C}{b_B^C} - 1 \leq 0 \\[2mm] g_2 = \dfrac{h_U^C}{h_B^C} - 1 \leq 0 \\[2mm] g_3 = \dfrac{n_U}{n_B} - 1 \leq 0 \\[2mm] g_4 = \dfrac{b^B}{b_B^C} - 1 \leq 0 \end{cases} \tag{14}$$

Strength Constraints:
$$\begin{cases} g_5 = \dfrac{M_u}{\phi M_n^+} - 1 \leq 0 \\[2mm] g_6 = \dfrac{|M_{ul}^-|}{|\phi M_n^-|} - 1 \leq 0 \\[2mm] g_7 = \dfrac{|M_{ur}^-|}{|\phi M_n^-|} - 1 \leq 0 \\[2mm] g_8 = \dfrac{\sqrt{P_u^2 + M_u^2}}{\sqrt{(\phi P_n)^2 + (\phi M_n)^2}} - 1 \leq 0 \end{cases} \tag{15}$$

3. ANT COLONY OPTIMIZATION

ACO is a population-based metaheuristic algorithm for solving discrete optimization problems proposed by Dorigo [11]. This algorithm has been inspired by the foraging behavior of ants in the nature. Blind ants can find the shortest path between food sources and their nests. They release pheromones on the path of their own colony. The next ants choose their path according to the level of pheromone in the environment, so the probability of choosing a path with a high pheromone level is higher than other paths.

In ACO algorithm, the ants start at the home point, travel through the various points from the first to the last one, and end at the destination point in each iteration. So, each ant can update the pheromones by considering the desirability of the created tour. The amount of pheromones in the path between points i and j, τ_{ij}, is updated as follows [18-19]:

$$\tau_{ij} = (1 - \rho).\tau_{ij} + \sum_{k=1}^{m} \Delta \tau_{ij}^k \tag{16}$$

$$\Delta\tau_{ij}^k = \begin{cases} \dfrac{C_Q.Q_0}{L_k}, & \text{if edge}(i,j) \text{ is in tour of ant } k \\ 0, & \text{otherwise} \end{cases} \tag{17}$$

where, $\rho \in (0,1]$ is the pheromones evaporation rate; m is the number of ants; $\Delta\tau_{ij}^k$ is the amount of pheromone that is secreted by the ant k in the path between points i and j; Q_0 is a constant; L_k is the length of the kth ant path; and C_Q represents the concentration of pheromones that determined according to the relative importance of the found solution.

The kth ant located at point i, uses the pheromone trail τ_{ij} to compute the probability of choosing j as the next point as follows [18-19]:

$$P_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^{\alpha}.\left(\dfrac{1}{d_{ij}}\right)^{\beta}}{\sum \tau_{ij}^{\alpha}.\left(\dfrac{1}{d_{ij}}\right)^{\beta}}, & \text{if } C_{ij} \in N_i^k \\ 0, & \text{otherwise} \end{cases} \tag{18}$$

where C_{ij} represents the path between points i and j; N_i^k is the collection of neighborhood points of ant k when located at point i; α and β are parameters that are determined according to the relative importance of pheromone; and d_{ij} shows the distance between points i and j.

4. ENHANCED ANT COLONY OPTIMIZATION

In the framework of enhanced ant colony optimization (EACO) algorithm, different strategies are used to satisfy the geometrical and strength constraints of RC frames without

using penalty function method. For the geometrical constraints, a strategy is adopted using a new probability function (*PF*).

The geometrical constraints for columns necessitates that, at each joint, the cross-sectional dimensions and the number of reinforcements in the upper column must not be greater than those of the bottom column. Considering that all the cross-sections in column database are points in ACO, the probability function for selecting a path between points i and j, PF_{ij}^C, is defined as follows:

$$PF_{ij}^C = \frac{N_C - |i - j|}{N_C} \cdot \langle n_{ci} - n_{cj} \rangle^0 \cdot \langle b_{ci} - b_{cj} \rangle^0 \cdot \langle h_{ci} - h_{cj} \rangle^0 \tag{19}$$

where N_C is the number of cross-sections in the column database; n_{cj} and n_{ci} are the numbers of reinforcing bars in upper and lower columns, respectively; b_{cj} and b_{ci} are the widths of upper and lower columns, respectively; h_{cj} and h_{ci} are the heights of upper and lower columns, respectively; and $\langle \ \rangle^0$ is the Singularity Function of zero order, which is defined as follows:

$$\langle x - a \rangle^0 = \begin{cases} 1, & x \geq a \\ 0, & x < a \end{cases} \tag{20}$$

For beams, the probability function for selecting a path between points i and j, PF_{ij}^B, is defined as follows:

$$PF_{ij}^B = \frac{N_B - |i - j|}{N_B} \cdot \langle b_{ci} - b_j^B \rangle^0 \tag{21}$$

where N_B is the number of cross-sections in the beam database.

In the ECBO algorithm, Eq. (18) is modified as follows for beams and columns:

$$P_{ij}^k = \begin{cases} \dfrac{\tau_{ij}^\alpha \cdot \left(\dfrac{1}{d_{ij}}\right)^\beta \cdot PF_{ij}^C}{\sum_{j \in N_i^k}\left(\tau_{ij}^\alpha \cdot \left(\dfrac{1}{d_{ij}}\right)^\beta \cdot PF_{ij}^C\right)}, & \text{for columns} \\[4ex] \dfrac{\tau_{ij}^\alpha \cdot \left(\dfrac{1}{d_{ij}}\right)^\beta \cdot PF_{ij}^B}{\sum_{j \in N_i^k}\left(\tau_{ij}^\alpha \cdot \left(\dfrac{1}{d_{ij}}\right)^\beta \cdot PF_{ij}^B\right)}, & \text{for beams} \end{cases} \tag{22}$$

where N_i^k for columns and beams are section databases of columns and beams, respectively.

In addition, another strategy is adopted to directly satisfy the strength constraints as follows [15]: i) Any feasible solution is better than any infeasible solution; ii) among two

solutions in feasible region, the one with better objective value is better; and iii) among two solutions in infeasible region, the one with smaller constraint violation is better.

5. NUMERICAL EXAMPLES

Five benchmark design examples of 4-, 6-, 9-, 12- and 20-story RC frames are presented to illustrate the efficiency of the proposed algorithm. For 4- and 12-story RC frames, C_C, C_S, and C_F are 105 \$/m^3, 7065 \$/m^3 and 92 \$/m^2, respectively [4] and the section database of beams is given in Table 1. For 6-, 9- and 20-story RC frames, C_C, C_S, and C_F are 54 \$/m^3, 4317.5 \$/m^3 and 50.5 \$/m^2, respectively [5, 16] and the section database of beams is given in Table 2. In addition, the column section database is given for all the RC frames in Table 3.

Table 1: Beam section database for 4- and 12-story RC frames

No.	Width (mm)	Depth (mm)	Area ($\times 10^2$mm^2)	Moment of inertia ($\times 10^6$mm^4)	Number of bars		Factored moment resistance (kN.m)	
					Center (D19)	End (D22)	Center	End
1	300	450	1350	2278.1	2	2	75.366	97.689
2	300	450	1350	2278.1	3	2	108.75	97.693
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
837	450	900	4050	27338	10	12	780.27	1210.1
838	450	900	4050	27338	12	12	921.59	1210.0

Table 2: Beam section database for 6-, 9- and 20-story RC frames

No.	Width (mm)	Depth (mm)	Area ($\times 10^2$mm^2)	Moment of inertia ($\times 10^6$mm^4)	Number of bars		Factored moment resistance (kN.m)	
					Center (D22)	End (D22)	Center	End
1	300	450	1350	2278.1	2	2	97.738	97.738
2	300	450	1350	2278.1	3	2	141.98	97.738
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
906	450	900	4050	27338	10	12	1026.2	1214.4
907	450	900	4050	27338	12	12	1213.4	1213.4

Table 3: Column section database for all RC frames

No.	Width (mm)	Depth (mm)	Num. bars (D25)	P_0 (kN)	P_1 (kN)	P_3 (kN)	P_5 (kN)	P_6 (kN)	M_2 (kN.m)	M_3 (kN.m)	M_4 (kN.m)	M_6 (kN.m)
1	300	300	4	1643.3	1314.7	429	692.7	202.39	35.51	82.00	71.07	88.38
2	300	300	6	1880.7	1504.6	405.7	1039.1	99.441	39.075	101.85	101.36	109.68
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
54	900	900	22	13128	10503	4961.3	3810	4222.2	1019.4	2220.0	1507.0	2805.9
55	900	900	24	13366	10693	4954.9	4156.3	4213.3	1040.7	2314.5	1640.7	2936.7

In all the design examples, the maximum value of the following demand-capacity ratio (*DCR*) for all beams and columns of optimum solutions are reported.

$$DCR = \begin{cases} \dfrac{\sqrt{P_u^2 + M_u^2}}{\sqrt{(\phi P_n)^2 + (\phi M_n)^2}} & \text{for columns} \\[2em] max\left\{ \dfrac{M_u}{\phi M_n^+} \quad \dfrac{|M_{ul}^-|}{|\phi M_n^-|} \quad \dfrac{|M_{ur}^-|}{|\phi M_n^-|} \right\} & \text{for beams} \end{cases} \tag{23}$$

In all the design examples, the assumed specified compressive strength of concrete and yield strength of reinforcement bars are f_c'=23.5 and f_y=392 MPa, respectively. In addition, 50 independent optimization runs are performed for each example and the results are compared with literature.

5.1 Four-story RC frame

In this example, lateral equivalent static earthquake loads (*E*) are applied at joints as shown in Fig. 2, and uniform gravity loads are assumed for a dead load *D* = 22.3 kN/m and a live load *L* = 10.7 kN/m. The results of ACO and EACO are compared in Table 4. The number of structural analyses and the best cost found by heuristic particle swarm ant colony optimization (HPSACO) in [4] are 8500 and 22207 $, respectively.
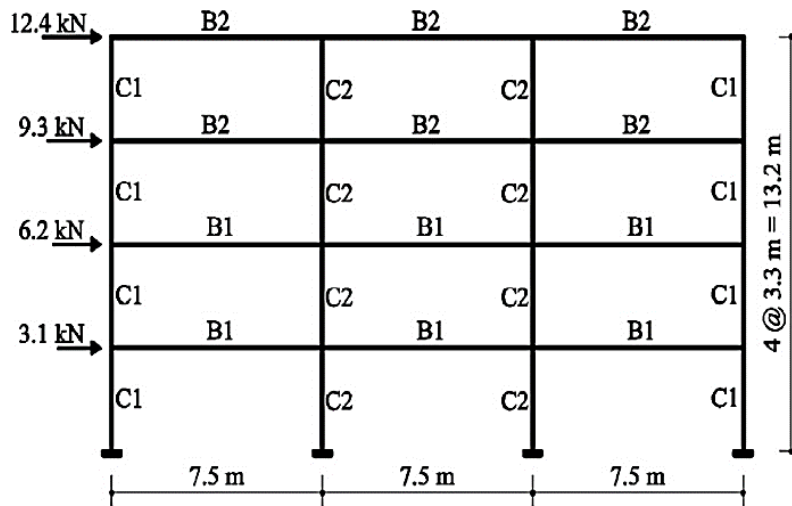
Figure 2. Four-story RC frame

Table 4: Optimization results of 4-story RC frame using ACO and EACO

Element		ACO				EACO			
		Dimensions		Reinforcements		Dimensions		Reinforcements	
Type	Group	Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	B1	300	550	3-D19	4-D22	300	450	5-D19	5-D22
	B2	300	450	8-D19	6-D22	300	450	5-D19	6-D22
Column	C1	350	350	8-D25		350	350	8-D25	
	C2	300	300	4-D25		300	300	4-D25	
Population size		200				50			

Iterations	46	32
Analyses	9200	1600
STD	32.123	24.127
Mean ($)	23503	21953
Best ($)	22384	21445
Max. DCR	0.9901	0.9997

The convergence histories corresponding to the best solutions found by ACO and EACO algorithms are compared in Fig. 3. The results show the superiourity of the EACO over ACO and HPSACO in terms of optimal cost and convergence rate.
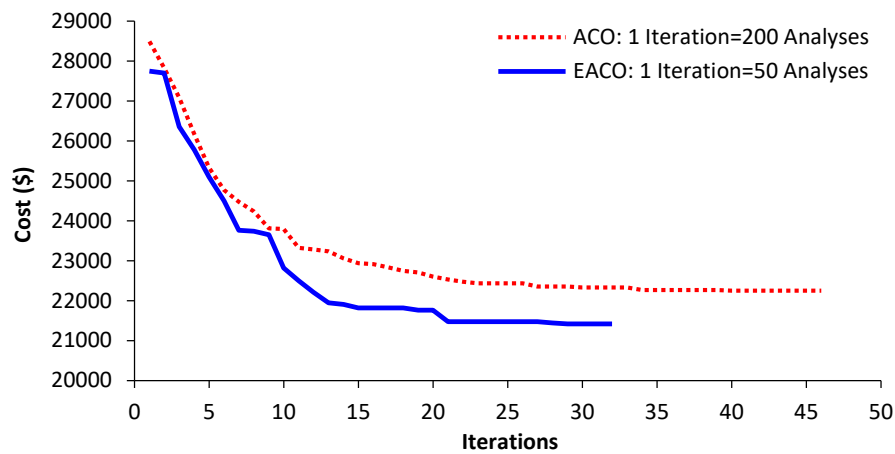

Figure 3. Convergence curves of the best designs found by ACO and EACO for 4-story frame

5.2 Six-story RC frame

In this example, lateral equivalent static earthquake loads (E) are applied at joints as shown in Fig. 4, and uniform gravity loads are assumed for a dead load $D = 16.5$ kN/m and a live load $L = 7.2$ kN/m. The results of ACO and EACO are compared in Table 5. The number of structural analyses and the best cost found by big bang-big crunch (BB-BC) in [5] are 29500 and 22182 $, respectively.

Figure 4. Six-story RC frame
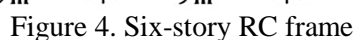
Fig. 5 illustrates the convergence histories related to the best solutions found by ACO and EACO algorithms. The results demonstrate the superiourity of the EACO over ACO and BB-BC in terms of optimal cost and convergence rate.

Table 5: Optimization results of 6-story RC frame using ACO and EACO

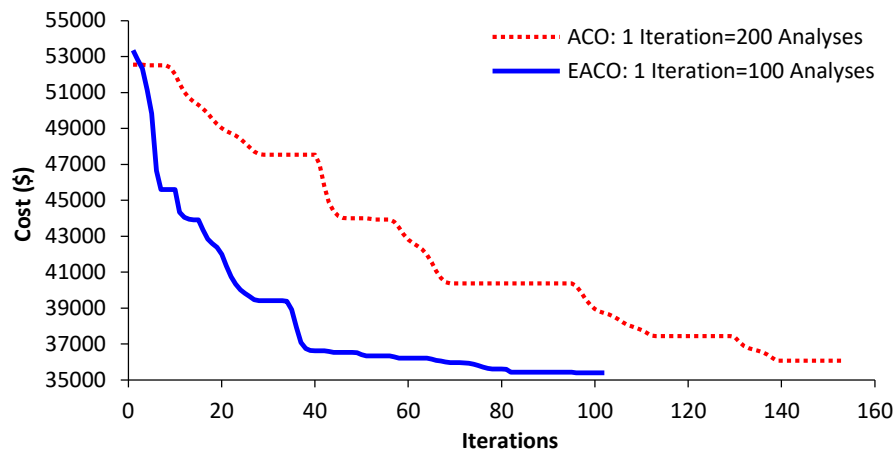| Element | | ACO | | | | EACO | | | |
| | | Dimensions | | Reinforcements | | Dimensions | | Reinforcements | |
Type	Group	Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	B1	300	450	3-D22	6-D22	300	450	3-D22	6-D22
	B2	300	450	3-D22	6-D22	300	450	4-D22	6-D22
	B3	300	450	3-D22	6-D22	300	500	3-D22	6-D22
Column	C1	450	450	12-D25		450	450	8-D25	
	C2	350	350	6-D25		350	350	4-D25	
	C3	400	400	10-D25		350	350	8-D25	
	C4	300	300	6-D25		300	300	4-D25	
Population size		200				100			
Iterations		102				78			
Analyses		20400				7800			
STD		129.157				68.306			
Mean ($)		24703				22541			
Best ($)		22163				21848			
Max. DCR		0.9725				0.9768			
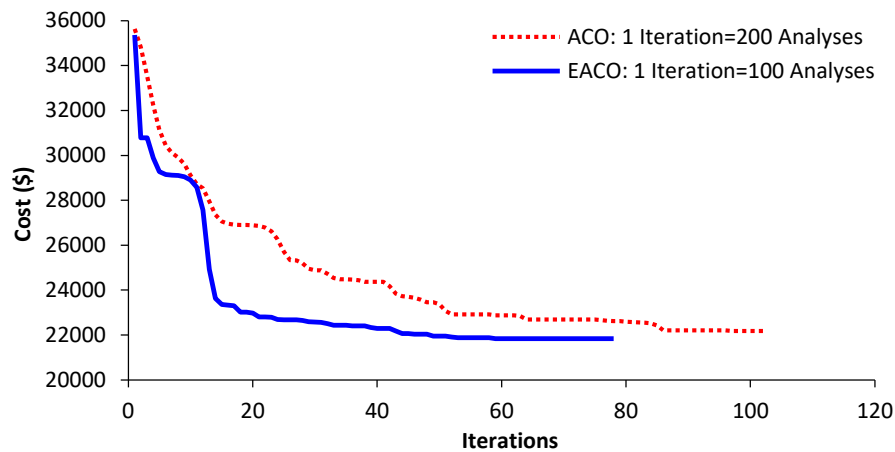
Figure 5. Convergence curves of the best designs found by ACO and EACO for 6-story frame

5.3 Nine-story RC frame

In the third example of this paper, lateral equivalent static earthquake loads (E) are applied at joints as shown in Fig. 6, and uniform gravity loads are assumed for a dead load $D = 16.5$ kN/m and a live load $L = 7.2$ kN/m. Table 6 reports the results of ACO and EACO algorithms. For this benchmark design example, the number of conducted structural analyses and the best cost found by big bang-big crunch (BB-BC) in [5] are 32000 and 35907 \$, respectively.

The convergence curves of the best runs of the ACO and EACO algorithms are illustrated in Fig. 7. The results show the superiourity of the EACO over ACO and BB-BC in terms of optimal cost and convergence rate.
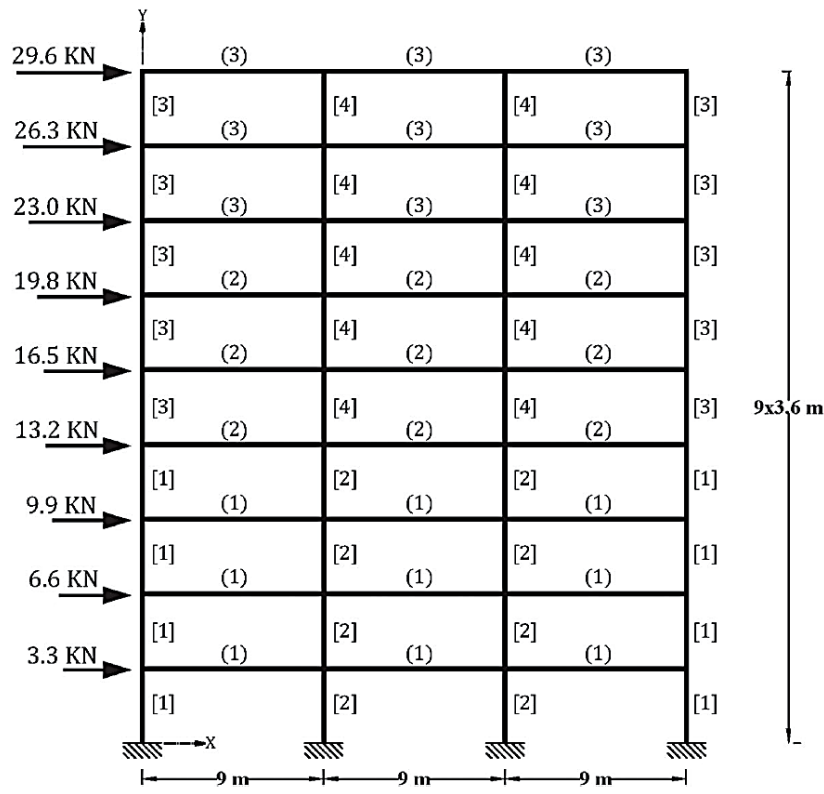
Figure 6. Nine-story RC frame

Table 6: Optimization results of 9-story RC frame using ACO and EACO

		ACO				EACO			
		Dimensions		Reinforcements		Dimensions		Reinforcements	
Element		Width	Depth	Positive	Negative	Width	Depth	Positive	Negative
Type	Group	(mm)	(mm)	moment	moment	(mm)	(mm)	moment	moment
Beam	B1	350	550	2-D22	6-D22	300	500	4-D22	6-D22
	B2	300	550	4-D22	6-D22	300	500	5-D22	6-D22
	B3	300	450	3-D22	6-D22	300	450	3-D22	6-D22
Column	C1	450	450	10-D25		450	450	10-D25	
	C2	400	400	10-D25		400	400	10-D25	
	C3	400	400	10-D25		400	400	10-D25	
	C4	350	350	6-D25		350	350	6-D25	
Population size		200				100			
Iterations		153				102			
Analyses		30600				10200			
STD		197.87				45.238			
Mean ($)		40123				36149			
Best ($)		36131				35388			
Max. *DCR*		0.9932				0.9975			

Figure 7. Convergence curves of the best designs found by ACO and EACO for 9-story frame

5.4 Twelve-story RC frame

Fig. 8 shows the 12-story RC frame and its lateral loads (E) applied at joints. Uniform gravity dead and live loads are $D = 22.3$ kN/m and $L = 10.7$ kN/m, respectively.
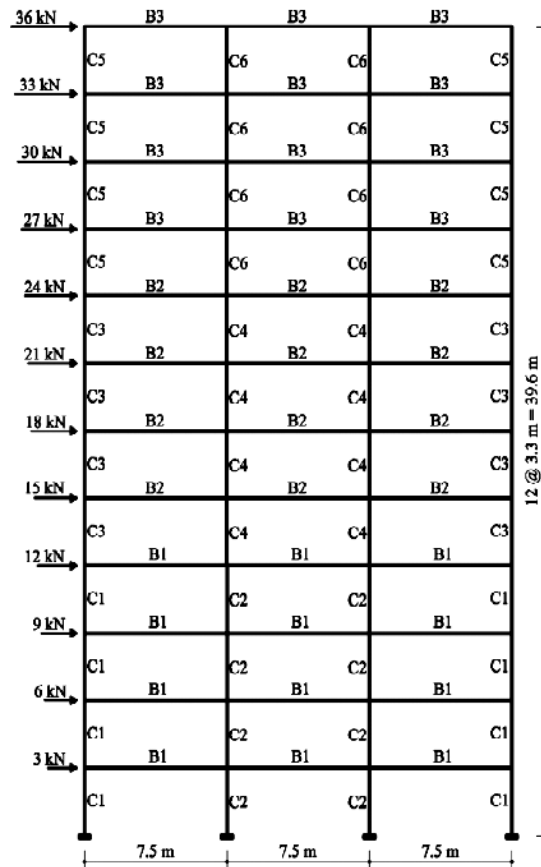

Figure 8. Twelve-story RC frame

The results of ACO and EACO are compared in Table 7. The number of structural analyses and the best cost found by heuristic big bang-big crunch (HBB-BC) in [4] are 54600 and 81138 $, respectively.

Table 7: Optimization results of 12-story RC frame using ACO and EACO

| Element | | ACO | | | | EACO | | | |
| | | Dimensions | | Reinforcements | | Dimensions | | Reinforcements | |
Type	Group	Width (mm)	Depth (mm)	Positive moment	Negative moment	Width (mm)	Depth (mm)	Positive moment	Negative moment
Beam	B1	300	600	4-D19	6-D22	300	600	4-D19	6-D22
	B2	300	550	4-D19	6-D22	300	550	4-D19	6-D22
	B3	300	550	5-D19	5-D22	350	550	3-D19	5-D22
Column	C1	500	500	8-D25		500	500	6-D25	
	C2	550	550	10-D25		550	550	10-D25	
	C3	400	400	6-D25		400	400	6-D25	
	C4	500	500	6-D25		500	500	6-D25	
	C5	400	400	6-D25		350	350	6-D25	
	C6	350	350	6-D25		350	350	6-D25	
Population size		250				125			
Iterations		192				147			
Analyses		48000				18375			
STD		196.453				68.598			
Mean ($)		87640				82343			
Best ($)		80404				79944			
Max. DCR		0.9685				0.9694			

For the ACO and EACO algorithms, the convergence curves of the best run are shown in Fig. 9. The results indicate the superiourity of the EACO over ACO and HBB-BC [4] in terms of optimal cost and convergence rate.
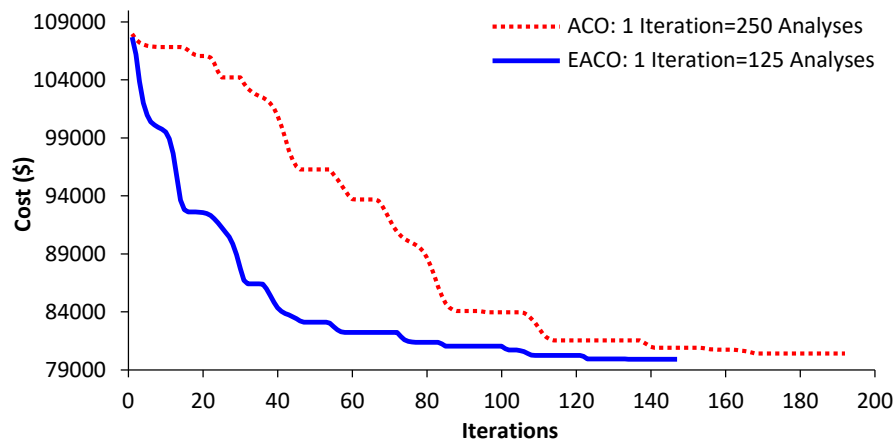

Figure 9. Convergence curves of the best designs found by ACO and EACO for 12-story frame

5.5 Twenty-story RC frame

Topology and lateral lods distribution of the 20-story RC frame are shown in Fig. 10.

Uniform gravity dead and live loads of $D = 16.5$ and $L = 7.2$ kN/m are applied to all beams.
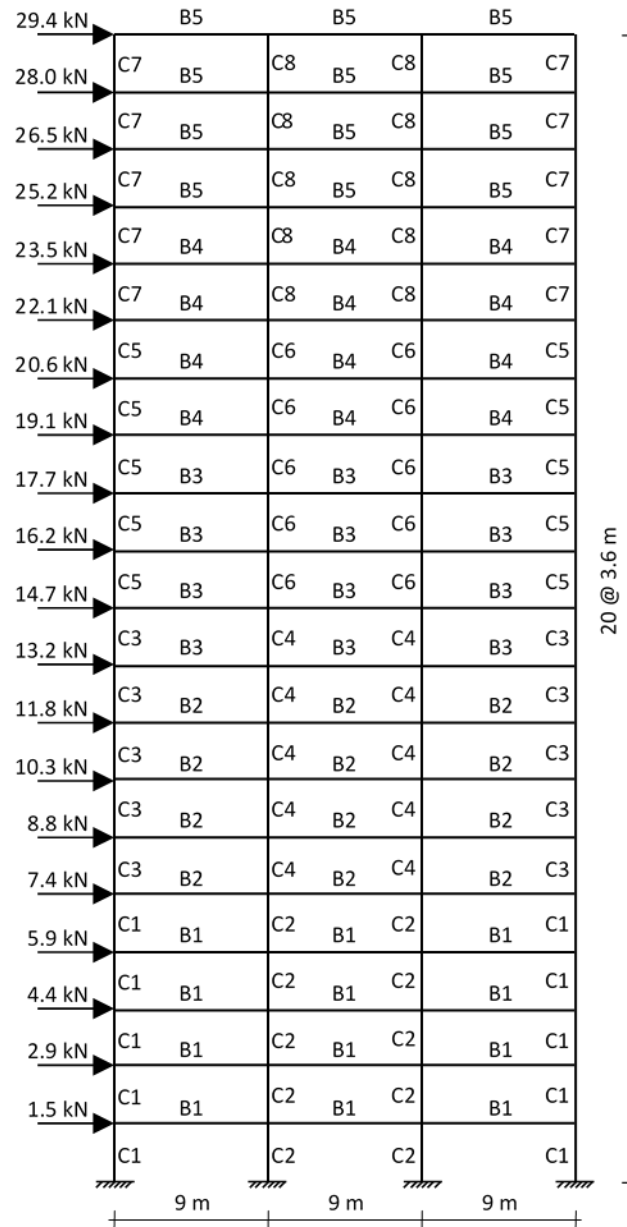

Figure 10. Twenty-story RC frame

This benchmark example is taken from [16], where the following load combinations are in accordance with ACI 318-99 [20] and genetic algorithm (GA) issed as the optimizer. The results of ACO and EACO algorithms are reported in Table 8. For this benchmark design example, the number of conducted structural analyses and the best cost found by GA in [20] are 360000 and 100833 $, respectively. The convergence curves of the best runs of the ACO and EACO algorithms are illustrated in Fig. 11.

$$\text{Load Case } 1 = 1.4D + 1.7L \tag{24}$$
$$\text{Load Case } 2 = 0.75(1.4D + 1.7L + 1.87E) \tag{25}$$
$$\text{Load Case } 3 = 0.75(1.4D + 1.7L - 1.87E) \tag{26}$$
$$\text{Load Case } 4 = 0.9D + 1.43E \tag{27}$$
$$\text{Load Case } 5 = 0.9D - 1.43E \tag{28}$$

Table 8: Optimization results of 20-story RC frame using ACO and EACO

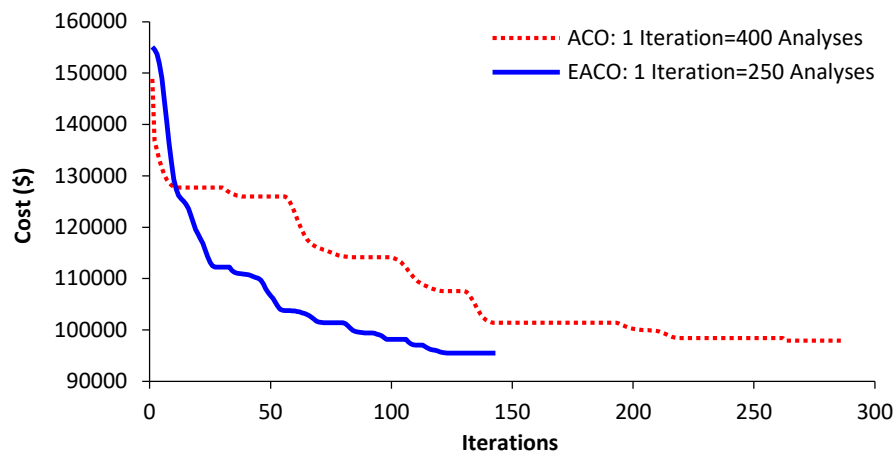|		ACO				EACO			
		Dimensions		Reinforcements		Dimensions		Reinforcements	
Element		Width	Depth	Positive	Negative	Width	Depth	Positive	Negative
Type	Group	(mm)	(mm)	moment	moment	(mm)	(mm)	moment	moment
Beam	B1	300	550	8-D22	8-D22	350	550	3-D22	9-D22
	B2	350	600	3-D22	8-D22	350	550	7-D22	8-D22
	B3	400	600	3-D22	7-D22	350	600	3-D22	8-D22
	B4	350	550	3-D22	9-D22	300	450	4-D22	8-D22
	B5	300	600	3-D22	5-D22	300	600	3-D22	5-D22
Column	C1	500	500	12-D25		500	500	12-D25	
	C2	850	850	18-D25		700	700	18-D25	
	C3	450	450	10-D25		500	500	12-D25	
	C4	600	600	8-D25		600	600	16-D25	
	C5	450	450	6-D25		450	450	6-D25	
	C6	550	550	8-D25		550	550	10-D25	
	C7	450	450	6-D25		450	450	6-D25	
	C8	400	400	6-D25		400	400	4-D25	
Population size		400				250			
Iterations		286				143			
Analyses		114400				35750			
STD		312.26				223.52			
Mean ($)		103729				96381			
Best ($)		97990				95482			
Max. *DCR*		0.9916				0.9980			

Figure 11. Convergence curves of the best designs found by ACO and EACO for 20-story frame

The results show the superiourity of the EACO over ACO and GA in terms of optimal cost and convergence rate.

6. CONCLUSIONS

An enhanced ant colony optimization (EACO) algorithm is proposed to deal with cost optimization of planar RC frames. In the proposed EACO, two strategies are adopted to directly satisfy the geometrical and strength constraints during the optimization process instead of using penalty function methods. To verify the effectiveness of the proposed EACO algorithm, five illustrative benchmark examples of planar 4-, 6-, 9-, 12- and 20-story RC frames are presented. For each design example, 50 independent optimization runs are performed using standard ACO and EACO and the results are compared with the literature. The main findings of this study are summarized as follows:

- For the 4-story RC frame, the best cost and the average cost found by EACO are 4.2% and 6.6% better than those of ACO. The number of structural analyses required by EACO is about 18% of the number of analyses required by ACO.
- For the 6-story RC frame, the best cost and the average cost found by EACO are 1.4% and 8.7% better than those of ACO. The number of structural analyses required by EACO is about 38% of the number of analyses required by ACO.
- For the 9-story RC frame, the best cost and the average cost found by EACO are 2.1% and 9.9% better than those of ACO. The number of structural analyses required by EACO is about 34% of the number of analyses required by ACO.
- For the 12-story RC frame, the best cost and the average cost found by EACO are 0.6% and 6.0% better than those of ACO. The number of structural analyses required by EACO is about 38% of the number of analyses required by ACO.
- For the 20-story RC frame, the best cost and the average cost found by EACO are 2.5% and 7.1% better than those of ACO. The number of structural analyses required by EACO is about 32% of the number of analyses required by ACO.
- For all the presented design examples, EACO outperforms the existing algorithms in the literature in terms of best cost and convergence rate.

Finally, it can be concluded that the proposed EACO is an efficient algorithm to solve cost optimization of planar RC frames.

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