



## VIBRATING PARTICLES SYSTEM-STATISTICAL REGENERATION MECHANISM ALGORITHM (VPS-SRM) FOR SIZING OPTIMIZATION OF STRUCTURES WITH DISCRETE VARIABLES

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### ABSTRACT

Vibrating particles system (VPS) is a swarm intelligence-based optimizer inspired by free vibration with a single degree of freedom systems. VPS is one of the well-known algorithms in structural optimization problems. However, its performance can be improved to find a better solution. This study introduces an improved version of the VPS using the statistical regeneration mechanism for the optimal design of the structures with discrete variables. The improved version is named VPS-SRM, and its efficiency is tested in the three real-size optimization problems. The optimization results reveal the capability and robustness of the VPS-SRM for the optimal design of the structures with discrete sizing variables.

**Keywords:** optimal design, metaheuristic, vibrating particles system, discrete variable, size optimization, frames, trusses.

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### 1. INTRODUCTION

In the recent decade, structural optimization has been one of the most popular research topics. Gradient-based methods and metaheuristic algorithms are the two main optimization methods. Due to simplicity and applicability, metaheuristic algorithms are more popular than gradient-based methods [1, 2]. Metaheuristic algorithms are applied in various fields, such as the optimum design of the truss and frame structures [3], reliability-based design

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optimization of the frames [4], design of cantilever retaining walls [5], and damage detection [6].

The single optimization approach cannot determine the optimal result for all optimization problems [7, 8]. Hence, engineers improve the performance of the optimization methods for their problems. For instance, Kaveh and Zakian [9] improved the performance of bat algorithm for optimum skeletal structure design. Nabati and Gholizadeh [10] introduced the modified version of the newton algorithm for the performance-based optimization of the steel frame. Kaveh and Talatahari [11] presented a new version of the charged system search for the optimum design of the truss structure. Al Thobiani *et al.* [12] Introduced the hybrid version of the particle swarm optimization and grey wolf algorithm for crack identification in structure. Javidi *et al.* [13] enhanced the performance of the crow search for optimum design of the structures. Alkayem *et al.* [14] presented a novel oppositional unified particle swarm gradient-based optimizer for structural damage detection problems. Kaveh and Zaerreza [15] proposed the improved version of the particle swarm optimization for the optimal design of the frames using the graph-theoretical force method. Gholizadeh *et al.* [16] present an improved version of the black hole for the optimum design of the planner structures.

In this paper, we focused on the vibrating particle system (VPS). This algorithm is introduced by Kaveh and Ilchi Ghazaan [17] and is used for a variety of optimization problems. In addition, researchers develop various improved versions of it. For example, Hoseini Vaez *et al.* [18] applied the VPS and its enhanced version for the reliability assessment of the truss structures. Kaur and Kumar [19] introduced the multi-objective version of the VPS for data clustering. Kaveh and Khosravian [20] improved the performance of the VPS for layout optimization of the trusses. Gnetchejo *et al.* [21] enhanced the performance of the VPS for parameter estimation of photovoltaic systems. Rabiei *et al.* [22] applied the VPS to optimize the reservoir system operation. Wedyan *et al.* [23] applied VPS for classification problem. Kaveh *et al.* [24] introduced the enhanced version of the VPS for structural damage detection problem.

The new enhanced version of the VPS using the statistical regeneration mechanism (SRM) is presented in this paper. This improved version is named VPS-SRM. In the VPS-SRM, 20 percent of the solutions are generated using the SRM, while the remaining of solutions are generated using the VPS. SRM utilizes the statistical information of the population; however, statistical information of the solution stored in the memory is used in the VPS-SRM. Also, when a new solution is generated using the SRM in VPS-SRM, first, the considered solution is replaced with the best solution obtained so far. Then, its variables are modified using the SRM. The performance of the VPS-SRM is tested in the three real size optimization problems with discrete sizing variables, including the 3-bay 15-story steel frame, 693-bar double-layer barrel vault, and 1016-bar double-layer grid. The optimization findings indicate that the VPS-SRM performs better than VPS and the other upgraded algorithms considered.

## 2. VIBRATING PARTICLES SYSTEMS

Vibrating particles systems (VPS) is a metaheuristic algorithm inspired by the free vibration

of the single degree of freedom systems, which was developed by Kaveh and Ilchi Ghazaan [17]. In this algorithm, the new position of the candidate solution is alternated based on bad and good candidate solutions selected for the considered solution and also the best candidate solution obtained by the algorithm so far. The steps of the VPS are provided as follows.

**Step 1: Initialization**

The algorithm parameters are set, and the optimization agents are randomly generated in the search space.

**Step 2: Select the good and bad optimization agents**

For each candidate solution, the good (GA) and bad agent (BA) is chosen. To do this, first, all of the agents are sorted based on their objective function. Then, from the fifty percent of agents with the good objective function, one of the agents is selected randomly as a GP. Form the remaining agents, one of them is chosen randomly as a BA.

**Step 3: Obtain a new position of the agent**

The new position of each optimization agent is obtained utilizing the following equation.

$$Vp_i^{new} = w_1 \times (D \times A \times r_1 + HA) + w_2 \times (D \times A \times r_2 + GA) + w_3 \times (D \times A \times r_3 + BA) \quad (1)$$

where  $Vp_i^{new}$  is the new position of the  $i$ th agent in the search space.  $r_1, r_2,$  and  $r_3$  are the random number which is generated between 0 and 1.  $w_1, w_2,$  and  $w_3$  are the parameter of the algorithm, which sum of them is one.  $HA$  is the best solution obtained so far. The parameter like  $p$  is defined by the user within (0,1), and the random number within (0,1) is generated for each agent. If the  $p < random\ number$ , then  $w_3$  is set to zero.  $D$  and  $A$  are defined as follows:

$$D = \left( \frac{Iter}{MaxIter} \right)^{-\alpha} \quad (2)$$

$$A = w_1 \times (HA - VP_i^{old}) + w_2 \times (GA - VP_i^{old}) + w_3 \times (BA - VP_i^{old}) \quad (3)$$

in which  $Iter$  is the current number of the iteration.  $MaxIter$  is the maximum number of the iteration.  $\alpha$  is the user-defined parameter, and  $VP_i^{old}$  is the position of the  $i$ th particle in the previous iteration.

**Step 4: Check the boundary of the search space**

The harmony search-based boundary handling approach is employed to ensure that the agents are in the search space. Hence, memory is considered, which is stored the best position obtained by the algorithm. The size of the memory is identical to the population size of the algorithm.

**Step 5: Check the termination condition**

The maximum number of iterations is considered as the termination condition of the algorithm. If the termination condition is satisfied, the optimization process is stopped, and the best solution stored in the memory is reported. Otherwise, the memory is updated, and

the algorithm goes to Step 2 for the next cycle of the optimization.

Details and MATLAB code of the VPS is provided by Kaveh and Bakhshpoori [25].

### 3. VPS-SRM OPTIMIZATION ALGORITHM

The statistical regeneration mechanism (SRM) developed by Kaveh et al. [26] is applied to enhance the performance of the VPS. The enhanced algorithm is named VPS-SRM. In the VPS-SRM, 80 percent of the optimization agent obtained new positions using the VPS, and the remaining population obtained their new positions utilizing the SRM. In order to apply the SRM, the mean and standard deviation of the solutions stored in the memory of the VPS is obtained. Then, the position of the considered agent is replaced with the best position of the best solution obtained so far. After that, in the first fifty percent of the optimization iteration, twenty percent of the positions are alternated using Eq (4). Otherwise, only one of its positions is modified as follows:

$$Vp_i^{new} = U(Mean - Std - Sigma, Mean + Std + Sigma) \quad (4)$$

where  $U$  is the operator that returns a random number generated from the continuous uniform distribution with lower and upper endpoints specified by  $Mean - Std - Sigma$  and  $Mean + Std + Sigma$ .  $Mean$  and  $Std$  are the average and standard deviation of the solutions stored in the memory of the VPS.  $Sigma$  is a parameter that helps the statistically regenerated mechanism to work efficiently when the entire population converges to the specified value and is defined as follows.

$$Sigma = \begin{cases} 5 & \text{If } Std < 0.01 \times (VP^{max} - VP^{min}) \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where  $VP^{max}$  and  $VP^{min}$  are the upper and lower bound of the search space. The value of the 5 is considered for the  $Sigma$  by testing the different functions and values. Due to using the rounding function to connect the discrete optimization problem to continuous optimization methods, using the constant value of the 5 it means that in Eq (4) at least 5 bigger or smaller sections than  $Mean$  are selected. For more details flowchart of the VPS-SRM is provided in Fig. 1.

### 4. DESIGN EXAMPLES

Performance of the VPS-SRM is tested on three design examples, consisting of the 3-bay 15-story steel frame, 693-bar double-layer barrel vault, and 1016-bar double-layer grid. The parameters of the VPS are the same as the parameters used by Kaveh and Ilchi Ghazaan [27]. The VPS part of the VPS-SRM also utilizes the same parameters. The maximum number of function evaluations for both optimization algorithms is set to 20000. To get the

statistical results, each of the algorithms in each example 30 independent runs are being considered.

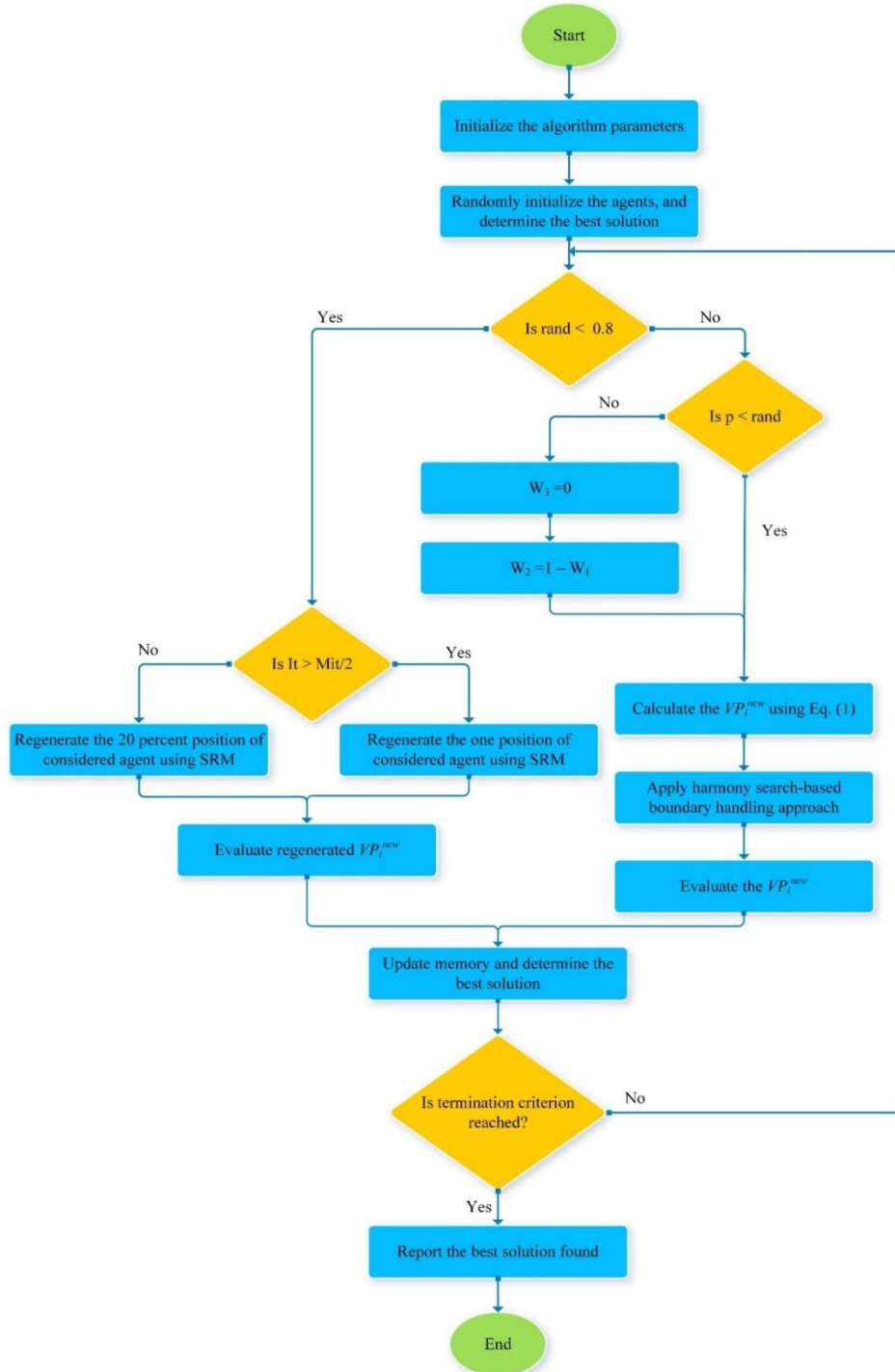


Figure 1. Flowchart of the VPS-SRM

4.1 3-bay 15-story steel frame

The first example considered in this study is the 3-bay 15-story steel frame. The structural elements are divided into 11 groups, as shown in Fig. 2. The cross-section of the members is selected from 267 W-section. The modulus of elasticity and the yield stress of the members are set to 29000 ksi and 36 ksi, respectively. Stress and displacement requirement is investigated according to the AISC-LRFD standards. Additionally, the top story drift is limited to 9.25 in. Detailed information on this design example can be found in Refs. [2, 28].

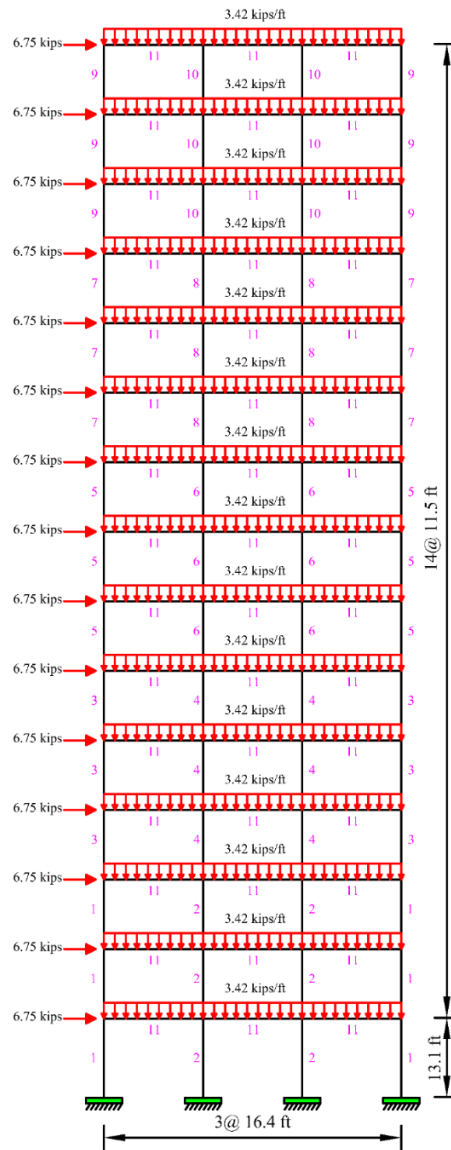


Figure 2. The 3-bay 15-story steel frame

The optimization result acquired by VPS-SRM, VPS, particle swarm optimization-statistical regeneration mechanism (PSO-SRM) [15], GA-based reduced search space

(GA-RSS) [29], enhanced whale optimization algorithm (EWOA) [30], An accelerated water evaporation optimization (Accelerated WEO) [31], and Hybrid Algorithm of Harmony Search, Particle Swarm and Ant Colony (HPSACO) [32] are provided in Table 1. According to this table, the optimum solution acquired by the VPS-SRM is better than other optimization methods. Also, considering the statistical results, VPS-SRM obtains better results than other considered algorithms. Stress ratio and inter-story drift are provided in Figs. 3 and 4. According to them, none of the optimization constraints is violated. The convergence histories of the VPS-SRM and VPS are given in Fig. 5.

Table 1: Comparison results of the VPS-SRM and VPS algorithms with other methods in the 3-bay 15-story steel frame

Element group	Optimal cross-sectional areas (W shapes)						
	HPSACO [32]	Accelerated WEO [31]	EWOA [30]	GA-RSS [29]	PSO-SRM [15]	Present study	
						VPS	VPS-SRM
1	W21×111	W14×99	W14×99	W33×118	W14×90	W12×87	W14×90
2	W18×158	W27×161	W27×161	W36×160	W36×170	W36×194	W36×170
3	W10×88	W27×84	W27×84	W14×90	W14×82	W18×76	W14×82
4	W30×116	W24×104	W24×104	W24×104	W24×104	W27×129	W24×104
5	W21×83	W14×61	W21×68	W24×76	W12×65	W12×65	W12×65
6	W24×103	W30×90	W18×86	W18×86	W18×86	W27×94	W18×86
7	W21×55	W16×50	W21×48	W14×48	W18×50	W21×48	W8×48
8	W27×114	W21×68	W14×68	W12×58	W14×61	W14×68	W14×61
9	W10×33	W14×34	W8×31	W14×30	W14×30	W14×34	W16×36
10	W18×46	W8×35	W10×45	W16×40	W10×39	W12×40	W18×35
11	W21×44	W21×44	W21×44	W21×44	W21×44	W21×44	W21×44
Best (lb)	95,850	87,537.96	88,090	91124.03	86950.79	90986.19	86916.96
Mean (lb)	N/A	88,893.09	90,784	95968.67	87705.73	96251.95	87442.06
SD (lb)	N/A	N/A	N/A	3212.52	722.49	3473.05	348.44

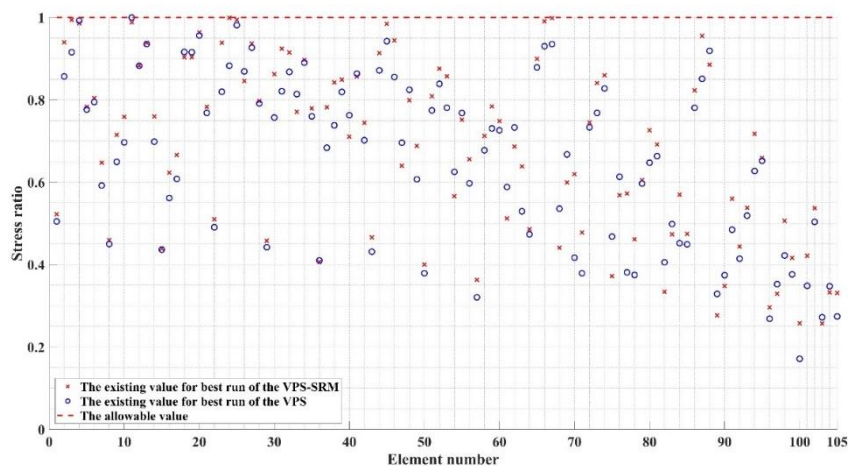


Figure 3. Stress ratio values calculated at the obtained optimal design by the VPS-SRM and VPS for the 3-bay 15-story frame structure

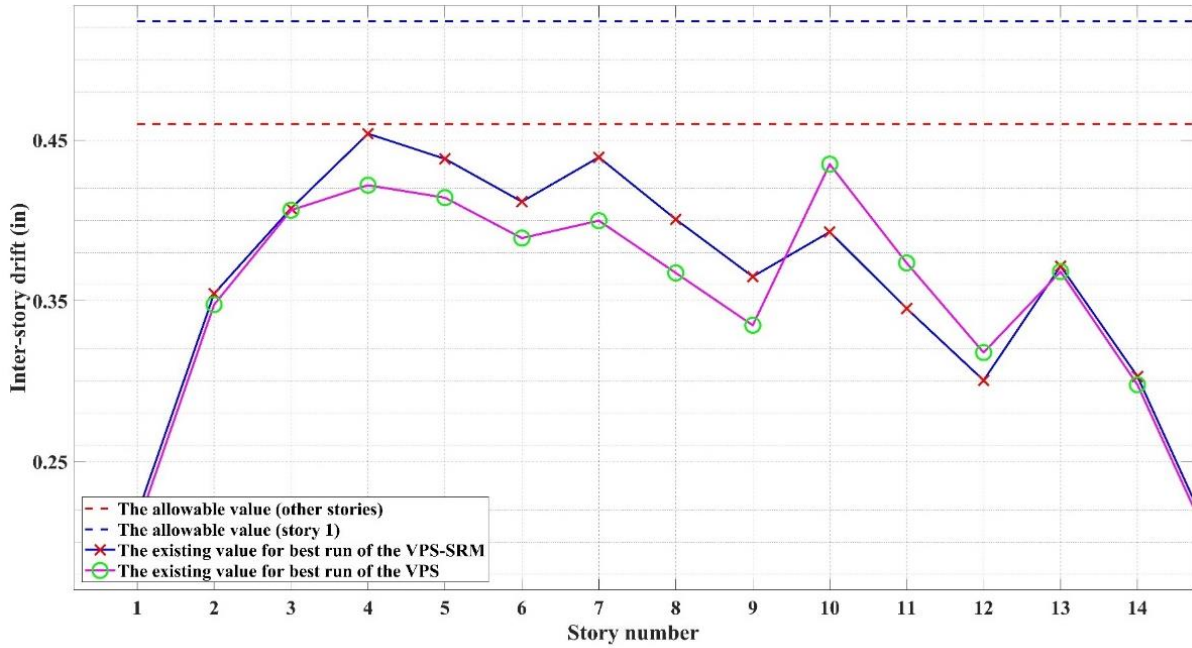


Figure 4. Inter-story drift values calculated at the obtained optimal design by the VPS-SRM and VPS for the 3-bay 15-story frame structure

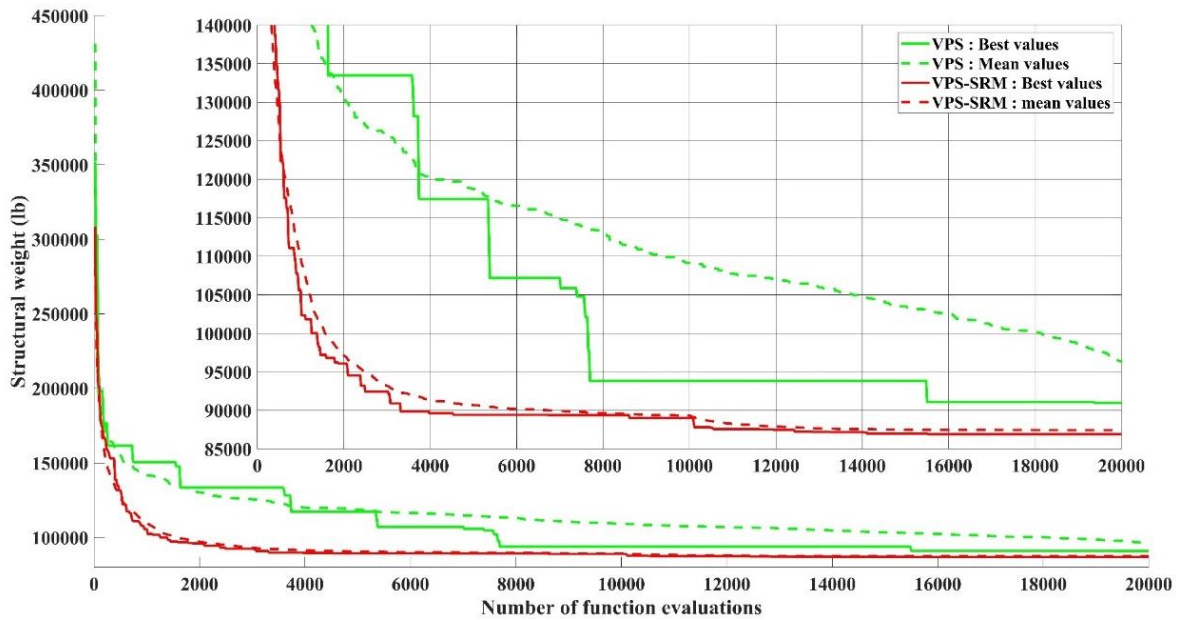


Figure 5. Convergence histories of the best and average of runs for the 3-bay 15-story frame structure

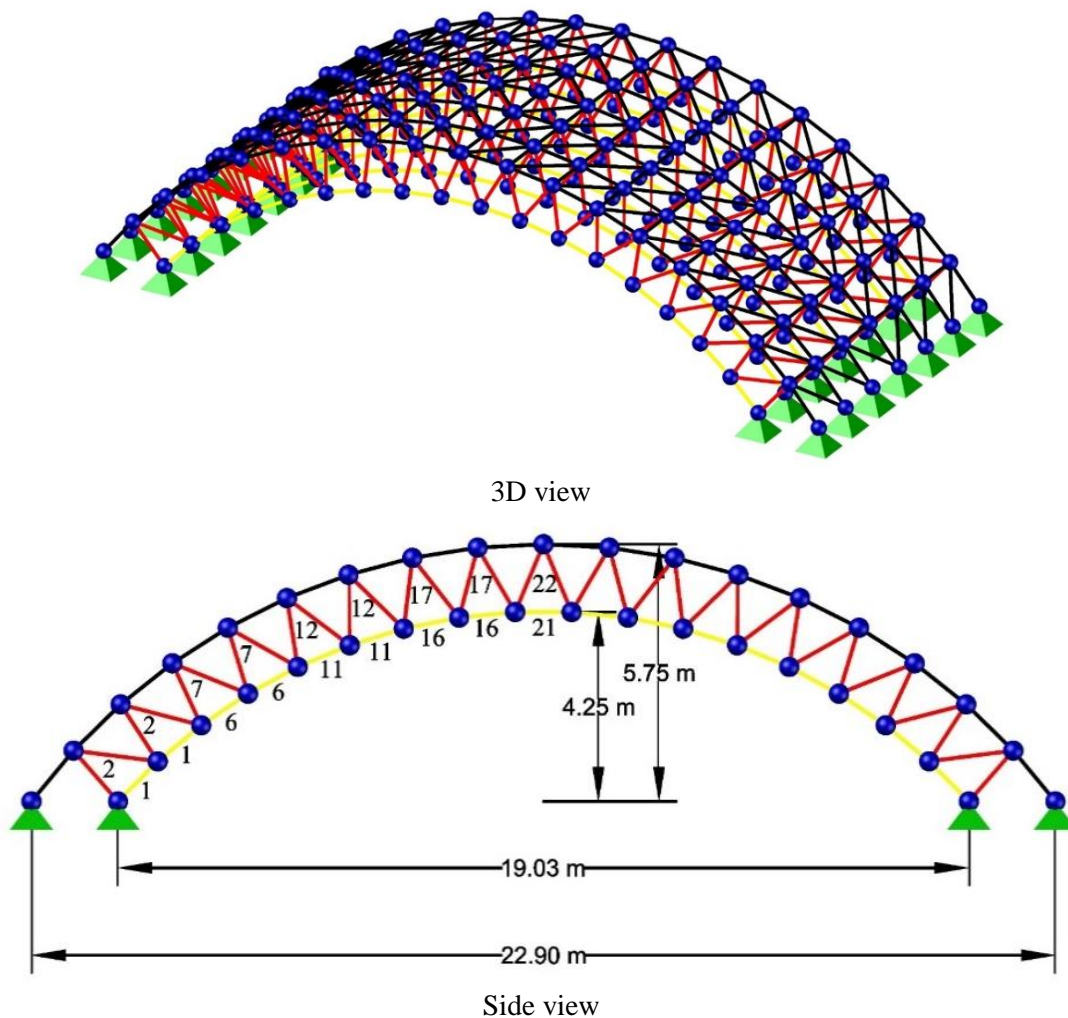
4.2 693-bar double-layer barrel vault

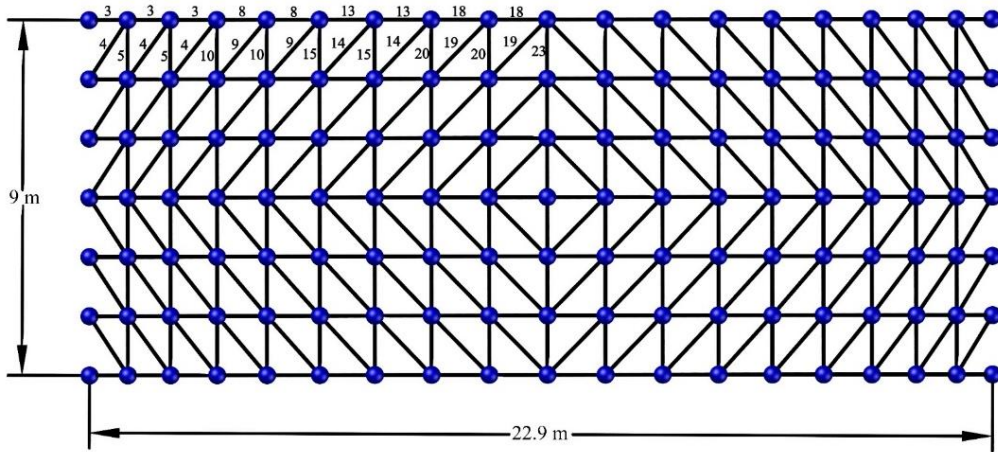
In the second example we investigate the performance of the VPS-SRM for the 693-bar



double-layer barrel vault. Due to structural symmetry, the structural members are divided into 23 groups, as shown in Fig. 6. There are two different load conditions applied to the top layer joints. The material density, modulus of elasticity, and yield stress of this steel structure are  $\rho = 0.283 \text{ lb/in}^3$ ,  $E = 29,000 \text{ ksi}$ , and  $F_y = 36 \text{ ksi}$ , respectively. The structural members are chosen from the pip section. Detailed information on this design example can be found in Refs [26, 28].

The optimization results obtained by the VPS-SRM, VPS, and other available methods is provided in Table 2. It can be seen from Table 2 that the VPS-SRM finds a better optimum weight (9,023.58 lb) than other existing methods, including MBB-BC [33] (10,595.33 lb), MCSS [34] (10,812.39 lb), IMCSS [34] (10,550.86 lb), ECBO [28] (9,240.5 lb), MDVC-UVPS [28] (9,091.1 lb), ESSOA [26] (9,053.4 lb), and VPS (9,066.28 lb). Also, the average and standard deviation of the 30 independent runs of the VPS-SRM is less than other methods. In all load cases, the members' stress ratio and displacement satisfy the constraints, as given in Figs. 7 and 8. The convergence histories of the VPS-SRM and VPS are shown in Fig. 9.





Top view

Figure 6. The 693-bar double-layer barrel vault

Table 2: Comparison of the results for the VPS-SRM and VPS algorithms with other methods in the 693-bar double-layer barrel vault

Element group	MBB-BC [33]	MCSS [34]	IMCSS [34]	ECBO [28]	MDVC-UVPS [28]	ESSEA [26]	Present study	
							VPS	VPS-SRM
1	EST 3½	EST 3	EST 3½	ST 4	ST 4	EST 3	EST 3	EST 3
2	ST 1	ST 1	ST 1	ST 1	ST 1	ST 1	ST 1	ST 1
3	ST ¾	EST ¾	EST 1	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾
4	ST 1	EST ½	ST ¾	ST 1	ST 1	ST 1	ST 1	ST 1
5	ST ¾	EST ½	ST 1	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾
6	EST 3½	EST 3	DEST 2	ST 3	ST 3½	EST 3	DEST 2	EST 3
7	ST 1	EST 1¼	ST 1	ST 1	ST 1	ST 1	ST 1	ST 1
8	ST 1	ST 1	ST 1¼	ST 1	ST 1	ST ¾	ST ¾	ST ¾
9	ST 1	ST ¾	EST ½	ST 1	ST 1	ST 1	ST 1	ST 1
10	ST ¾	EST ½	ST ½	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾
11	ST 3	EST 2½	ST 3	EST 2	EST 2½	EST 2½	ST 3	ST 3
12	ST 1½	EST 1½	EST 1¼	ST 1¼	ST 1	EST 1	EST 1	EST 1
13	EST 1½	ST 2½	EST 2	EST 2	ST 1½	EST 1	ST 1¼	ST 1¼
14	ST 1	ST ¾	ST ½	ST 1	ST 1	ST 1	ST 1	ST 1
15	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾
16	EST 1¼	ST 1¼	EST 1¼	ST 1	EST 1¼	EST 1½	ST 2	ST 2
17	ST 1¼	ST 1½	ST 1½	ST 1	ST 1	ST 1	EST 1	ST 1
18	ST 3	ST 3	ST 3	ST 3	EST 2	ST 2	EST 1½	ST 2
19	ST 1	EST ¾	ST ¾	ST 1	ST 1	ST 1	ST 1	ST 1
20	ST ¾	ST ½	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾	ST ¾
21	ST 1	ST 1¼	ST 1	ST ¾	ST 1	ST 1	ST 1	ST 1

22	ST $\frac{3}{4}$	EST $\frac{3}{4}$	EST 1	ST $\frac{3}{4}$	ST 1	ST 1	ST 1	ST $\frac{3}{4}$
23	ST $\frac{3}{4}$	ST $\frac{3}{4}$	EST $\frac{3}{4}$	ST $\frac{3}{4}$	ST $\frac{3}{4}$	ST $\frac{3}{4}$	ST $\frac{3}{4}$	ST $\frac{3}{4}$
Best (lb)	10,595.33	10,812.39	10,550.86	9,240.5	9,091.1	9,053.4	9,066.28	9,023.58
Mean (lb)	N/A	N/A	N/A	9,577	9,475	9,265.6	9,525.05	9,119.91
SD (lb)	N/A	N/A	N/A	505	765	111.5	209.22	77.45

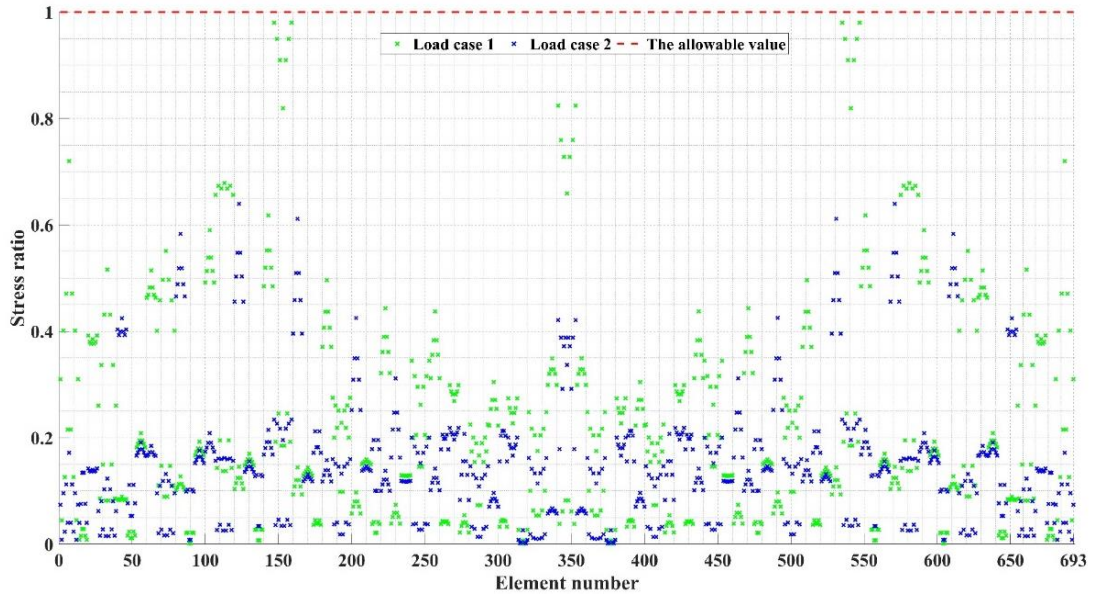


Figure 7. Stress ratios in two different loading conditions found by the VPS-SRM for the 693-bar double-layer barrel vault

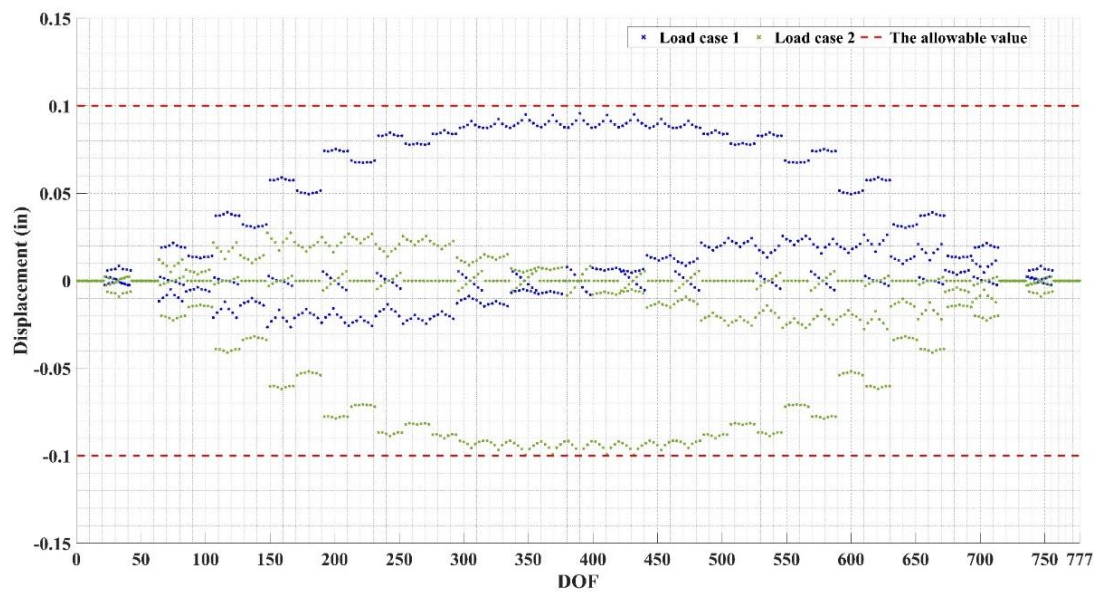


Figure 8. Displacement values in two different load conditions found by the VPS-SRM for the 693-bar double-layer barrel vault

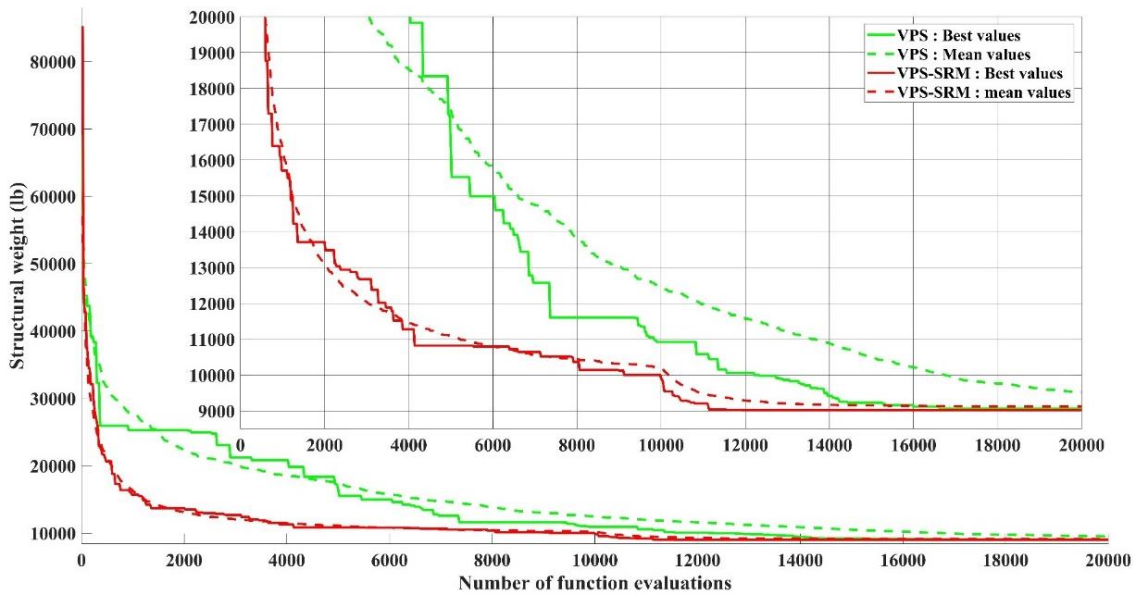
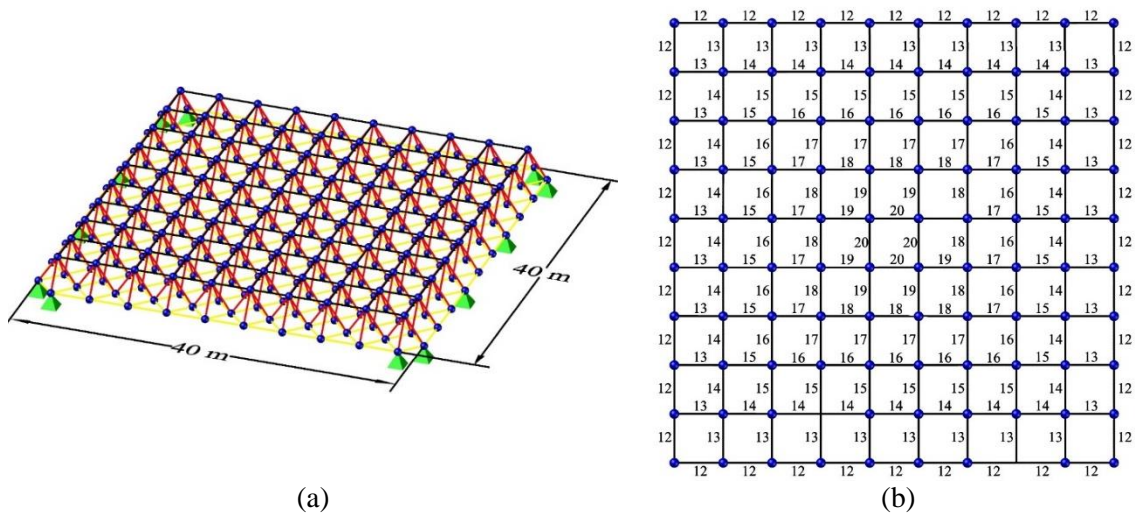


Figure 9. Convergence histories of the best and average of runs for the 693-bar double-layer barrel vault

4.3 The 1016-bar double layer grid

The last example is the 1016-bar double-layer grid, as given in Fig 10. The structural members are divided into 25 groups, and they are chosen from the pipe steel section same as in the previous example. The single load condition is applied to the top layer joint. The material density, modulus of elasticity, and yield stress of this steel structure are 7833.413 kg/m<sup>3</sup>, 205 GPa, and 248.2 MPa, respectively. This design example can be found in detail in Refs [26, 28].



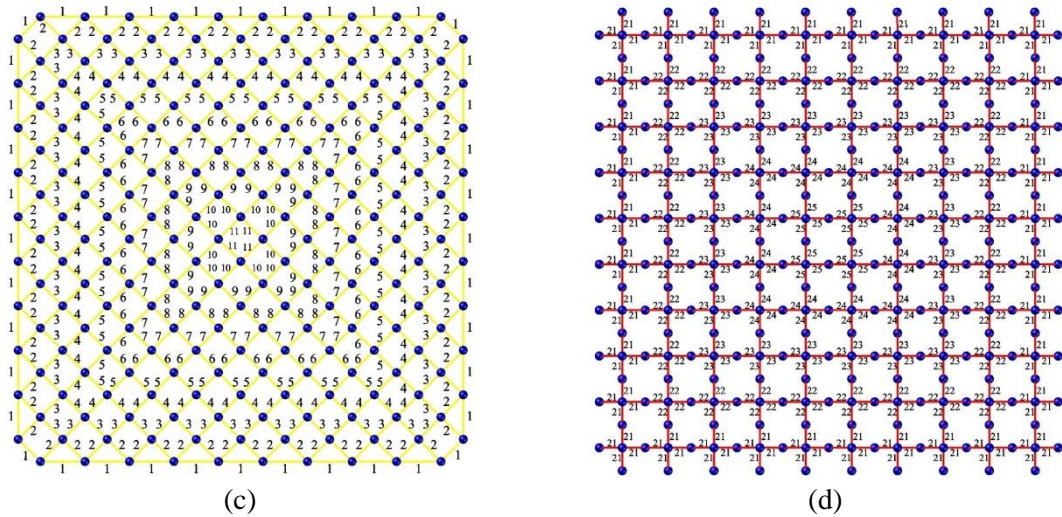


Figure 10. Schematic of the 1016-bar double layer grid; (a) 3D view, (b) top layer members, (c) bottom layer members, and (d) web members.

The result acquired by the VPS-SRM, VPS, and the other existing methods are compared in Table 3. According to this table, VPS-SRM acquired the lightest weight (63,464.17 kg) among all other methods, namely ECBO [28] with a weight of 67,839 kg, MDVC-UVPS [28] with a weight of 65,826 kg, ESSOA [26] with a weight of 67,079 kg, ERao-1 [35] with a weight of 64,971 kg, ERao-2 [35] with a weight of 64,597 kg, PRSSOA [36] with a weight of 67,407 kg, and VPS with a weight of 67,978.24 kg. Moreover, VPS-SRM obtained a better average weight than other considered methods. The stress ratio and displacement of the members in the best run of the VPS-SRM is provided in Figs. 11 and 12, respectively. According to these figures, the displacement constraint has the control roles of the optimization process, and its values in some of the members are very close to the limits. The convergence histories of the VPS-SRM and VPS are given in Fig. 13.

Table 3: Comparison of the results of the VPS-SRM and VPS algorithms with other methods in the 1016-bar double-layer grid

Element group	ECBO [28]	MDVC-UVPS [28]	ESSOA [26]	ERao-1 [35]	ERao-2 [35]	PRSSOA [36]	Present study	
							VPS	VPS-SRM
1	EST 5	DEST 4	ST 6	ST 6	ST 6	EST 5	EST 5	EST 5
2	EST 5	DEST 3	ST 5	EST 4	EST 4	EST 4	ST 5	ST 5
3	ST 3	ST 3½	EST 3	ST 3½	ST 3½	EST 3	EST 3	ST 3
4	ST 3 ½	ST 2½	EST 2 ½	ST 2½	ST 2½	ST 2 ½	ST 3	ST 2½
5	ST 2 ½	ST3	ST 3	EST 2½	ST 2½	ST 3	ST 2½	ST 2½
6	ST 2	EST 1½	EST 1 ½	EST 1½	EST 1	EST 1 ½	EST 1	ST 1½
7	DEST 2	EST 1½	EST 1 ½	EST 2	EST 2	EST 1	EST 1½	ST 1½
8	DEST 2	EST 2½	ST 2 ½	DEST 2	ST 3	ST 2 ½	EST 2½	ST 3
9	EST 2	ST 3½	EST 3	ST 3	DEST 2	EST 2	DEST 2	ST 3
10	ST 6	DEST 2	EST 2 ½	EST 3	EST 2½	ST 3 ½	EST 3½	DEST 2

11	ST 2	DEST 2½	EST 4	DEST 2	ST 2½	ST 4	ST 2	ST 3
12	EST 8	EST 8	ST 10	ST 12	ST 12	ST 10	DEST 5	DEST 6
13	EST 3 ½	EST 4	ST 4	ST 4	ST 4	ST 6	ST 4	ST 4
14	ST 5	ST 4	ST 5	ST 5	ST 5	ST 5	ST 6	ST 5
15	ST 4	ST 5	EST 4	ST 5	ST 5	ST 5	EST 4	ST 5
16	EST 5	ST 4	ST 6	ST 6	EST 5	ST 5	ST 6	EST 4
17	ST 5	ST 6	EST 4	ST 6	EST 4	ST 6	EST 4	ST 6
18	EST 5	ST 6	ST 5	EST 4	ST 6	EST 5	ST 5	DEST 4
19	EST 5	EST 6	EST 6	EST 5	EST 5	EST 5	ST 6	DEST 4
20	ST 8	EST 6	EST 6	EST 6	DEST 5	DEST 4	DEST 5	DEST 4
21	ST 5	ST 5	ST 6	ST 5	ST 5	ST 6	ST 6	ST 5
22	ST 3	ST 3½	ST 3 ½	ST 3	ST 3½	ST 3 ½	EST 3	ST 3
23	EST 2 ½	EST 2½	ST 3 ½	ST 3½	ST 3½	ST 3 ½	ST 4	ST 3
24	ST 5	ST 2½	EST 2 ½	ST 3½	ST 2½	ST 2 ½	ST 2½	ST 2½
25	ST 4	ST 2½	EST 1 ½	EST 2	EST 1½	EST 1 ½	EST 1½	EST 1½
Best (kg)	67,839	65,826	67,079	64,971	64,597	67,407	67,978.24	63,464.17
Mean (kg)	73,042	70,488	70,408	67,200	66,955	70,054	80,937.91	65,167.95
SD	9,158	5,018	2703	1,189	1,071	1,864	5,316.05	1,081.63

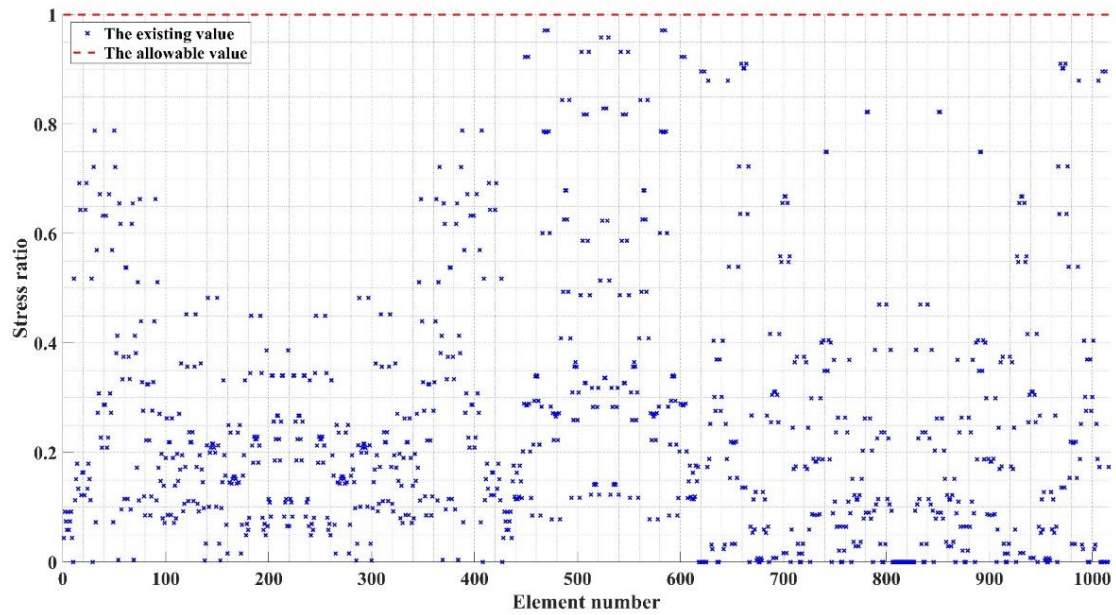


Figure 11. Stress ratios found by the VPS-SRM for the 1016-bar double-layer grid.

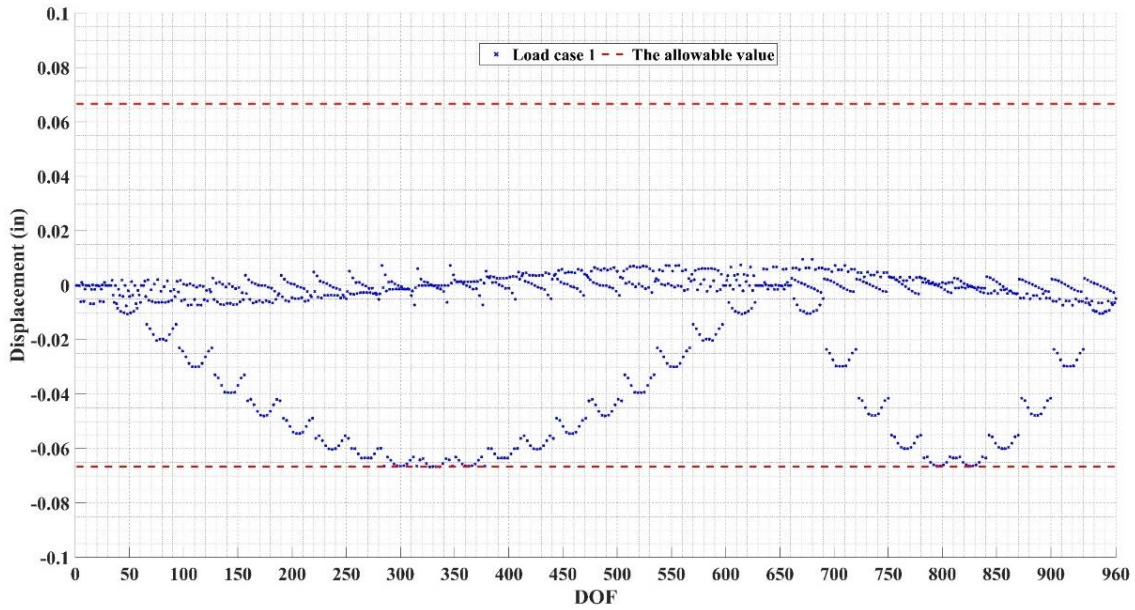


Figure 12. Displacement values found by the VPS-SRM for the 1016-bar double-layer grid.

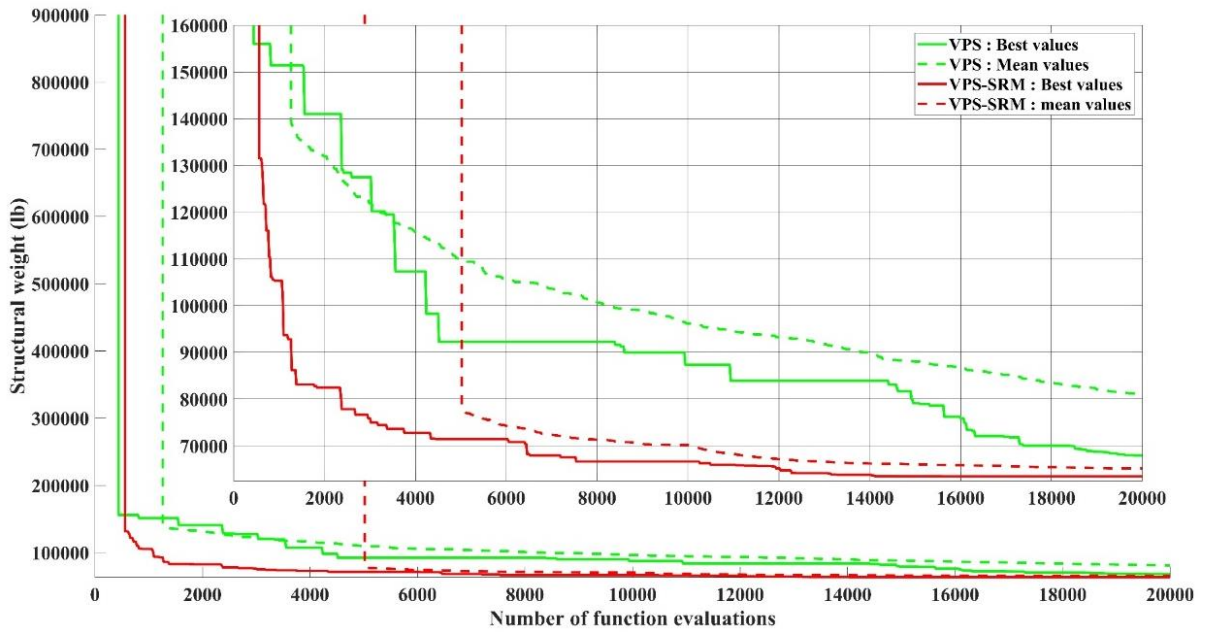


Figure 13. Convergence histories of the best and average of runs for the 1016-bar double-layer grid

### 5. CONCLUSIONS

This paper presented the new version of the VPS algorithm named VPS-statistical regeneration mechanism (VPS-SRM). In this method, SRM is utilized to improve the

performance of the VPS. SRM helps the algorithm explore the search space more efficiently in the first fifty percent of the iteration. In the remaining iterations, the exploitation of the algorithm is improved. Performance of the VPS-SRM is tested in the three benchmark examples consisting of the 3-bay 15-story steel frame, the 693-bar double-layer barrel vault, and the 1016-bar double-layer grid. VPS-SRM found the better optimum result than VPS and other enhanced algorithms are considered. Also, the statistical results obtained by the VPS-SRM are better than other methods considered in this paper. This shows that VPS-SRM is comparable to or better than many other optimization techniques.

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