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DESIGN OPTIMIZATION OF CABLE-STAYED BRIDGES USING MOMENTUM SEARCH ALGORITHM

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ABSTRACT

Design optimization of cable-stayed bridges is a challenging optimization problem because a large number of variables is usually involved in the optimization process. For these structures the design variables are cross-sectional areas of the cables. In this study, an efficient metaheuristic algorithm namely, momentum search algorithm (MSA) is used to optimize the design of cable-stayed bridges. The MSA is inspired by the Physics and its superiority over many metaheuristics has been demonstrated in tackling several standard benchmark test functions. In the current work, the performance of MSA is compared with that of two other metaheuristics and it is shown that the MSA is an efficient algorithm to tackle the optimization problem of cable-stayed bridges.

Keywords: cable-stayed bridge; structural optimization; metaheuristic algorithm; particle swarm optimization.

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1. INTRODUCTION

Due to their excellent stability, beautiful appearance, relatively low design and maintenance costs, cable-stayed bridge (CSB) structures have become very popular among structural engineers and designers in recent decades. The CSBs are one of the most economical options to build bridges with large spans. These bridges rely on durable steel cables as the main structural elements, which in terms of cable arrangement, the most common types of these bridges are fan cable bridge, semi-fan and harp bridges [1]. Due to the structural system of this type of bridges, their stiffness is higher than suspension bridges. The forces resulting from the weight of the deck and other loads are supplied and transmitted through the cables to the bridge pylons and these forces are transferred through the pylons to the bridge foundation and the pile group and from there to the bed. Therefore, the most

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important points in designing CSBs are the arrangement of the cables, the shape of pylons, how the forces are applied to the pylons and the control of the bridge deck [1-3]. Due to the expansion of the use of this type of bridges in transportation and life line and significant progress of design knowledge, the need for optimal design of these structures to improve their structural behaviour and reduce the construction and maintenance costs are a very important task [4].

In general, there are two types of optimization techniques: gradient-based methods and metaheuristics. Many of gradient-based methods have difficulties when dealing with complex and discrete optimization problems, and they usually converge to local optima. In order to overcome these difficulties, it is necessary to use global search algorithms such as metaheuristics. Metaheuristics are designated based on stochastic natural phenomena and they have attracted a great deal of attention during the last two decades. As the metaheuristic optimization techniques require no gradient computations, they are simple for computer implementation. During the recent years, researchers have designed many metaheuristic algorithms and many successful applications of them have been reported in optimization literature. The purpose of this study is to optimally design CSBs using momentum search algorithm (MSA) [5] as an an efficient metaheuristic algorithm. The MSA is designed based on the law of conservation of momentum. This algorithm includes a set of masses considering the conservation of momentum and kinetic energy of bodies in an *n*-dimensional space. At each iteration of MSA, all bodies are moved toward the optimal solution by colliding an external body to them. The superiority of the MSA over many metaheuristics including particle swarm optimization (PSO), genetic algorithm (GA), gravitational search algorithm (GSA), grey wolf optimizer (GWO), teaching–learning-based optimization (TLBO), grasshopper optimization algorithm (GOA), emperor penguin optimizer (EPO) and spotted hyena optimizer (SHO) has been demonstrated in tackling several standard benchmark test functions [5].

In the present work, the performance of MSA is compared with that of PSO [6], colliding bodies optimization (CBO) [7] and enhanced colliding bodies optimization (ECBO) [8] in solving the optimization problem of CSBs. The numerical results indicate that the obtained optimal design and convergence rate of the MSA is better than those of other algorithms used in literature for tackling the optimization problem of CSBs.

2. OPTIMIZATION OF CABLE-STAYED BRIDGES

Due to the high construction cost of CSBs, their optimization is of high importance and it is a challenging task. CSBs are large, complex and statically indeterminate structures that consist of three main components: deck, pylon and cables [1]. The general behaviour of such structures is influenced by the interaction of many design parameters such as the length of the main span, bridge height, number of cables and their arrangement, pylon shape and materials used in the bridge structure, load distribution and stiffness among structural components [2]. Every design effort has a high volume of calculations. Analyses should be performed considering the effects of geometric nonlinearities such as cable swelling due to its weight (sag effect), large deformations and P-delta [3]. The design must estimate all the performance condition and resistance specified in the design codes and regulations. All of

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these are repetitive, tedious, time consuming and expensive. The uncertainty of most cable-stayed bridges, the large number of design variables, the restrictions imposed by design codes, the geometric nonlinear behaviour, the enormous impact of cable retraction forces make optimization solutions difficult to obtain using traditional design methods. Previously, some algorithms have been proposed in existing papers to solve the optimization problem of CSBs. Simoes and Negrao [4] proposed an entropy-based optimization algorithm to optimize the cost of a cable-stayed bridge. The position of the cables and their connection to the deck and pylons, the cross-sectional characteristics of the deck, the pylons and the cables were considered as design variables. In their study, post tensioning forces were not considered. In addition, they assumed that the number of cables and the main span length were fixed and predetermined. Long et al. [9] used an algorithm to optimize the cost of composite deck (steel-concrete) cable-stayed bridges. In their study, the effect of cable post tensioning forces was not considered and only the dimensions of the sections of the bridge components were considered as design variables. Pylon height, main span length and number of traction cables were assumed as fixed parameters. Simoes and Negrao [10] used a convex scalar function to minimize the cost of a bridge equipped with a box deck. This function is a combination of the cross-sectional dimensions of different bridge components and the forces of traction cables. Chen et al. [11] reported a method for calculating the cables strength and showed that this method is very sensitive to the choice of constraints, which must be chosen very carefully to obtain a practical output. Lute et al. [12] examined the ability of a supportive vector machine which is a learning method for regression, to reduce the computation time of a genetic algorithm for optimizing CSBs. In their study, the number of traction cables is considered as a predetermined variable and also the impact of post tensioning forces has not been considered. Hassan [13] proposed the use of genetic algorithm and B-Spline technique to optimize the weight of cables and the post tensioning force in stretched cable bridge.

Figs. 1 to 3 show all the parameters describing a CSB. Dimensions (TL_1, TL_2, H_a) are considered as the main variables in the optimization process and other dimensions remain constant and are defined by the user which include the total length of the bridge (L), middle span (M), side spans (S) and the heights of the tower under the deck (H_b), which are determined acording to topographic and navigation conditions. The width of the deck also includes a fixed distance between obstacles because it is controlled according to traffic needs and the number of lanes. The main beam (I) has six dimensions B_{FT} ، B_{FB} ، t_{FT} ، t_{FB} ، t_w ، H_G which are considered as secondary variables in this study. Instead of optimizing the six main dimensions of the I-shaped steel girder, its moment of inertia is optimized to minimize its surface area. Other secondary variables are the pre-tensioning forces of the cables and their cross section, number and type of cable arrangement.

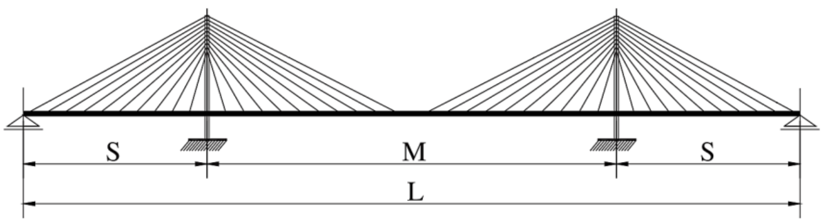

Figure 1. Geometry of cable-stayed bridge

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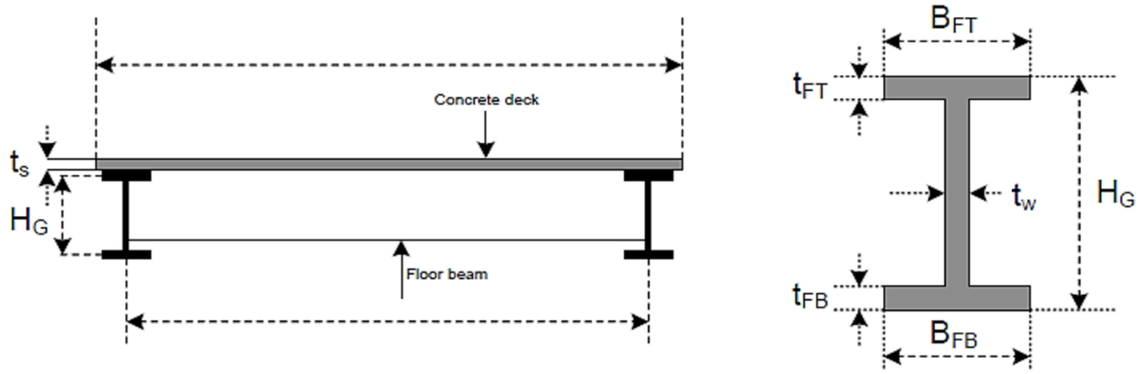
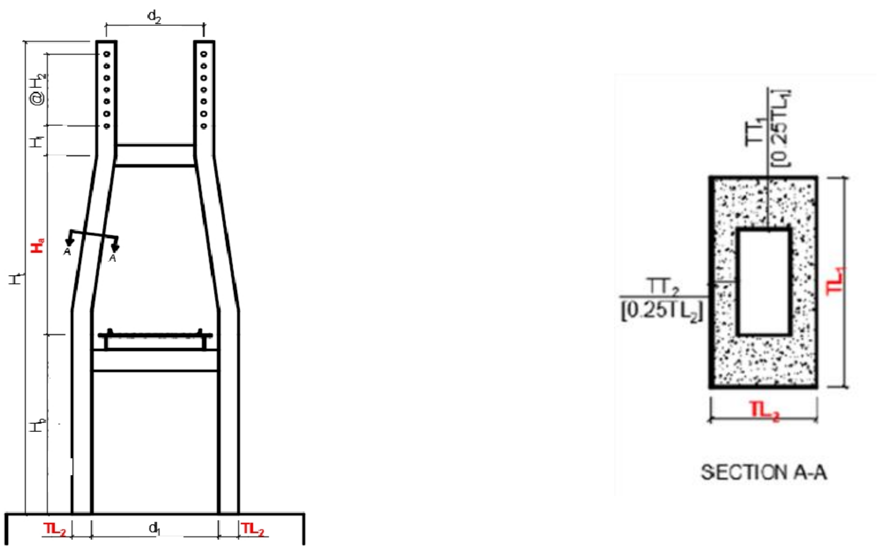


(a) (b)

Figure 2. (a) Cross-section of the bridge deck and (b) Steel main girder



(a) (b)

Figure 3. (a) dimensions of the pylon, (b) cross section of the pylon

In order to achieve small displacements and better flexural moment distribution in decks and towers, there are six main variables listed below:

(1) Total number of cables (2×4×N) so that N is the number of cables in the side village (or in half of the main span)

(2) Moment of inertia of the steel beam of the deck about its main axis

(3) The thickness of the concrete slab (t_s);

(4) Tower height above the deck surface (H_a);

(5) External dimension of the tower cross-section in the longitudinal direction (TL_1);

(6) External dimension of the tower cross-section in the transverse direction (TL_2);

If the concept of primary and secondary variables is not considered, the total number of design variables varies between 23 and 35 for the number of cables N=6 and N=12, and also when using bridge symmetry for larger spans, traction cables number and design variables

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become even more. Since these parameters are more dependent on the characteristics and conditions of the bridge, the number of variables can be reduced from 23 or 35, compared to the work of Hassan [13], to 6 or 4. In addition, the ratio of the side span (*S*) to the main span of the bridge (*M*), represented by β, varies between 0.40 and 0.60 in the real world CSBs, the total number of design variables will be 6. In this case, L is assumed to be constant and the ratio β will be drawn from interval [0.4, 0.6].

$$M = \frac{L}{2\beta + 1} \tag{1}$$

$$\beta = \frac{S}{M} \tag{2}$$

Three types of loads are applied to the CSB structure including [14]: dead loads of the total weight of the structure (6.5 kN/m^2), initial post tensioning force of the cables, and live loads (9 kN/m^2) according to the patters of Fig. 4.

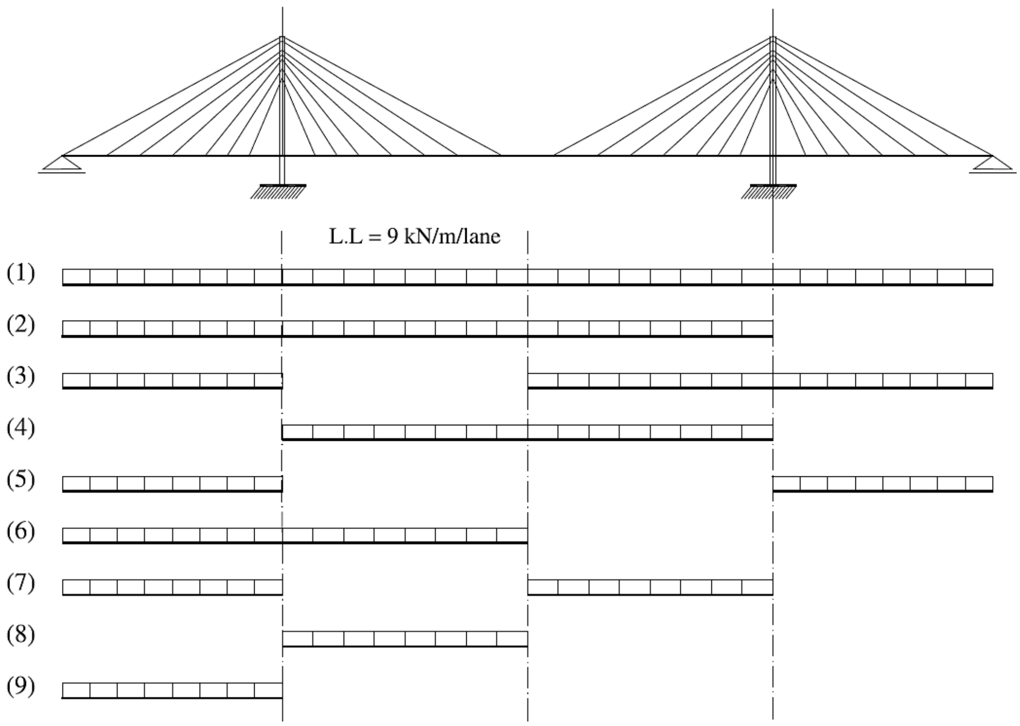

Figure 4. Live load cases

2.1 Arrangment of cables

There are three main arrangements for cables in CSBs namely, Harp, Fan and Semi-Fan as shown in Fig. 5. In the harp arrangement, the cables are made nearly parallel by attaching them to different points on the pylon. In the Fan pattern, all the stay cables are attached to a single point at top of each pylon. The relatively steep slope of the stay cables results in

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smaller cable cross section in comparison to the harp type. Moreover, the horizontal cable forces in the deck in this arrangement is less than the harp type [2]. In the Semi-Fan arrangement, the cables are distributed over the upper part of the pylon, which are more steeply inclined close to the pylon. Among the three types of cable bridge arrangement, the Harp arrangement has the most tensile cable forces in all its cables. Semi-fan arrangement has the minimum forces of traction cable and fan arrangement is between these two arrangements. Several modern cable-stayed bridges have been built around the world using semi-fan arrangement due to its efficiency [14-15]. In the current study, Semi-fan arrangement is used for arrangement of cables in CSBs.

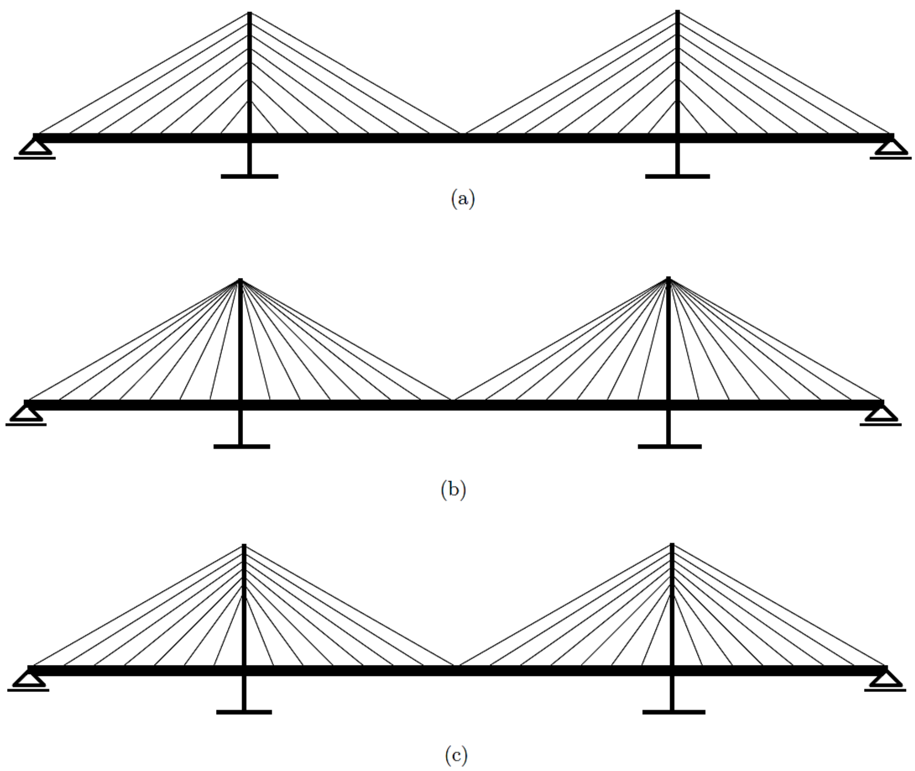

Figure 5. (a) Harp, (b) Fan, and (c) Semi-Fan arrangements of cables in cable-stayed bridges

2.1 Objective function and design constraints

In the present study, the objective function of the CSB optimization problem is the weight of steel of the stay cables expressed as follows:

$$W = \sum_{i=1}^{nc} \gamma_i L_i A_i \tag{3}$$

where γ_i, L_i, and A_i are the unit weight, length, and cross-sectional area of *i*th stay cable, respectively; and *nc* is the number of stay cables.

The design constraints for stay cable stress, vertical deflection of the deck, and horizontal deflection of the pylons are defined as follows:

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$$g_i^{\sigma} = \frac{\sigma_i}{\sigma_{all}} - 1 \leq 0 \tag{4}$$

$$g_i^{\delta} = \frac{\delta_i}{\delta_{all}} - 1 \leq 0 \tag{5}$$

$$g_j^{\Delta} = \frac{\Delta_j}{\Delta_{all}} - 1 \leq 0 \tag{6}$$

where σ_i and σ_{all} are the maximum tensile stress in *i*th stay cable for all the considered load cases and the allowable stress, respectively; δ_i and δ_{all} are the maximum vertical deflection of the deck at the cable position *i* for all the considered load cases and the allowable vertical deflection, respectively; Δ_j and Δ_{all} are the maximum horizontal deflection of the tops of the *j*th pylon in the longitudinal direction of the bridge deck for all the considered load cases and the allowable horizontal deflection, respectively.

2.2 Optimization problem formulation

The optimization problem of CBFs can be expressed in standard mathematical terms as follows:

$$\text{Minimize: } W \tag{7}$$

$$\text{Subject to:} \begin{cases} g_i^{\sigma} \leq 0 \text{ , } i = 1,2, \dots , N \\ g_i^{\delta} \leq 0 \text{ , } i = 1,2, \dots , N \\ g_j^{\Delta} \leq 0 \text{ , } j = 1,2, \dots , Np \end{cases} \tag{8}$$

where Np is the number of pylons.

In this study, to transform the above-mentioned constrained optimization problem of CSBs into an unconstrained one the exterior penalty function method (EPFM) is employed as follows:

$$\emptyset = W + r_p \sum_{k=1}^{nc} [max\{0 \quad g_k\}]^2 \tag{9}$$

where $\emptyset$, r_p and k are the pseudo objective function, positive penalty parameter and the number of design constraints, respectively.

3. METAHEURISTIC ALGORITHMS

Metaheuristics are applied to a very wide range of problems and they mimic natural metaphors to solve complex optimization problems. In the current study, PSO, CBO, ECBO

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and MSA metaheuristics are applied to solve the optimization problem of CSBs. Theoretical background of these algorithms are explained below.

3.1 Particle swarm optimization

Eberhart and Kennedy [6] proposed PSO to simulate the motion of bird swarms. The position of each particle is updated based on the social behavior of the swarm, which adapts to its environment by returning to promising regions of design space previously discovered and searching for better positions over time. Numerically, the position of the *i*th particle, X_i, at iteration t+1 is updated as follows:

$$X_i^{t+1} = X_i^t + V_i^{t+1} \tag{10}$$

$$V_i^{t+1} = \omega V_i^t + c_1 r_1 \left(P_i^t - X_i^t\right) + c_2 r_2 \left(G_{best}^t - X_i^t\right) \tag{11}$$

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{t_{\max}}.t \tag{12}$$

where V_i^t is the velocity vector at iteration t; r_1 and r_2 represents random numbers between 0 and 1; P_i^t represents the best ever particle position of particle i; G_{best}^t corresponds to the global best position in the swarm up to iteration t; c_1, and c_2 are social parameters; $\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum values of ω, respectively; and $t_{\max}$ is the number of maximum iterations.

3.2 Colliding bodies optimization

Colliding bodies optimization (CBO) is a meta-heuristic search algorithm that is developed by Kaveh and Mahdavi [7] based on the collision between objects. Collisions between bodies are governed by the laws of momentum and energy. When a collision occurs in an isolated system, the total momentum of the system of objects is conserved. Provided that there are no net external forces acting upon the objects, the momentum of all objects before the collision equals the momentum of all objects after the collision. In this optimization technique, one object collides with other object and they move towards a minimum energy level. The CBO is simple in concept and does not depend on any internal parameter. In CBO, each solution candidate is considered as a colliding body (CB). Each CB, has a mass defined as follows:

$$m_i(t) = \frac{1}{f_i(t)} \tag{13}$$

where $f_i(t)$ is the fitness value of *i*th body.

In order to select pairs of objects for collision, CBs are divided into two equal groups:

(a) Stationary group; $i_S = 1,2,\dots,\frac{n}{2}$ and (b) Moving group; $i_M = \frac{n}{2}+1, \frac{n}{2}+2, \dots, n$

The velocities of stationary and moving bodies before collision are evaluated as follows:

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$$V_{i_S} = 0 \tag{14}$$

$$V_{i_M} = X_{i_S} - X_{i_M} \tag{15}$$

The velocities of stationary and moving bodies after collision are evaluated as follows:

$$V'_{i_S} = \left(\frac{(1+\varepsilon)\, m_{i_M}}{m_{i_S} + m_{i_M}} \right) V_{i_M} \tag{16}$$

$$V'_{i_M} = \left(\frac{(m_{i_M} - \varepsilon\, m_{i_S})}{m_{i_S} + m_{i_M}} \right) V_{i_M} \tag{17}$$

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{18}$$

where $iter$ and $iter_{\max}$ are the current iteration number and the total number of iterations for optimization process, respectively; ε is the coefficient of restitution (COR).

The new position of each CB is calculated as follows:

$$X_{i_S}^{\text{new}} = X_{i_S} + \overline{R}_{i_S} . V'_{i_S} \tag{19}$$

$$X_{i_M}^{\text{new}} = X_{i_M} + \overline{R}_{i_M} . V'_{i_M} \tag{20}$$

where $\overline{R}_{i_S}$ and $\overline{R}_{i_M}$ are random vectors uniformly distributed in the range of [-1,1].

3.3 Enhanced colliding bodies optimization

Enhanced CBO (ECBO) has been proposed by Kaveh and Ilchi [8] to improve convergence rate and reliability of CBO by adding a memory to save some of the best solutions during the optimization process and also utilizing a mutation operator to decrease the probability of trapping into local optima. The basic steps of ECBO are summarized as follows:

1. The initial positions of all colliding bodies (CBs) are determined randomly in an m-dimensional search space using Eq. (21).

$$X_i^0 = X_{\min} + R.(X_{\max} - X_{\min})\, ,\, i = 1,2,...,n \tag{21}$$

in which X_i^0 is the initial solution vector of the ith CB. Here, $X_{\min}$ and $X_{\max}$ are respectively the lower and upper bounds of design variables; R is a random vector in the interval [0, 1]; n is the number of CBs.

2. The value of mass for each CB is evaluated using Eq. (13).

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3. Colliding memory (CM) is utilized to save a number of historically best CB vectors and their related mass and objective function values. Solution vectors which are saved in CM are added to the population and the same number of current worst CBs are deleted. Finally, CBs are sorted according to their masses in a decreasing order.
4. CBs are divided into two groups namely, Stationary and Moving groups.
5. The velocities of CBs before collision are evaluated using Eqs. (14) and (15).
6. The velocities of CBs after collision are evaluated using Eqs. (16) to (18).
7. The new position of each CB is calculated using Eqs. (19) and (20).
8. A parameter like *pro* within (0, 1) is introduced and it is specified whether a component of each CB must be changed or not. For each CB, *pro* is compared with rn_i (i=1,…,n) which is a random number uniformly distributed within (0, 1). If $rn_i < pro$, one dimension of the ith CB is selected randomly and its value is regenerated in interval [$X_{\min}$, $X_{\max}$]. In order to protect the structures of CBs, only one dimension is changed. In the framework of ECBO, the value of *pro* is considered to be 0.3.

When a stopping criterion is satisfied, the optimization process is terminated and the best design found is considered as the optimal solution.

3.4 Momentum search algorithm

Momentum search algorithm (MSA), is based on two important physics' laws: momentum conservation law and kinetic energy conservation law [16]. The MSA includes a number of bodies called solution bodies in a closed system considering the conservation of momentum and kinetic energy of masses. At every iteration of this algorithm, the position of bodies represents the possible solutions. As the mass of the bodies reflects their fitness value during the optimization process, heavier bodies are associated with better solutions. At each iteration, an external body collides separately with all solution bodies and moves them in a random direction which is not in reverse with the direction toward the iteration's best solution. The better solutions are moving slower than the worse solutions which are lighter. The mass and the speed of the external body are reduced during each iteration. In this way, the algorithm preserves two important concepts in the heuristic algorithms: exploration and exploitation [5]. The MSA has three main steps. The first step includes forming an artificial time-discrete and closed system and positioning for all bodies considering constraints of the problem. The second step includes implementing motion and conservation laws. The third step includes letting time to pass in discrete steps until a stopping criterion is met. The flowchart and pseudocode of MSA have been provided in [5]. The mentioned steps are briefly explained in the following..

In the first step, an artificial time-discrete and closed system is considered to form a specified space for placing a limited number of bodies. This space includes an n-dimensional coordinate system in which each point can be a solution of the problem. If there are m motionless solution bodies in the initial population, the position of ith body in time t is shown by $X_i(t)$ and its dth components in a n-dimensional design space is shown by $x_i^{(d)}(t)$. At iteration t, the mass of ith body is defined as follows [5]:

$$m_i(t) = \frac{f_i(t) - w_i(t)}{b_i(t) - w_i(t)} \tag{22}$$

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where $f_i(t)$ is the fitness value of *i*th body; $w_i(t)$ and $b_i(t)$ are the maximum and minimum values of fitness at iteration *t*.

At every iteration, all solution bodies are motionless, and there is a separate external body in the space called external body. Such body collides with all other bodies and changes their positions toward better ones. Since we know by the passage of time, all bodies approach to the sub-optimum point, it is essential for bodies to search the space with smaller and more accurate steps. In order to achieve this task, the mass and velocity of external body decrease in time with the maximum mass of unity for external body. The mass of the external body at iteration *t* is calculated as follows [5]:

$$M(t) = 1 - \frac{t-1}{T-1} \tag{23}$$

where T is the maximum number of iterations.

The speed of the external body should also get decreased over time. The speed of the external body which collides with *i*th body at iteration *t* is as follows [5]:

$$U_i(t) = r_1 . M(t) . U_{max} . sign\big(X_b(t) - X_i(t)\big) \tag{24}$$

where r_1 is a random number drawn from a uniform distribution between 0 and 1; U_{max} is the maximum speed of external body; $X_b(t)$ is the best solution found up to iteration *t*.

After collision with *i*th body at time *t*, by implementing the momentum and kinetic energy conservation laws, the speed of bodies is determined as follows [5]:

$$V_i(t) = \frac{2M(t)}{m_i(t) + M(t)} . U_i(t) \tag{25}$$

The new position of *i*th body at time *t* is determined as follows [5] in which where r_2 is a random number drawn from a uniform distribution between 0 and 1.

$$X_i(t+1) = X_i(t) + r_2 . V_i(t) \tag{26}$$

4. NUMERICAL RESULTS

In this study, the geometry of the chosen bridge is similar to the Quincy Bayview Bridge, located in Illinois, USA [17] as shown in Fig. 1. In this case M=285.6 m, S=128.1 m and H=87 m. Cross-section of the bridge deck, steel main girder and dimensions of the pylons are shown in Figs 2 and 3, respectively and the values of variables related to them are taken same as those in [13]. By the way, the cross-sectional area of forty cables shown in Fig. 1 are the design variables of the CSB optimization problem. In the structural model of CSBs,

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the cables are defined as truss elements. The flexural rigidity of the cables is ignored and the equivalent cable's modulus of elasticity used to account for the sag effect is as follows [13]:

$$E_{eq} = \frac{E}{1 + \frac{(w.l)^2.A.E}{12F^3}} \tag{27}$$

where E_{eq} is the equivalent modulus of elasticity; E is the cable elastic modulus; w is the weight per unit length of the cable; l is horizontal projected length of a cable; A is the cross-sectional area of the cable; and F is the tension force of the cable. In this work, the unit weight of stay cables is 77 kN/m^3. According to [15,17,18], the allowable tensile stress in cables, the allowable vertical deflection of the deck, and the allowable horizontal deflection of the pylons are considered as 1600 MPa, M/550 = 0.5192 m, and H/550, respectively during the optimization process. The finite element model for structural analysis of CSBs is validated using the optimal design reported in [13].

For PSO, CBO, ECBO and MSA metaheuristics, the population size and maximum number of optimization iterations are considered as 50 and 200, respectively. In the current work, 20 independent optimization runs are performed and the results obtained by PSO, CBO, ECBO and MSA are compared with that of reported in [13] in Table 1. In addition, the convergence curves of PSO, CBO, ECBO and MSA for their best designs are compared in Fig. 6.

The numerical results indicate that the optimal weight of the best design found by MSA is 30.82%, 3.41%, 1.99% and 0.61% lighter than those found by GA[13], PSO, CBO and ECBO algorithms, respectively. Furthermore, the comparison of the convergence curves in Fig. 6 shows that the convergence rate of MSA is better than that of other metaheuristic algorithms. Therefore, it can be concluded that the MSA outperforms the other algorithms in terms of best weight, mean weight, worst weight, standard deviation (std) and convergence rate over 20 independent optimization runs.

The tensile stress of cables for the best optimal design found by the MSA for the nine live load cases is shown in Fig. 7 and it can be seen that all the stresses are less than the allowable value of 1600 MPa. The vertical deflections of the deck of the best optimal design obtained by the MSA resulting from the nine live load cases are plotted in Fig. 8. The allowable vertical deflection of the deck is equal to 0.5192 m while the maximum vertical deflections of the deck of the best optimal design found by the MSA is 0.5024 m. The horizontal deflections of the pylons of the best optimal design obtained by the MSA are plotted in Fig. 9. The allowable horizontal deflection of the pylons is 0.1582 m while the maximum horizontal deflection of the pylons of the best optimal design found by the MSA is 0.1562 m. These results demonstrate that the tensile stress of the cables constraints, the vertical deflection of the deck constraint and the horizontal deflection of the pylons constraints are satisfied for the best optimal design obtained by the MSA metaheuristic. Moreover, the optimal designs found by PSO, CBO and ECBO metaheuristic algorithms are also feasible.

Table 1: Results of CSB optimization

Cable Number		Cable Cross-Sectional Area (m^2)				
		GA [13]	PSO	CBO	ECBO	MSA
1		0.0043	0.0023	0.0026	0.0023	0.0025
2		0.0041	0.0023	0.0022	0.0022	0.0022
3		0.0038	0.0017	0.0018	0.0018	0.0017
4		0.0034	0.0016	0.0015	0.0015	0.0014
5		0.0030	0.0015	0.0014	0.0014	0.0013
6		0.0026	0.0010	0.0012	0.0013	0.0011
7		0.0022	0.0015	0.0013	0.0014	0.0014
8		0.0018	0.0014	0.0015	0.0014	0.0013
9		0.0014	0.0014	0.0015	0.0015	0.0014
10		0.0009	0.0014	0.0013	0.0011	0.0014
11		0.0007	0.0011	0.0012	0.0012	0.0010
12		0.0009	0.0010	0.0012	0.0013	0.0011
13		0.0001	0.0011	0.0012	0.0012	0.0011
14		0.0013	0.0013	0.0011	0.0012	0.0010
15		0.0015	0.0010	0.0011	0.0011	0.0011
16		0.0016	0.0016	0.0011	0.0011	0.0010
17		0.0018	0.0018	0.0013	0.0013	0.0014
18		0.0019	0.0016	0.0018	0.0018	0.0017
19		0.0019	0.0017	0.0018	0.0018	0.0017
20		0.0020	0.0019	0.0018	0.0017	0.0019
21		0.0020	0.0019	0.0018	0.0017	0.0019
22		0.0019	0.0017	0.0018	0.0018	0.0017
23		0.0019	0.0016	0.0018	0.0018	0.0017
24		0.0018	0.0018	0.0013	0.0013	0.0014
25		0.0016	0.0016	0.0011	0.0011	0.0010
26		0.0015	0.0010	0.0011	0.0011	0.0011
27		0.0013	0.0013	0.0011	0.0012	0.0010
28		0.0001	0.0011	0.0012	0.0012	0.0011
29		0.0009	0.0010	0.0012	0.0013	0.0011
30		0.0007	0.0011	0.0012	0.0012	0.0010
31		0.0009	0.0014	0.0013	0.0011	0.0014
32		0.0014	0.0014	0.0015	0.0015	0.0014
33		0.0018	0.0014	0.0015	0.0014	0.0013
34		0.0022	0.0015	0.0013	0.0014	0.0014
35		0.0026	0.0010	0.0012	0.0013	0.0011
36		0.0030	0.0015	0.0014	0.0014	0.0013
37		0.0034	0.0016	0.0015	0.0015	0.0014
38		0.0038	0.0017	0.0018	0.0018	0.0017
39		0.0041	0.0023	0.0022	0.0022	0.0022
40		0.0043	0.0023	0.0026	0.0023	0.0025
Structural analyses		10000	10000	10000	10000	10000
Weight (KN)	Best	1339.2	959.24	945.29	932.15	926.52
	Mean	-	997.70	993.71	984.52	972.77
	Worst	-	1123.6	1061.5	1052.0	1021.0
	std	-	38.72	35.62	30.44	25.63

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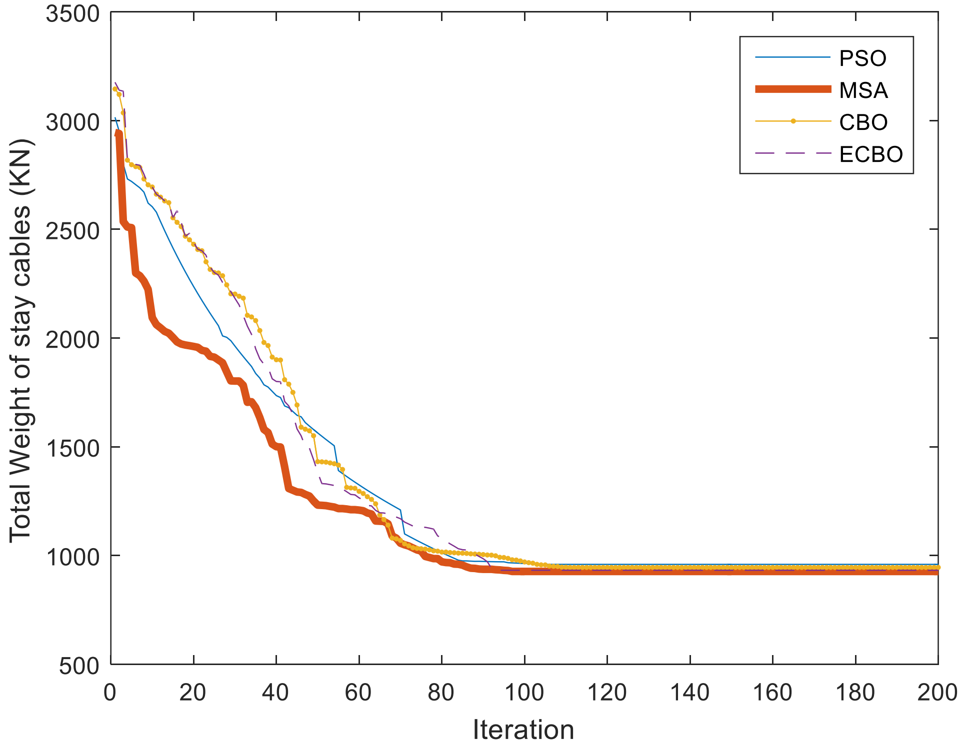

Figure 6. Convergence curves of PSO, CBO, ECBO and MSA for their best designs

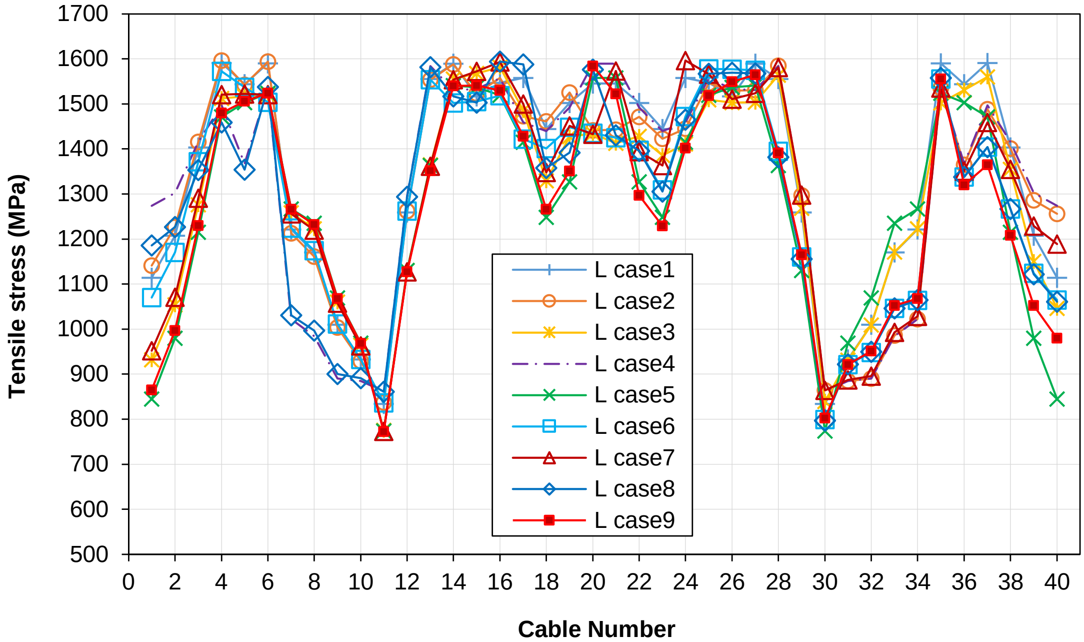

Figure 7. Tensile stress of the cables for the best optimal design found by MSA

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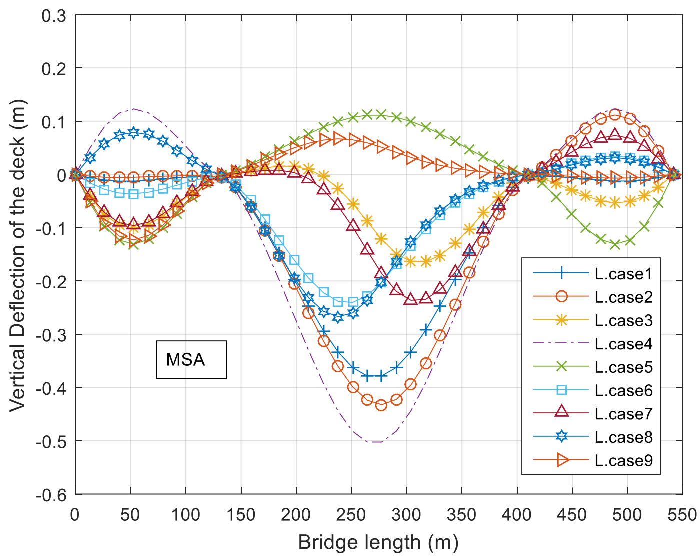

Figure 8. Vertical deflection of the deck for the best optimal design found by MSA

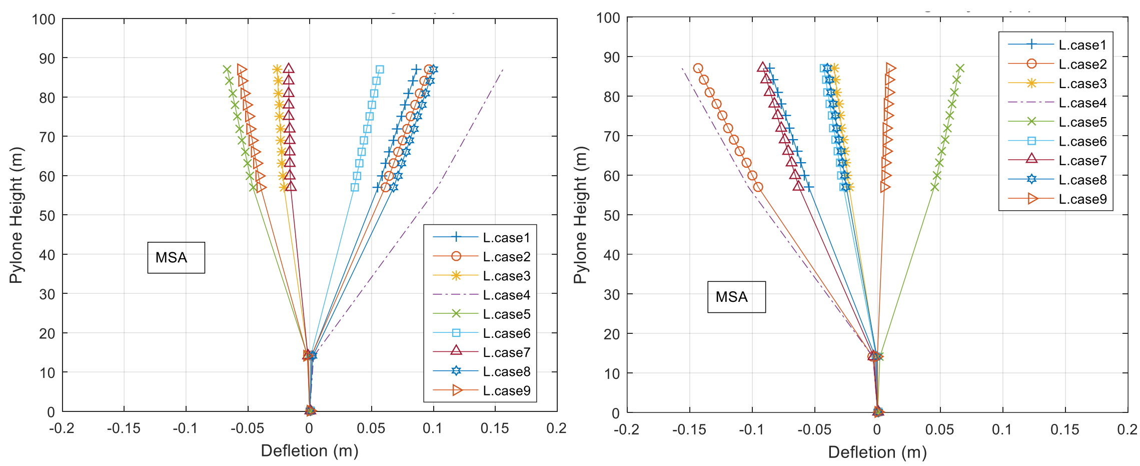

Figure 9. Horizontal deflections of the pylons for the best optimal design found by MSA

5. CONCLUSIONS

The current study is devoted to address a challenging optimization problem of cable-stayed bridges. The cross-sectional areas of the cables are treated as the design variables of the optimization problem. The design constraints include the stay cables stress, the vertical deflection of the deck, and the horizontal deflection of the pylons. Because there is a large number of design variables, an efficient optimization algorithm should be used to deal with this optimization problem. An efficient metaheuristic algorithm namely, momentum search algorithm (MSA) is used to solve the optimization problem of cable-stayed bridges. The MSA metaheuristic is based on two important physics' laws: momentum conservation law and kinetic energy conservation law. The performance of MSA is compared with that of four well-known optimization algorithms including GA, PSO, CBO and ECBO. The optimal

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weight of the cables found by MSA is 30.82%, 3.41%, 1.99% and 0.61% lighter compared to GA, PSO, CBO and ECBO, respectively. The obtained numerical results demonstrate that best weight, mean weight, worst weight, standard deviation (std) and convergence rate of the MSA are better than those of GA, PSO, CBO and ECBO.

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