



RELIABILITY ANALYSIS OF OPTIMALLY DESIGNED DOUBLE LAYER BARREL VAULTS

M. H. Seyyed Jafari and S. Gholizadeh^{*,†}
Department of Civil Engineering, Urmia University, Urmia, Iran

ABSTRACT

The present work deals with optimization and reliability assessment of double layer barrel vaults. In order to achieve the optimization task an improved colliding bodies optimization algorithm is employed. In the first phase of this study, different forms of double layer barrel vaults namely, square-on-square, square-on-diagonal, diagonal-on-diagonal and diagonal-on-square are considered and designed for optimal weight by the improved colliding bodies optimization algorithm. In the second phase, in order to account for the existing uncertainties in action and resistance of the structures, the reliability of the optimally designed double layer barrel vaults is assessed using importance sampling method by taking into account a limit-state function on the maximum deflection of the structures. The results demonstrate that the minimum reliability index of the optimal designs is 0.92 which means that all the optimally designed double layer barrel vaults are reliable and safe against uncertainties.

Keywords: Structural optimization; Reliability; Double layer barrel vault; Metaheuristic

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1. INTRODUCTION

The primary objective of this study is to evaluate the reliability of optimally designed double layer barrel vaults with different forms considering uncertainties in structural capacity and demands. The designer must verify the serviceability and ultimate conditions dominated by the action and resistance related parameters. The intrinsic random nature of material properties and actions must be actually considered in the design process of structures and the probability of failure must be computed from the joint probability distribution of the random variables associated with the action and resistance. Theory and methods for structural reliability are actually useful tools for evaluating the safety of complex structures. Recent

^{*}Corresponding author: Department of Civil Engineering, Urmia University, Urmia, P.O. box 165, Iran

[†]E-mail address: s.gholizadeh@urmia.ac.ir (S. Gholizadeh)

developments allow anticipating that their application will gradually increase, even in the case of common structures [1]. Monte Carlo Simulation (MCS) is a simulation method for reliability analysis. The main concept of simulation techniques is to simulate a probabilistic phenomenon numerically and then observe the frequency of a certain event in that phenomenon [2]. The implementation of these simulation techniques is easy, but in the case of small failure probabilities, the required number of simulations is prohibitively high and drastically increases the computational cost of these simulation techniques. So, MCS method can be applied to many practical problems allowing the direct consideration of any type of probability distribution for the random variables. It is able to compute the probability of failure with the desired precision and it is easy to implement. However, its computational burden is high as MCS requires a great number of structural analyses [3]. During the recent years a few studies have been conducted on the reliability analysis of different kinds of space structures. Gordini et al [4] investigated the effects of initial member length and its imperfection on the load-bearing capacity of double-layer space domes. Hadidi et al [5] used the response surface method to evaluate the reliability of structures. In his research, the efficiency of the proposed method has been increased by using the exponential alternative model instead of the quadratic function. Tahammoli and Gordini [6] investigated the effect of initial curvature coefficients on the load-bearing capacity of double layer grids with different types of supports. Biabani and Kalatjari [7] proposed a computational framework for evaluating the system reliability of truss structures and simultaneously optimizing size and geometry of the structures under reliability constraints.

Optimization of structures is an integral part of the design and construction process of structures. The major concern in this field is designing the cheapest possible structures with the minimum amount of used material. To this end, modern optimization methods can be effectively employed in the field of structural engineering. In recent years, metaheuristic algorithms have been widely used in the field of civil engineering to solve large-scaled and complex optimization problems. Regarding the optimization of space structures, one can refer to the works done by Gholizadeh et al [8], Kaveh and Rezaei [9], Kamyab and Selajgeh [10], and Saka and Kameshki [11]. One of the efficient metaheuristics developed by Kaveh and Mahdavi [12] is colliding bodies optimization (CBO) algorithm which is designed based on the governing laws of one dimensional collision between two bodies from the physics. An enhanced CBO (ECBO) algorithm was proposed by Kaveh and Ilchi Ghazaan [13] to improve performance and convergence rate of original CBO algorithm. Since the number of metaheuristic algorithms is very large and each of them is suitable for solving a specific class of optimization problems, so in this research, an improved colliding bodies optimization algorithm (ICBO) [8] is used to efficiently tackle the optimization problem of double layer barrel vaults.

In the first phase of the proposed methodology of the present work, double layer barrel vaults having different forms including square-on-square, square-on-diagonal, diagonal-on-diagonal and diagonal-on-square are designed for optimal weight by ICBO metaheuristic algorithm taking into account different load cases. In the second phase, the reliability of the optimally designed double layer barrel vaults is assessed using MCS method by taking into account different limit-state functions. The obtained numerical results demonstrate that the minimum reliability index of the optimally designed double layer barrel vaults is 0.92 which means that all the optimally designed double layer barrel vaults are reliable and safe against uncertainties.

2. DOUBLE LAYER BARREL VAULTS

In the case of structures with many members, one of the most important issues that should be addressed prior to optimization is to find the best form. The vertical section of a typical double layer barrel vault is depicted in Fig. 1 defining its height (H), span (S), and layer thickness (h). As regard S is usually predefined, one of the most important issues in the form finding process is to find the best values of H and h . In the present study, four basic forms including square-on-square (SS), square-on-diagonal (SD), diagonal-on-diagonal (DD) and diagonal-on-square (DS), with $S=42.0$ m and length (L) of 60 m, shown in Fig. 2, are considered.

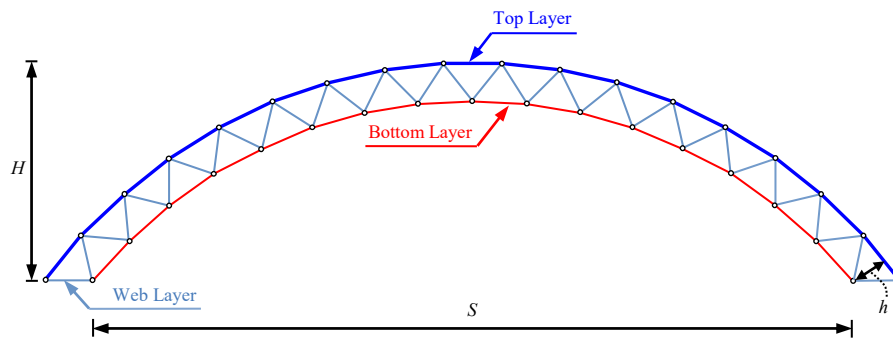


Figure 1. The vertical section of a typical double layer barrel vault

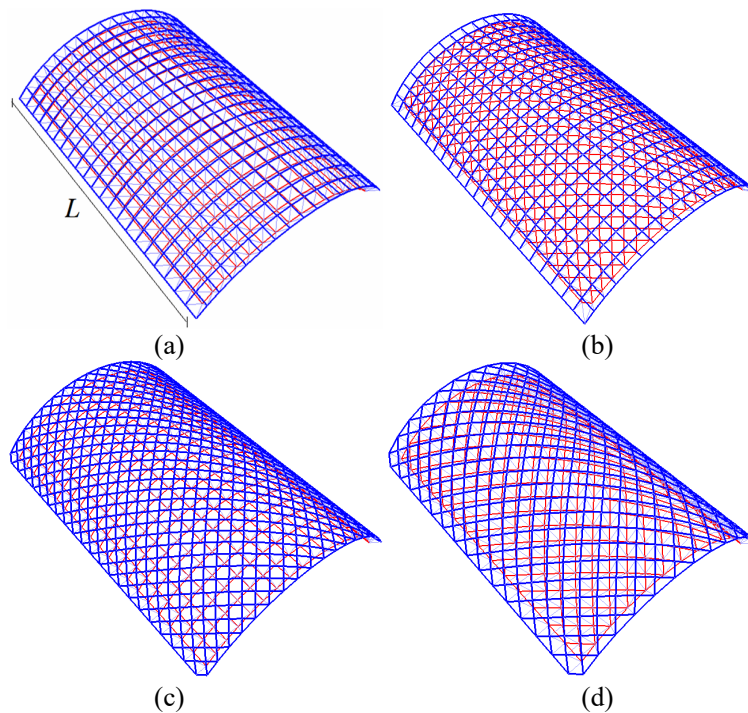


Figure 2. Double layer barrel vaults with (a) SS, (b) SD, (c) DS, and (d) DD forms

In the design process of these structures, dead, symmetrical and non-symmetrical snow, earthquake and wind loads are considered according to Standard No. 400 [14], Standard No. 2800 [15], Eurocode 1, Part 1.3 [16] and Part 1.4 [17]. For optimal design of double layer barrel vault considering nonlinear behavior service load combinations are employed. The design dead load is determined on the basis of the actual loads that may be expected to act on the structure of constant magnitude. In this study, a uniform dead load of 70 kg/m² is considered for estimated weight of sheeting, structural members, and nodes of barrel vault. The snow load for arched roofs is calculated according to Eurocode 1, Part 1.3 [16]. Snow loads acting on a sloping surface is assumed to act on the horizontal projection of that surface and can be computed as follows:

$$S = C_e C_t S_k \mu_i \tag{1}$$

where C_e , C_t , S_k and μ_i are exposure coefficient, thermal coefficient, flat roof snow load and shape coefficient, respectively. In this study, $C_e = 1.0$, $C_t = 1.0$ and $S_k = 1.5$ kN/m². The shape coefficient μ_i is computed for symmetrical and non-symmetrical snow load based on the values of α and δ , shown in Fig. 3, as follows:

$$\begin{cases} \mu_0 = 0.8 \\ \mu_1 = 0.4 \\ \mu_2 = 0.8 + 0.4[(\alpha - 15)/15] \\ \mu_3 = \mu_2(60 - \delta)/30 \end{cases} \tag{2}$$

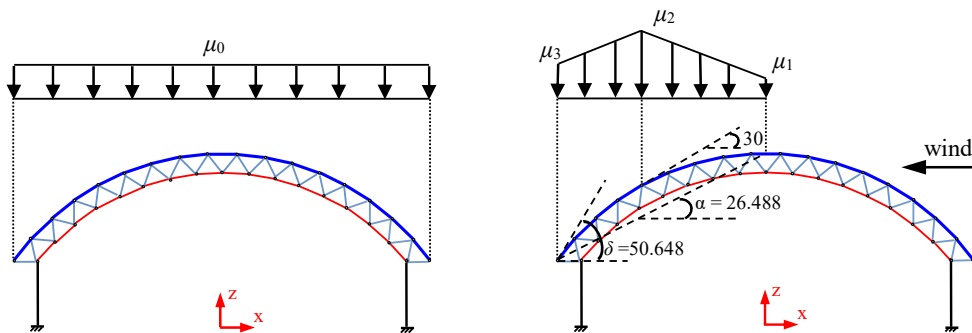


Figure 3. Distribution of symmetrical and non-symmetrical snow loads

In the case of wind load in arched roofs, different loads are applied in the windward quarter, center half and leeward quarter of the roof as depicted in Fig. 4. Wind induced loads are computed according to Eurocode 1, Part 1.4 [17] as follows:

$$W = C_e q_b C_{pe} \tag{3}$$

where C_e and q_b are exposure coefficient and basic wind pressure, respectively and C_{pe} is external pressure coefficient which determines the distribution of wind load.

In the present work, $C_e = 2.0$, $q_b = 0.6 \text{ kN/m}^2$. Based on Fig. 4, values of C_{pe} in zones A, B, C, D, E, F, G, I and J are -0.45, 0.55, 0.4, -0.75, 0.5, 1.1, 1.2, 0.8, and 0.5, respectively. The value of e in Fig. 4 is determined as follows:

$$e = \min(S, 2(H + H_c)) \tag{4}$$

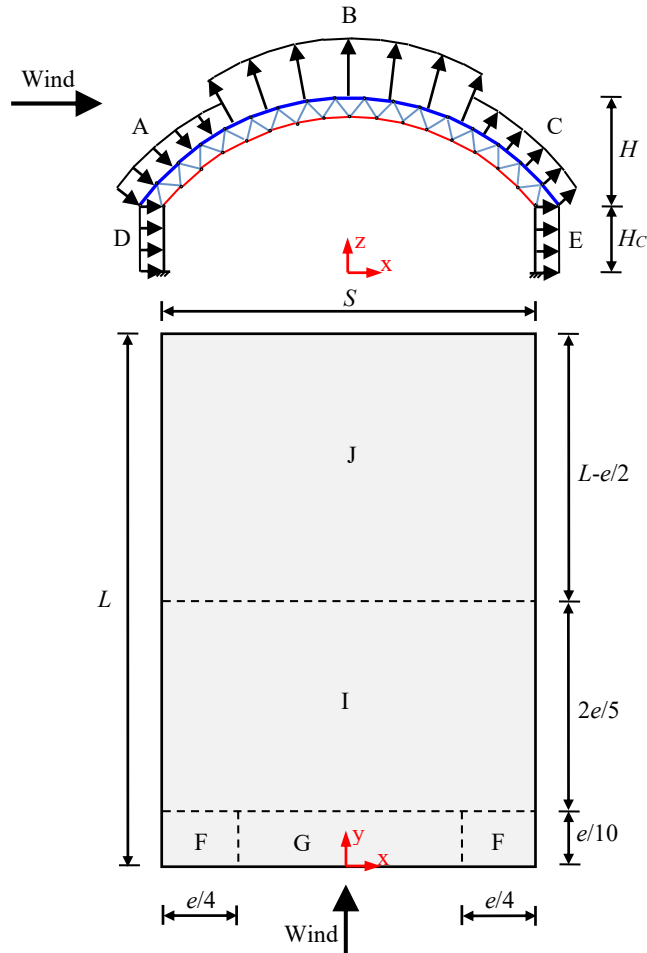


Figure 4. Distribution of wind load in x and y directions

Earthquake equivalent static loads in x and y directions are applied according to Standard No. 2800 [15]. Vertical earthquake load acting on a sloping surface shall be assumed to act on the horizontal projection of that surface. The vertical earthquake load is calculated using the following equation in agreement with Standard No. 400 [15] requirements.

$$E_v = 0.2(D + 0.5 S) \tag{5}$$

In order to model nonlinear behavior of members of barrel vault an element with plasticity and large deflection capabilities is utilized. In elasto-plastic analysis the von-mises yield function is used as yield criterion, flow rule in this model is associative and the hardening rule is bi-linear kinematics hardening in tension. In compression, according to FEMA-274 [18], it is assumed that the element buckles at its corresponding buckling stress state and its residual stress is about 20% of the buckling stress.

3. OPTIMIZATION PROBLEM FORMULATION

The main aim of the optimization problem of double layer barrel vaults considering nonlinear behaviors is to minimize the weight of the structure, subject to some constraints. The first constraint limits the maximum deflection of the structure. The second constraint is checked to ensure the overall stability of the structure during the optimization process. For a double layer barrel vault with ne members collected in ng groups, if the design variables associated with each design group are selected from a given profile list, the nonlinear optimization problem can be formulated as follows:

$$\text{Minimize: } w(X) = \sum_{i=1}^{ne} \rho_i A_i \sum_{j=1}^{nm} L_j \quad (6)$$

$$\text{Subject to: } g_d(X) = \frac{\Delta_{\max}}{\Delta_{\text{all}}} - 1 \leq 0 \quad (7)$$

$$g_s(X) = \frac{f_{\text{app}}}{f_u} - 1 \leq 0$$

$$X = \{X_1 \quad X_2 \quad \dots \quad X_i \quad \dots \quad X_{ng}\}^T \quad (8)$$

where w represents the weight of the frame; ρ_i and A_i are weight of unit volume and cross-sectional area of the i th group section, respectively; nm is the number of elements collected in the i th group; L_j is the length of the j th element in the i th group; $g_d(X)$ is the maximum deflection constraint; Δ_{\max} is the maximum deflection of the structure and Δ_{all} is its allowable value; $g_s(X)$ is the stability constraint; f_{app} is applied load and f_u is ultimate load of the structure which can be determined by incremental nonlinear analysis; X_i is an integer value expressing the sequence numbers of steel sections assigned to the i th group.

In this study, the constraints of the optimization problems are handled using the concept of exterior penalty function method (EPFM). In this case, the pseudo unconstrained objective function for optimization processes is expressed as follows:

$$\Phi_{NL}(X) = w(X) \left(1 + r(\max\{0, g_d(X)\})^2 + r(\max\{0, g_s(X)\})^2 \right) \quad (9)$$

where $\Phi_{NL}(X)$ is pseudo objective function; and r is the penalty parameter.

4. IMPROVED COLLIDING BODIES OPTIMIZATION

Colliding bodies optimization (CBO) is a meta-heuristic search algorithm that has been developed by Kaveh and Mahdavi [12]. In this technique, one object collides with other object and they move towards a minimum energy level. The CBO is simple in concept and does not depend on any internal parameter. Kaveh and Ilchi Ghazaan [13] proposed enhanced CBO (ECBO) to improve convergence rate and reliability of CBO by adding a memory to save some of the best solutions during the optimization process and also utilizing a mutation operator to decrease the probability of trapping into local optima. In order to improve the convergence rate of CBO a different computational strategy, in comparison with ECBO, has been proposed in [8] and an improved colliding bodies optimization (ICBO) has been introduced to tackle space structures optimization problems. In the framework of ICBO the global best body up to current iteration is saved based on this important point that during the optimization process the best solutions should not be lost and should be passed onto the next generations. Furthermore, a simple mechanism has been included in ICBO to escape from local optima. The basic steps of ICBO is as follows:

1. The initial positions of all colliding bodies (CBs) are determined randomly in an m -dimensional search space using Eq. (10).

$$X_i^0 = X_{\min} + R.(X_{\max} - X_{\min}), i = 1, 2, \dots, n \quad (10)$$

in which X_i^0 is the initial solution vector of the i th CB. Here, X_{\min} and X_{\max} are respectively the lower and upper bounds of design variables; R is a random vector in the interval $[0, 1]$; n is the number of CBs.

2. Each colliding body (CB), has a specified mass defined using Eq. (11).

$$m_i = \frac{1}{F(X_i)} \quad (11)$$

where $F(X_i)$ is the objective function value of the i th CB.

3. In order to select pairs of objects for collision, CBs are divided into two equal groups:

(a) Stationary group: $i_S = 1, 2, \dots, n/2$

(b) Moving group: $i_M = (n/2)+1, \dots, n$

4. The velocities of stationary and moving bodies before collision are evaluated using Eq. (12).

$$V_{i_S} = 0, V_{i_M} = X_{i_S} - X_{i_M} \quad (12)$$

5. The velocities of stationary and moving bodies after collision are evaluated using Eqs. (13) to (15)

$$V'_{i_S} = \left(\frac{(1 + \varepsilon)m_{i_M}}{m_{i_S} + m_{i_M}} \right) V_{i_M} \quad (13)$$

$$V'_{i_M} = \left(\frac{(m_{i_M} - \varepsilon m_{i_S})}{m_{i_S} + m_{i_M}} \right) V_{i_M} \quad (14)$$

$$\varepsilon = C^0 - \frac{iter}{iter_{max}} \quad (15)$$

where $iter$ and $iter_{max}$ are the current iteration number and the total number of iterations for optimization process, respectively; ε is the coefficient of restitution the best value of C^0 should be determined by performing sensitivity analysis.

6. The position of CBs in design space are updated using Eqs. (16) to (17)

$$X_{i_S}^{iter+1} = X_{i_S}^{iter} + \bar{R}_{i_S} \cdot V'_{i_S} + \alpha^{iter} \cdot \bar{R}_i \quad (16)$$

$$X_{i_M}^{iter+1} = X_{i_M}^{iter} + \bar{R}_{i_M} \cdot V'_{i_M} + \alpha^{iter} \cdot \bar{R}_i \quad (17)$$

$$\alpha^{iter} = \alpha^{iter-1} \cdot \alpha_{damp} \quad (18)$$

where \bar{R}_{i_S} and \bar{R}_{i_M} are random vectors uniformly distributed in the range of $[-1,1]$; \bar{R}_i is a random vector in the interval $[-0.5, 0.5]$. In this work α_{damp} is considered to be 0.995 and in order to find the best value of α^0 sensitivity analysis should be conducted.

7. This process continues until one of the termination criteria is met.

The results of sensitivity analysis conducted in [8] demonstrated that the best values of C^0 and α^0 are 3.0 and 2.0, respectively.

5. STRUCTURAL RELIABILITY ANALYSIS

Deterministic structural optimization without considering the uncertainties in structural capacity and demands results in an unreliable design and therefore cannot provide a fine balance between cost and safety. In this case, it is not possible to ensure that the structural performance will be fulfilled during the lifetime of structures, because the uncertainty in actions and resistances affect the structural response. An appropriate framework for modeling uncertainty is probability theory which allows calculating the reliability index of structures. In the past decades, to deal with the randomness in actions and resistances, semi-probabilistic, approximate probabilistic, and exact probabilistic methods have been widely used [19]. In exact probabilistic methods the probability of failure is determined based on the joint probability distribution of the random variables. Monte Carlo simulation (MCS) is a simulation method categorized in exact probabilistic methods that allows the consideration of any probability distribution function for random variables. The major advantage of MCS is that accurate solutions can be obtained for almost every problem, however its computational cost is excessive in many cases [20].

In order to solve a reliability problem, random design variables need to be defined. For

the optimally designed double layer barrel vaults the random variables taken in the present study are represented as follows:

$$Z = \{E, f_y, S\}^T \quad (19)$$

where Z is vector of random variables; E and f_y are respectively Young's modulus and yield strength; and S is snow load.

A reliability problem is normally formulated using a limit state function. Limit state function for optimally designed nonlinear double layer barrel vaults is defined as follows:

$$G(Z) = \delta_T - \delta_S(Z) \quad (20)$$

where G is a limit state function; δ_T and δ_S are the target vertical displacement and maximum displacement of double layer barrel vaults.

The non-performance probability, P_f , is defined as a function of the defined limit state functions for the problem at hand. Estimation of the non-performance probability requires the evaluation of the multiple integral over the failure domain, i.e. $G(Z) < 0$, as follows:

$$P_f = \int \int_{G(Z)} \dots \int F_Z(Z) dZ \quad (21)$$

where $F_Z(Z)$ is the joint probability density function of Z .

The total exceedance probability, P_{f_E} , is defined as a series system when one of the limit state functions fails:

$$P_{f_E} = P \left(\bigcup_{i=1}^{n_l} \{G_i(Z) \leq 0\} \right) \quad (22)$$

where n_l is the number of the limit state functions.

As in the present work there is only one limit state function for each structure, Eq. (22) can be rewritten as follows:

$$P_{f_E} = P(G(Z) \leq 0) \quad (23)$$

Computation of total exceedance probability requires integration of a multi-normal distribution function. This integral can be estimated by the MCS method and it allows the determination of an estimate of P_{f_E} given by

$$P_{f_E} = \frac{1}{N} \sum_{i=1}^N I_i(Z) \quad (24)$$

$$I(Z) = \begin{cases} 1 & \text{if } G(Z) \leq 0 \\ 0 & \text{if } G(Z) > 0 \end{cases} \quad (25)$$

where N is the number of independent samples generated based on the probability distribution for each random variable.

Finally, the reliability index (RI) for the problem at hand is determined as follows:

$$RI = 1 - Pf_E \quad (26)$$

The standard MCS method is the most straightforward approach to evaluate the reliability of structures. However, this approach is in general time-consuming, especially if the failure probability is small (e.g. $\leq 10^{-6}$) and/or the number of structures to be analyzed is large. The efficiency of the MCS can be amended by employing the importance sampling (IS) technique which its mathematical background is well described in literature [19–21]. The IS-based simulation samples the failure domain more frequently and therefore achieves a higher efficiency in estimating the failure probability compared with the simple MCS [21–24]. In this work, Rt [25] software is used to perform IS-based reliability analysis of structures.

5. PROPOSED METHODOLOGY

The outlines of methodology presented in this study to evaluate the reliability index RI of optimally designed double layer barrel vaults are as follows:

Design optimization of double layer barrel vaults is achieved using ICBO algorithm according to the formulation provided in section 2. The cross-sections of structural members of double layer barrel vaults are selected from a set of available Pipe profiles given in Table 1. In this case, 16 models with 3.0 m modulation and different height to span (H/S) ratios are generated as given in Table 2.

Table 1: The available list of standard Pipe profiles

No.	Profile	Area (cm ²)
1	D48×2.9	4.1089
2	D60×3.0	5.3721
3	D76×3.0	6.8801
4	D89×3.0	8.1053
5	D114×4.0	13.823
6	D114×5.0	17.121
7	D140×4.0	17.090
8	D140×5.0	21.206
9	D168×5.0	25.604
10	D168×6.0	30.536

Table 2: Different models of double layer barrel vaults

No.	Form	Height (m)	Span (m)	H/S	Model name
1	square-on-square	8.4	42	0.2	SS20
2	square-on-square	12.6	42	0.3	SS30
3	square-on-square	14.7	42	0.35	SS35
4	square-on-square	16.8	42	0.4	SS40
5	diagonal-on-diagonal	8.4	42	0.2	DD20
6	diagonal-on-diagonal	12.6	42	0.3	DD30
7	diagonal-on-diagonal	14.7	42	0.35	DD35
8	diagonal-on-diagonal	16.8	42	0.4	DD40
9	square-on-diagonal	8.4	42	0.2	SD20
10	square-on-diagonal	12.6	42	0.3	SD30
11	square-on-diagonal	14.7	42	0.35	SD35
12	square-on-diagonal	16.8	42	0.4	SD40
13	diagonal-on-square	8.4	42	0.2	DS20
14	diagonal-on-square	12.6	42	0.3	DS30
15	diagonal-on-square	14.7	42	0.35	DS35
16	diagonal-on-square	16.8	42	0.4	DS40

Reliability assessment of the optimally designed double layer barrel vaults is carried out using IS method. The probability density function, mean value and standard deviation of each random parameter are given in Table 3.

Table 3: Properties of the random variables for steel lattice domes

Random Variable	Probability density function	Mean value	Standard deviation
E	Lognormal	2.1e6 kg/cm ²	2.1e5
f_y	Lognormal	2400 kg/cm ²	480
S (snow load)	Normal	250 kg/m ²	50

The limit state function considered in this study is as follows:

$$G = \Delta_{\max} - \delta \quad (27)$$

where Δ_{\max} is the maximum deflection of double layer barrel vault determined by performin structural analysis; δ is a limiting value which is considered as $L/360$ in which L is span length of the barrel vault.

6. NUMERICAL RESULTS

The optimal weight of different models of double layer barrel vaults obtained by ICBO metaheuristic are given in Table 4. The results of IS-based reliability assessment of these

optimal designs, considering 2000 samples, are given in Table 5.

Table 4: Results of optimization of double layer barrel vaults

No.	Model name	Number of element	Number of joint	Optimal weight (kg)
1	SS20	2720	718	35937.07
2	SS30	2720	718	39794.06
3	SS35	2720	718	42426.51
4	SS40	2720	718	45821.55
5	DD20	3597	1061	46563.49
6	DD30	3597	1061	48928.82
7	DD35	3597	1061	51385.31
8	DD40	3597	1061	54803.07
9	SD20	3293	1021	40952.24
10	SD30	3293	1021	44447.81
11	SD35	3293	1021	49586.98
12	SD40	3293	1021	52679.35
13	DS20	3433	1057	36946.54
14	DS30	3433	1057	39110.86
15	DS35	3433	1057	41981.89
16	DS40	3433	1057	46550.41

Table 5: Results of reliability analysis

No.	Model name	Coefficient of Variation (CoV)	P_f	RI
1	SS20	0.0663	0.041	0.959
2	SS30	0.0638	0.038	0.962
3	SS35	0.0579	0.036	0.964
4	SS40	0.0537	0.032	0.968
5	DD20	0.0461	0.067	0.933
6	DD30	0.0426	0.057	0.943
7	DD35	0.0399	0.050	0.95
8	DD40	0.0376	0.043	0.957
9	SD20	0.0661	0.080	0.920
10	SD30	0.0624	0.077	0.923
11	SD35	0.0596	0.074	0.926
12	SD40	0.0563	0.072	0.928
13	DS20	0.0646	0.047	0.953
14	DS30	0.0594	0.045	0.955
15	DS35	0.0521	0.044	0.956
16	DS40	0.0498	0.043	0.957

The obtained RI values for different models are depicted in Fig. 5 indicating that the minimum and maximum RI values are 0.920 and 0.968, respectively which means that all the optimally designed double layer barrel vaults are reliable and safe against uncertainties.

In addition, it can be observed that by increasing the H/S ratio for all forms, RI increases.

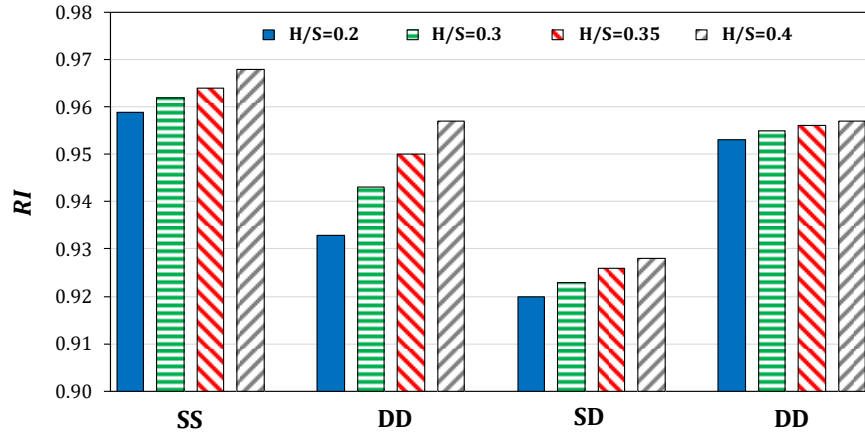


Figure 5. Reliability indices of optimally designed double layer barrel vaults

7. CONCLUSIONS

This study is devoted to reliability analysis of optimally designed double layer barrel vaults. An improved colliding bodies optimization algorithm is used to implement the optimization process. Different forms of double layer barrel vaults including square-on-square (SS), square-on-diagonal (SD), diagonal-on-diagonal (DD) and diagonal-on-square (DS) are optimized. The reliability of the optimally designed double layer barrel vaults is assessed using importance sampling (IS) method. The obtained numerical results indicate that the maximum reliability index among the different forms of double layer barrel vaults belongs to SS form and the minimum to SD form. Moreover, the minimum and maximum indices of are 0.920 and 0.968, respectively which implies that all the optimally designed double layer barrel vaults are reliable and safe against uncertainties.

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