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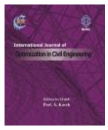

SIZING OPTIMIZATION OF TRUSS STRUCTURES WITH NEWTON META-HEURISTIC ALGORITHM

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ABSTRACT

This study is devoted to discrete sizing optimization of truss structures employing an efficient discrete evolutionary meta-heuristic algorithm which uses the Newton gradient-based method as its updating scheme and it is named here as Newton Meta-heuristic Algorithm (NMA). In order to enable the NMA population-based meta-heuristic to effectively explore the discrete design space, a term containing the best solution found is added to the basic updating rule of the algorithm. The efficiency of the proposed NMA metaheuristic is illustrated by presenting five benchmark discrete truss optimization problems and comparing the results with literature. The numerical results demonstrate that the NMA is a robust and powerful meta-heuristic algorithm for dealing with the discrete sizing optimization problems of steel trusses.

Keywords: Discrete optimization; Sizing optimization; Truss structures; Meta-heuristic; PSO.

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1. INTRODUCTION

Saving in energy and material consumption is an important factor in the field of green engineering and usually from an economical viewpoint, the structure with minimum weight is defined as the best structure. In order to find such designs, structural optimization techniques can be effectively used. In the last decade, many optimization techniques have been developed and successfully applied to a wide range of structural optimization problems including sizing, layout and topology optimization problems [1-3]. Meta-heuristics are the most general kinds of stochastic optimization algorithms and they are now recognized as one of the most practical approaches for solving a wide range of optimization problems. The main idea behind designing these meta-heuristic algorithms is to solve complex optimization

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problems where other optimization methods have failed to be effective. The practical advantage of meta-heuristics lies in both their effectiveness and general applicability. In recent years, meta-heuristic algorithms are emerged as the global search approaches, which are responsible to tackle the complex optimization problems.

Most of the meta-heuristic algorithms are developed based on natural phenomena. Every meta-heuristic method consists of a group of search agents that explore the design space based on randomization and some specified rules inspired the laws of natural phenomena. For example, Genetic Algorithms (GA) [4], Biogeography-Based Optimization (BBO) [5], and Differential Evolution (DE) [6] are developed based on the Darwin's principle of survival of the fittest. Gravitational Search Algorithm (GSA) [7], Colliding Bodies Optimization (CBO) [8] and Center of Mass Optimization (CMO) [9] are Physics-based meta-heuristic algorithms. Particle Swarm Optimization [10] (PSO), Ant Colony Optimization [11] (ACO), Bat algorithm [12] (BA), Dolphin Echolocation Algorithm (DEA) [13] and Finite Defirence Algorithm [14] (FDA) are recognized as popular Swarm intelligence meta-heuristics.

One of the newly developed meta-heuristic algorithms is the Newton Meta-heuristic Algorithm (NMA), which is proposed by Author [15]. The NMA requires that the generated solutions fluctuate outwards or towards the best solution found so far using Newton's method for find an optimum of a function. It was demonstrated in that the NMA is able to effectively solve the continuous optimization problems.

Optimization of truss structures is very popular in the area of structural optimization and over the last decades, various algorithms have been proposed for solving these problems. There is a significant number of meta-heuristics employed for truss optimization with discrete variables in the literature such as: Discrete Heuristic Particle Swarm Ant Colony Optimization (DHPSACO) [16], Improved Dolphin Echolocation Algorithm (IDEA) [17], Improved Mine Blast Algorithm (IMBA) [18], Adaptive Elitist Differential Evolution (AEDE) [19], and Improved Fireworks Algorithm (IFWA) [20].

In the present study, a newton meta-heuristic algorithm (NMA) is proposed to handle the truss structures optimization with discrete design variables, that uses the approximate gradient of pseudo objective function during its search is proposed. In the other words, the main updating rule of the proposed meta-heuristic is derived from summation of two terms. The first term is derived from the Newton gradient-based method using the approximate derivatives of objective function and constraints. The second term includes the difference between the current solution and the best solution obtained so far. Since the proposed meta-heuristic applies the Newton's method in an evolutionary framework, it is named as Newton Meta-heuristic Algorithm (NMA). In order to demonstrate the merit of the proposed NMA for solving discrete structural optimization problems, five benchmark discrete truss optimization problems are presented and the results of NMA are compared with those of other global search algorithms reported in literature.

2. TRUSS OPTIMIZATION PROBLEM

For the optimization problem of trusses, objective function is the structural weight and some limitations are usually considered on nodal displacements and element stress as the design constraints. Formulation of truss structures optimization problem is as follows:

$$\text{Minimize}: W=\sum_{i=1}^{n}\gamma_i l_i X_i, \quad i=1,2,...,n \tag{1}$$

$$\text{Subject to}: \begin{cases} g_j^d=\dfrac{d_j}{\bar{d}_j}-1, & j=1,2,...,m \\ g_k^s=\dfrac{\sigma_k}{\bar{\sigma}_k}-1, & k=1,2,...,n \end{cases} \tag{2}$$

$$X_i^L \le X_i \le X_i^U \tag{3}$$

where W is structural weight; γ_i, l_i and X_i are the density of material, element length and cross-sectional area of ith element, respectively; displacement and stress constraints are represented by g_j^d and g_k^s, respectively; d_j and σ_k are j^{th} node displacement and k^{th} element stress, respectively; $\bar{d}_j$ and $\bar{\sigma}_k$ are their allowable values; n and m are numbers of elements and nodes, respectively.

The following exterior penalty function (EPF) is employed to handle the constraints of the above constrained optimization problem.

$$\Phi=W\times\left(1+r_p\sum_{j=1}^{m}\left(\max\{0,g_j^d\}\right)^2+r_p\sum_{k=1}^{n}\left(\max\{0,g_k^s\}\right)^2\right) \tag{4}$$

where Φ is pseudo unconstrained objective function; and r_p is a penalty parameter. In this study, r_p is linearly increased from 1.0 at the first iteration to 10^6 at the last one during the optimization process.

3. NMA METAHEURISTIC ALGORITHM

In order to find an optimum of a function *f(x)* the Newton’s method can be effectively used. As the derivative is zero at an optimum point, local optima may be found by applying Newton’s method to the derivative. In this case, the iteration can be formulated as follows:

$$x_i^{t+1}=x_i^t-\frac{f'(x_i^t)}{f''(x_i^t)} \tag{5}$$

where x_i^t and x_i^{t+1} are the values of x_i at iterations t and $t+1$, respectively; $f'(x_i^t)$ and $f''(x_i^t)$ are the first and second order derivatives of function f at point x_i^t, respectively.

Determining the explicit form of derivatives for many real-world problems is impossible or at least is very difficult therefore, numerical approximations of derivatives can be effectively applied. In this study, to calculate the numerical approximations of the derivatives, three points x_{i-1}^t, x_i^t, and x_{i+1}^t for which $f(x_{i-1}^t) < f(x_i^t) < f(x_{i+1}^t)$ are selected. Moreover it is assumed that:

$$x_i^t - x_{i-1}^t = \kappa\left(x_{i+1}^t - x_{i-1}^t\right) \tag{6}$$

$$x_{i+1}^t - x_i^t = (1-\kappa)\left(x_{i+1}^t - x_{i-1}^t\right) \tag{7}$$

where κ is a positive parameter.

The second-order Taylor expansion of the function f around x_i is as follows:

$$f(x) = f(x_i^t) + \left(x - x_i^t\right) f'(x_i^t) + \frac{\left(x - x_i^t\right)^2}{2} f''(x_i^t) \tag{8}$$

By assuming $\lambda = x_{i+1}^t - x_{i-1}^t$ and using Eqs. (5) to (7), $f(x_{i-1}^t)$ and $f(x_{i+1}^t)$ can be calculated as follows:

$$f(x_{i+1}^t) = f(x_i^t) + \lambda(1-\kappa) f'(x_i^t) + \frac{\lambda^2(1-\kappa)^2}{2} f''(x_i^t) \tag{9}$$

$$f(x_{i-1}^t) = f(x_i^t) - \lambda\,\kappa\, f'(x_i^t) + \frac{\lambda^2\kappa^2}{2} f''(x_i^t) \tag{10}$$

Simultaneously solving Eqs (9) and (10) for $f'(x_i^t)$ and $f''(x_i^t)$ yields:

$$f'(x_i^t) = \frac{\kappa^2 f(x_{i+1}^t) + (1-2\kappa) f(x_i^t) - (1-\kappa)^2 f(x_{i-1}^t)}{\kappa(1-\kappa)\left(x_{i+1}^t - x_{i-1}^t\right)} \tag{11}$$

$$f''(x_i^t) = \frac{2\kappa f(x_{i+1}^t) - 2 f(x_i^t) + 2(1-\kappa) f(x_{i-1}^t)}{\kappa(1-\kappa)\left(x_{i+1}^t - x_{i-1}^t\right)^2} \tag{12}$$

By putting Eqs (11) and (12) in Eq. (5), the iteration will be as follows:

$$x_i^{t+1} = x_i^t + \left(\frac{\kappa^2 f(x_{i+1}^t) + (1-2\kappa) f(x_i^t) - (1-\kappa)^2 f(x_{i-1}^t)}{2\kappa f(x_{i+1}^t) - 2 f(x_i^t) + 2(1-\kappa) f(x_{i-1}^t)}\right)\left(x_{i-1}^t - x_{i+1}^t\right) \tag{13}$$

In this study, a new population-based meta-heuristic optimization algorithm is proposed to deal with discrete structural optimization problems based on a modified version of Eq. (13) as the updating rule of position of particles in design space. The proposed new and simple optimization algorithm is named as Newton Meta-heuristic Algorithm (NMA). The basic concepts of the proposed NMA metaheuristic are explained in details below.

For an optimization problem with *m* design variables, an initial population of *n* particles is randomly generated in the design space.

$$\mathbf{P}^0 = [X_1^0 \quad X_2^0 \quad \dots \quad X_i^0 \quad \dots \quad X_n^0] \tag{14}$$

$$X_i^0 = \{x_{1i}^0 \quad x_{2i}^0 \quad \dots \quad x_{ji}^0 \quad \dots \quad x_{mi}^0\}^{\mathrm{T}} \tag{15}$$

where $\mathbf{P}^0$ is initial population; X_i^0 is the i^{th} particle of initial population; and x_{ij}^0 is the j^{th} design variable of i^{th} particle of initial population.

At iteration *t*, objective function values of the particles are evaluated and the population is sorted in an ascending order based on the objective values:

$$\mathbf{P}_S^t = [X_1^t \quad X_2^t \quad \dots \quad X_{i-1}^t \quad X_i^t \quad X_{i+1}^t \quad \dots \quad X_n^t] \tag{16}$$

$$f(X_1^t) < f(X_2^t) < \dots < f(X_{i-1}^t) < f(X_i^t) < f(X_{i+1}^t) < \dots < f(X_n^t) \tag{17}$$

where $\mathbf{P}_S^t$ is the sorted population at iteration *t*; and *f(.)* is the objective function of the optimization problem.

If the following equation is used to update the position of particles in the design space, the algorithm will prematurely converge to a local optimum:

$$X_i^{t+1} = X_i^t + round\left(\Gamma(X_{i-1}^t - X_{i+1}^t)\right) \tag{18}$$

$$\Gamma = \frac{\kappa^2 f(X_{i+1}^t) + (1-2\kappa) f(X_i^t) - (1-\kappa)^2 f(X_{i-1}^t)}{2\kappa f(X_{i+1}^t) - 2 f(X_i^t) + 2(1-\kappa) f(X_{i-1}^t)} \tag{19}$$

$$\kappa = \frac{\left\| X_i^t - X_{i-1}^t \right\|}{\left\| X_{i+1}^t - X_{i-1}^t \right\|} \tag{20}$$

where *round(.)* function rounds real numbers to the nearest integer; and ||.|| denotes a vector norm. In order to improve the performance of the NMA meta-heuristic, the following equation is proposed for updating the particles' position:

$$X_i^{t+1} = X_i^t + round\left((\frac{t}{t_{\max}}) R_1^t \, \Gamma(X_{i-1}^t - X_{i+1}^t) + (1 - \frac{t}{t_{\max}}) R_2^t (X_B - X_i^t) \right) \tag{21}$$

where R_1^t and R_2^t are vectors of random numbers drawn from [0,1] at iteration *t*; the maximum number of iterations is represented by t_{max}; and X_B is the global best solution obtained so far.

The local and global search abilities of the proposed NMA meta-heuristic come from second and third terms in Eq. (21), respectively and the results of this study reveal that coefficients of ($1-t/t_{max}$) and (t/t_{max}) provide a fine balance between exploitation and exploration of NMA. Flowchart of the proposed NMA is depicted in Fig. 1.

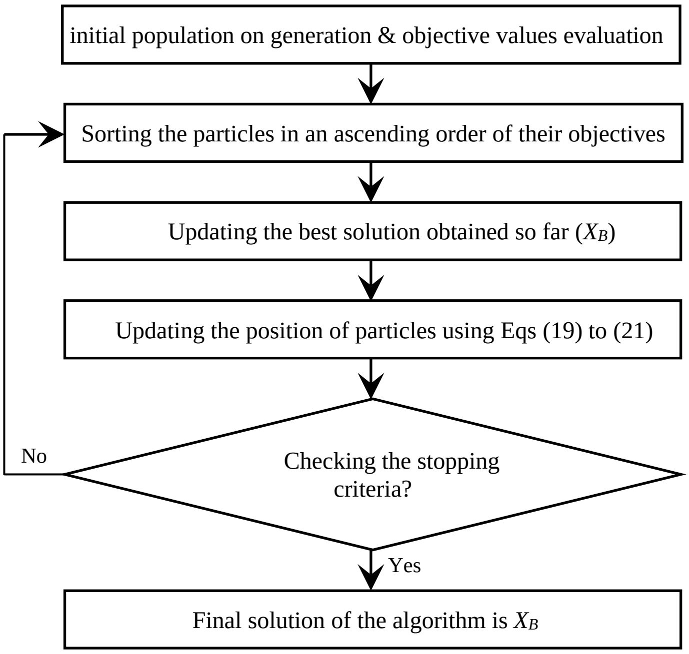


Figure 1. Flowchart of NMA

4. NUMERICAL RESULTS

In order to illustrate the merit of the proposed NMA, a number of popular discrete benchmarks truss optimization problems are presented and the obtained results are compared with those of literature. For the presented examples, 50 independent optimization runs are performed and the best weight (Best), average weight (Average) and the standard deviation (SD) of optimal weights are reported.

Example 1: 10-bar planar truss

The 10-bar truss shown in Fig. 2 is one of the most extensively studied problems. The vertical load in nodes 2 and 4 is equal to 10^5 lb. The Young's modulus and density of material are 10^4 ksi and 0.1 lb/in^3, respectively.

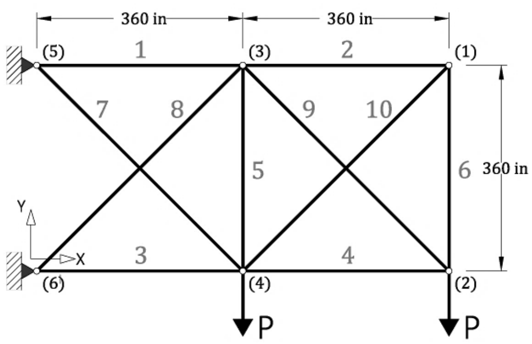


Figure 2. 10-bar truss

The allowable stress for all members is specified as 25 ksi in both tension and compression. The maximum displacements of all free nodes in the x and y directions are limited to ±2 in. In this example, the discrete design variables are selected from the following list: [1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50] (in^2).

Cross-sectional areas of elements 1 to 10 (i.e. A_1 to A_{10}) are considered as the design variables. In the optimization process, 50 particles are involved and the maximum number of iterations is chosen to be 100. The optimization result of NMA is compared with those of HPSO [21], HHS [22], and AEDE [19] in Table 1. In addition, the best and mean convergence curves of NMA is shown in Fig. 3.

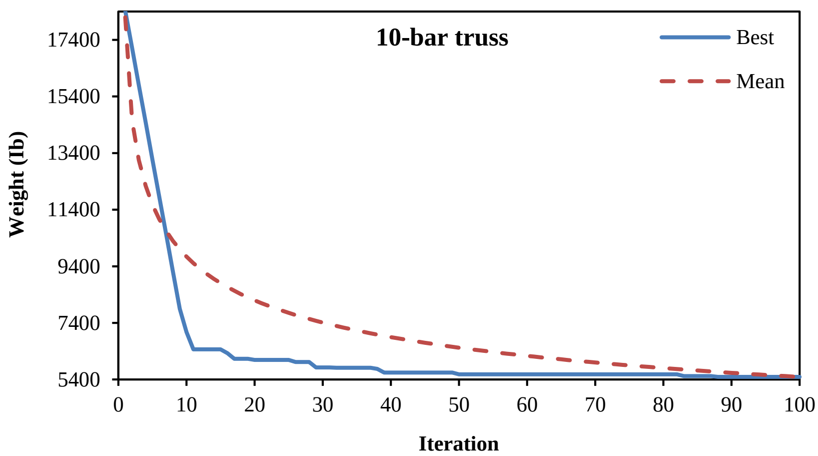


Figure 3. Mean and Best convergence curves for 10-bar truss

Table 1: Results of optimization for the 10-bar truss

Design variables	HPSO [20]	HHS [21]	AEDE [18]	This Study
				NMA
A_1	30.0	33.5	33.5	33.5
A_2	1.62	1.62	1.62	1.62
A_3	22.9	22.9	22.9	22.9
A_4	13.5	14.2	14.2	14.2
A_5	1.62	1.62	1.62	1.62
A_6	1.62	1.62	1.62	1.62
A_7	7.97	7.97	7.97	7.97
A_8	26.5	22.9	22.9	22.9
A_9	22.0	22.0	22.0	22.0
A_{10}	1.80	1.62	1.62	1.62
Best (lb)	5531.98	5490.74	5490.74	5490.74
Average (lb)	N/A	5493.49	5502.62	5490.91
SD (lb)	N/A	10.46	20.78	0.00
Analyses	50000	5000	2550	2880

NMA, AEDE and HHS find the best optimal design among other algorithms. However, the statistical results of NMA, in terms of Average and SD are very better than those of AEDE and HHS.

Example 2: 25-bar spatial truss

The 25-bar spatial truss structure, shown in Fig. 4, is one of the popular design examples in literature. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi. The structure includes 25 members, which are divided into eight groups, as follows: (1) A_1, (2) A_2–A_5, (3) A_6–A_9, (4) A_{10}– A_{11}, (5) A_{12}–A_{13}, (6) A_{14}–A_{17}, (7) A_{18}–A_{21} and (8) A_{22}–A_{25}. The allowable stress of the members is ±40 ksi and all nodes are subjected to displacement limitation of ±0.35 in.

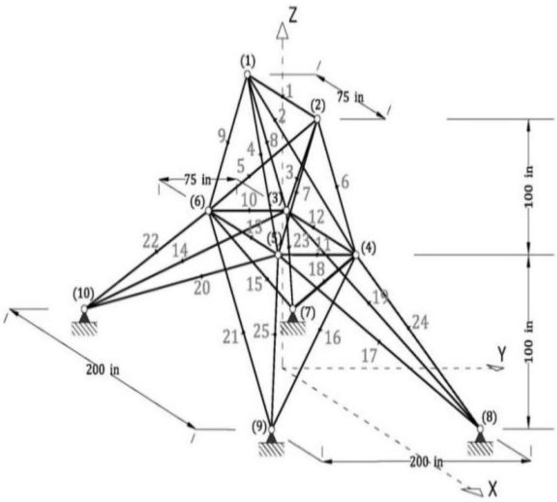


Figure 4. 25-bar spatial truss

The design variables will be selected from the set: [0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 2.0, 2.1, 2.2, 2.3, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4] (in^2). The loads applied to the truss are given in Table 2.

Table 2: Loading conditions for the 25-bar truss (kips)

Node	F_x	F_y	F_z
1	1.0	–10.0	–10.0
2	0.0	–10.0	–10.0
3	0.5	0.0	0.0
6	0.6	0.0	0.0

The optimization result of this study, by considering 50 particles and 100 iterations, is compared with those of HPSO [21], ECBO [23] and AEDE [19] in Table 3. In addition, Fig. 5 compares the best and mean convergence curves of NMA.

The numerical results indicate that also all algorithms converge to the best optimal design however, the Average, SD and convergence rate of NMA are better in comparison with other algorithms. The computational effort of HPSO is significantly more than that of other algorithms.

Table 3: Results of optimization for the 25-bar truss

Design variables	HPSO[20]	ECBO[22]	AEDE[18]	This Study
				NMA
A_1	0.1	0.1	0.1	0.1
A_2–A_5	0.3	0.3	0.3	0.3
A_6–A_9	3.4	3.4	3.4	3.4
A_{10}–A_{11}	0.1	0.1	0.1	0.1
A_{12}–A_{13}	2.1	2.1	2.1	2.1
A_{14}–A_{17}	1.0	1.0	1.0	1.0
A_{18}–A_{21}	0.5	0.5	0.5	0.5
A_{22}–A_{25}	3.4	3.4	3.4	3.4
Best (lb)	484.85	484.85	484.85	484.85
Average (lb)	-	485.89	485.01	484.94
SD (lb)	-	-	0.273	0.00
Analyses	25000	7050	1678	250

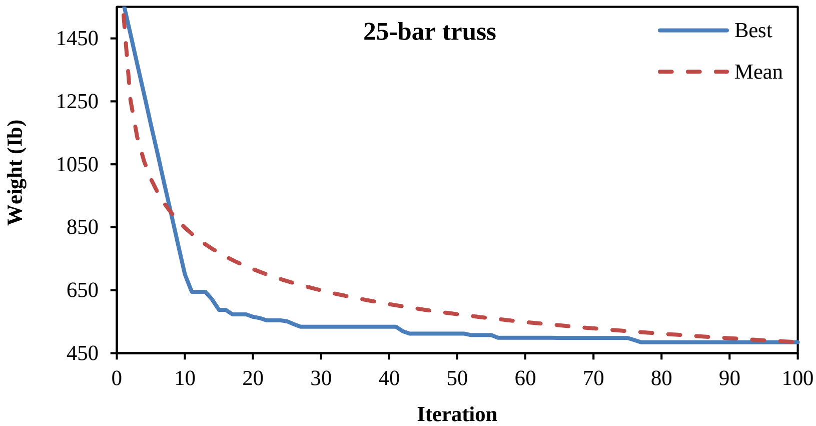
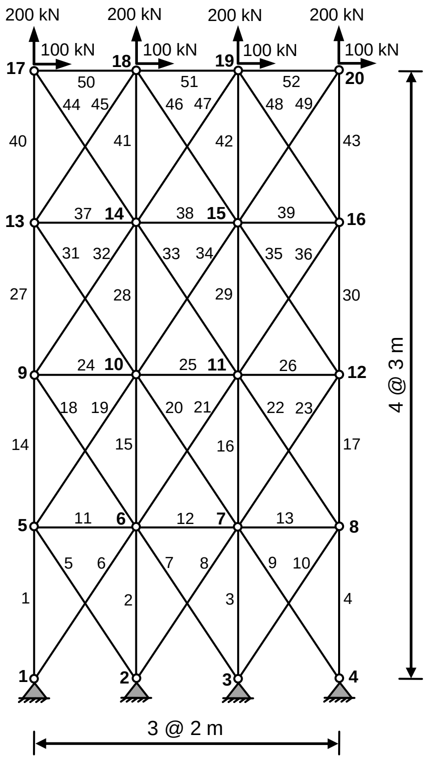


Figure 5. Mean and Best convergence curves for 25-bar truss

Example 3: 52-bar planar truss

Another popular benchmark truss optimization problem is the 52-bar truss shown in Fig. 6 in which P_x=100 kN and P_y=200 kN. The Young's modulus, the material density and the allowable stress are 207 GPa, 7860 kg/m^3 and ±180 MPa, respectively. Element groups are as: (1) A_1–A_4, (2) A_5–A_{10}, (3) A_{11}–A_{13}, (4) A_{14}–A_{17}, (5) A_{18}–A_{23}, (6) A_{24}–A_{26}, (7) A_{27}–A_{30}, (8) A_{31}–A_{36}, (9) A_{37}–A_{39}, (10) A_{40}–A_{43}, (11) A_{44}–A_{49}, and (12) A_{50}–A_{52} which are selected from Table 4 during the optimization process. In this example, population size and maximum number of iterations are 100 and 20, respectively.



Group	Members
1	1 – 4
2	5 – 10
3	11 – 13
4	14 – 17
5	18 – 23
6	24 – 26
7	27 – 30
8	31 – 36
9	37 – 39
10	40 – 43
11	44 – 49
12	50 – 52

Figure 6. 52-bar truss

Table 4: Available cross-sectional areas of the AISC

No	mm^2	in^2	No	mm^2	in^2	No	mm^2	in^2	No	mm^2	in^2
1	71.613	0.11	17	1008.38	1.56	33	2477.41	3.84	49	7419.340	11.
2	90.968	0.14	18	1045.15	1.62	34	2496.76	3.87	50	8709.660	13.
3	126.45	0.19	19	1161.28	1.80	35	2503.22	3.88	51	8967.724	13.
4	161.29	0.25	20	1283.86	1.99	36	2696.76	4.18	52	9161.272	14.
5	198.06	0.30	21	1374.19	2.13	37	2722.57	4.22	53	9999.980	15.
6	252.25	0.39	22	1535.48	2.38	38	2896.76	4.49	54	10322.56	16.
7	285.16	0.44	23	1690.31	2.62	39	2961.28	4.59	55	10903.20	16.
8	363.22	0.56	24	1696.77	2.63	40	3096.76	4.80	56	12129.00	18.
9	388.38	0.60	25	1858.06	2.88	41	3206.44	4.97	57	12838.68	19.
10	494.19	0.76	26	1890.31	2.93	42	3303.21	5.12	58	14193.52	22.
11	506.45	0.78	27	1993.54	3.09	43	3703.21	5.74	59	14774.16	22.
12	641.28	0.99	28	2019.35	3.13	44	4658.05	7.22	60	15806.42	24.
13	645.16	1.0	29	2180.64	3.38	45	5141.92	7.97	61	17096.74	26.
14	792.25	1.22	30	2238.70	3.47	46	5503.21	8.53	62	18064.48	28.
15	816.77	1.26	31	2290.31	3.55	47	5999.98	9.30	63	19354.80	30.
16	939.99	1.45	32	2341.93	3.63	48	6999.98	10.8	64	21612.86	33.

Table 5 compares the optimization result of the present study is those of obtained by HPSO [21], IMBA [18] and AEDE [19]. Comparison of convergence curves of best and mean is shown in Fig. 7.

Table 5. Results of optimization for the 52-bar truss

Design variables	HPSO[20]	IMBA[17]	AEDE[18]	This Study NMA
A_1–A_4	4658.055	4658.055	4658.055	4658.055
A_5–A_{10}	1161.288	1161.288	1161.288	1161.288
A_{11}–A_{13}	363.225	494.193	494.193	494.193
A_{14}–A_{17}	3303.219	3303.219	3303.219	3303.219
A_{18}–A_{23}	939.998	939.998	939.998	939.998
A_{24}–A_{26}	494.193	494.193	494.193	494.193
A_{27}–A_{30}	2238.705	2238.705	2238.705	2238.705
A_{31}–A_{36}	1008.385	1008.385	1008.385	1008.385
A_{37}–A_{39}	388.386	494.193	494.193	494.193
A_{40}–A_{43}	1283.868	1283.868	1283.868	1283.868
A_{44}–A_{49}	1161.288	1161.288	1161.288	1161.288
A_{50}–A_{52}	792.256	494.193	494.193	494.193
Best (kg)	1905.49	1902.605	1902.605	1902.605
Average (kg)	-	1903.076	1906.735	1903.07
SD (kg)	-	1.13	6.679	1.326
Analyses	100000	4750	3402	10000

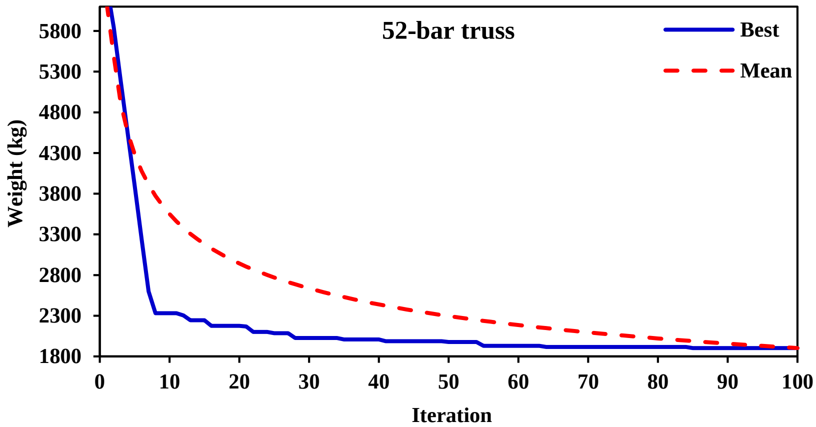


Figure 7. Mean and Best convergence curves for 52-bar truss

It can be seen that IMBA, AEDE and NMA converge to the same best. In this example, IMBA is the best algorithm in terms of Average and SD and the second best algorithm is NMA.

Example 4: 72-bar spatial truss

The 72-bar spatial truss is shown in Fig. 8. In this example, there are 16 groups of elements as follows: (1) A_1–A_4, (2) A_5–A_{12}, (3) A_{13}–A_{16}, (4) A_{17}–A_{18}, (5) A_{19}–A_{22}, (6) A_{23}–A_{30} (7) A_{31}–A_{34}, (8) A_{35}– A_{36}, (9) A_{37}–A_{40}, (10) A_{41}–A_{48}, (11) A_{49}–A_{52}, (12) A_{53}–A_{54}, (13) A_{55}– A_{58}, (14) A_{59}–A_{66} (15) A_{67}–A_{70}, (16) A_{71}–A_{72}. The modulus of elasticity and material density are 10^4 ksi and 0.1 lb/in^3, respectively. During the optimization process, the design variables are selected from the database of Table 4. The allowable stress in elements is ±25 ksi and the allowable horizontal displacement is ±0.25 in. In addition, there are two loading conditions given in Table 6.

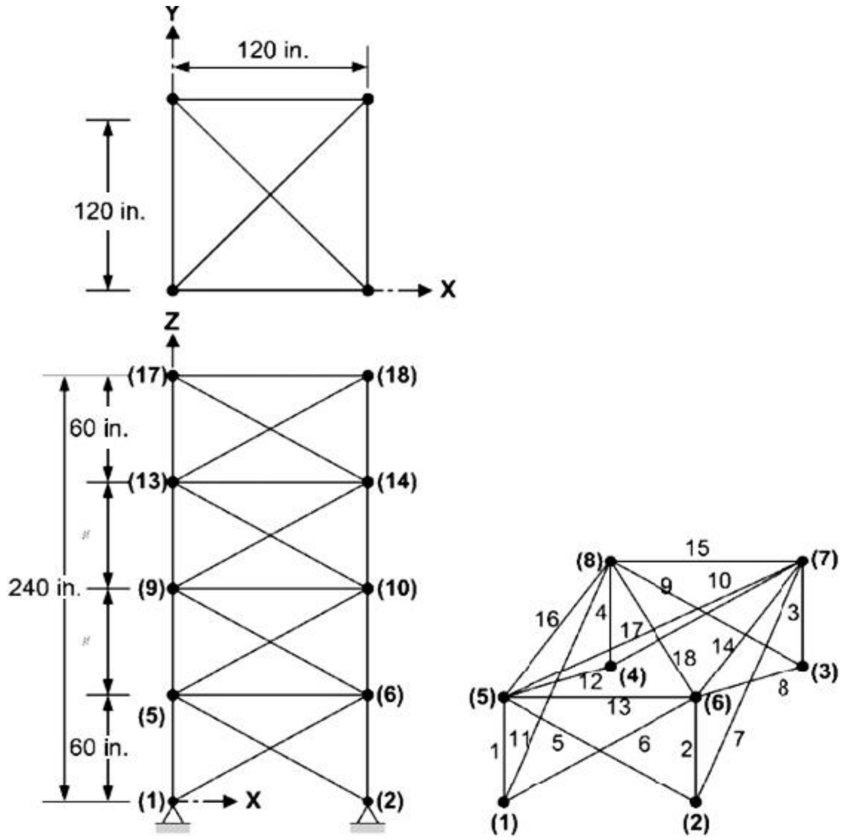


Figure 8. 72-bar truss

Table 6: Loading conditions for the 72-bar truss

Node	Loading condition 1			Loading condition 2		
	F_x	F_y	F_z	F_x	F_y	F_z
17	5.0	5.0	–5.0	0.0	0.0	–5.0
18	0.0	0.0	0.0	0.0	0.0	–5.0
19	0.0	0.0	0.0	0.0	0.0	–5.0
20	0.0	0.0	0.0	0.0	0.0	–5.0

In the optimization process the population size and maximum number of iterations are considered to be 50 and 200, respectively. The results obtained in the present study are compared with those of HPSO [21], IMBA [18] and AEDE [19] in Table 7. Furthermore, convergence curves of best and mean are compared in Fig. 9.

These results reveal that, NMA is competitive in comparison with other algorithms of literature. The statistical results of IMBA are slightly better than those of NMA however at very high computational effort.

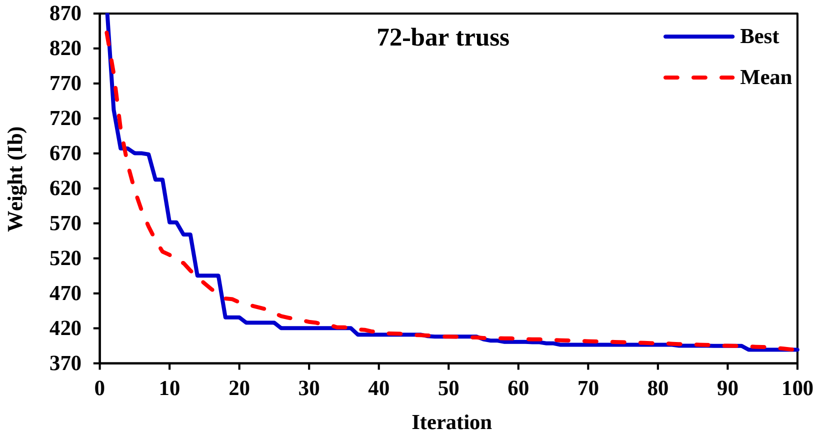


Figure 9. Mean and Best convergence curves for 72-bar truss

Table 7: Results of optimization for the 72-bar truss

Design variables	HPSO[20]	IMBA[17]	AEDE[18]	This Study NMA
A_1–A_4	4.97	1.990	1.990	1.990
A_5–A_{12}	1.228	0.442	0.563	0.563
A_{13}–A_{16}	0.111	0.111	0.111	0.111
A_{17}–A_{18}	0.111	0.111	0.111	0.111
A_{19}–A_{22}	2.88	1.228	1.228	1.228
A_{23}–A_{30}	1.457	0.563	0.442	0.442
A_{31}–A_{34}	0.141	0.111	0.111	0.111
A_{35}–A_{36}	0.111	0.111	0.111	0.111
A_{37}–A_{40}	1.563	0.563	0.563	0.563
A_{41}–A_{48}	1.228	0.563	0.563	0.563
A_{49}–A_{52}	0.111	0.111	0.111	0.111
A_{53}–A_{54}	0.196	0.111	0.111	0.111
A_{55}–A_{58}	0.391	0.196	0.196	0.196
A_{59}–A_{66}	1.457	0.563	0.563	0.563
A_{67}–A_{70}	0.766	0.391	0.391	0.391
A_{71}–A_{72}	1.563	0.563	0.563	0.563
Best (lb)	933.09	389.33	389.33	389.33
Average (lb)	N/A	389.82	390.91	389.75
SD (lb)	N/A	0.84	1.161	0.928
Analyses	50000	50000	4160	5000

Example 5: 200-bar planar truss

Fig. 10 depicts the 200-bar truss and the grouping details of its members. The material density, elastic modulus and the allowable stress for elements are 0.283 lb/in^3, 30 Msi, and ±10 ksi, respectively.

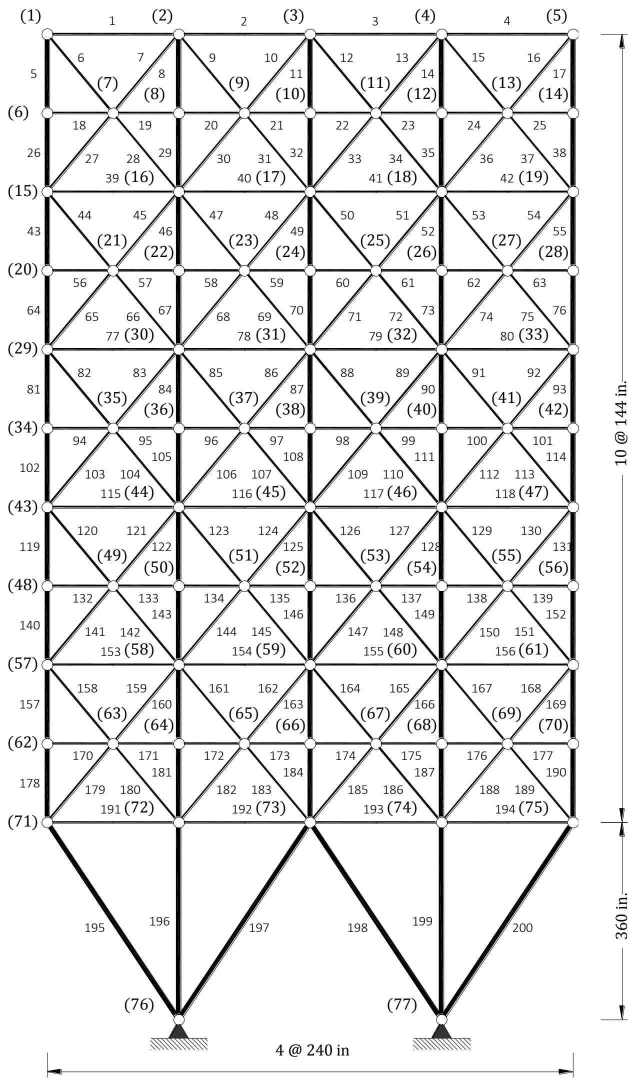
The structure is subjected to the following loading conditions:

(I) 1 kip in the positive *x* direction at nodes 1, 6, 15, 20, 29, 34, 43, 48, 57, 62 and 71;

(II) 10 kips in the negative *y* direction at nodes 1 to 5; 6 to 14 with step 2; 15 to 19; 20 to

28 with step 2; 29 to 33; 34 to 42 with step 2; 43 to 47; 48 to 56 with step 2; 57 to 61; 62 to 70 with step 2; and 71 to 75;

(III) Loading conditions (I) and (II) are applied simultaneously.



Group	Members
1	1, 2, 3, 4
2	5, 8, 11, 14, 17
3	19, 20, 21, 22, 23, 24
4	18, 25, 56, 63, 94, 101, 132, 139, 170, 177
5	26, 29, 32, 35, 38
6	6, 7, 9, 10, 12, 13, 15, 16, 27, 28, 30, 31, 33, 34, 36, 37
7	39, 40, 41, 42
8	43, 46, 49, 52, 55
9	57, 58, 59, 60, 61, 62
10	64, 67, 70, 73, 76
11	44, 45, 47, 48, 50, 51, 53, 54, 65, 66, 68, 69, 71, 72, 74, 75
12	77, 78, 79, 80
13	81, 84, 87, 90, 93
14	95,96, 97, 98, 99, 100
15	102, 105, 108, 111, 114
16	82, 83, 85, 86, 88, 89, 91, 92, 103, 104, 106, 107, 109, 110, 112, 113
17	115, 116, 117, 118
18	119, 122, 125, 128, 131
19	133, 134, 135, 136, 137, 138
20	140, 143, 146, 149, 152
21	120, 121, 123, 124, 126, 127, 129, 130, 141, 142, 144, 145, 147, 148, 150, 151
22	153, 154, 155, 156
23	157, 160, 163, 166, 169
24	171, 172, 173, 174, 175, 176
25	178, 181, 184, 187, 190
26	158, 159, 161, 162, 164, 165, 167, 168, 179, 180, 182, 183, 185, 186, 188, 189
27	191, 192, 193, 194
28	195, 197, 198, 200
29	196, 199

Figure 10. 200-bar truss

Design variables' discrete database for the optimization of 200-bar truss is S={0.100, 0.347, …, 28.080, 33.700 } in^2 [19-25]. In this example, the number of particles and the maximum number of iterations for NMA are 200 and 50, respectively. The results of NMA over fifty independent optimization runs are compared with those of elitist self-adaptive step-size search (ESASS) [25], AEDE [19] and IFWA [20] in Table 8. It can be observed that NMAoutperforms ESASS, AEDE, and IFWA metaheuristics in terms of Best, SD, and Average optimal weights spending almost the same computational cost. Furthermore, Fig. 11 illustrates mean and best convergence curves for 50 independent optimization runs of

200-bar truss indicating very close and good convergence rate of NMA in the all runs.

These results reveal that, NMA is competitive in comparison with other algorithms of literature. The statistical results of NMA are slightly better than those of others however at very high computational effort.

Table 8. Results of optimization for the 200-bar truss

Member group	ESASS	AEDE	IFWA	This Study
				NMA
1	0.100	0.100	0.347	0.100
2	0.954	0.954	0.954	0.954
3	0.100	0.347	0.100	0.347
4	0.100	0.100	0.100	0.100
5	2.142	2.142	2.142	2.142
6	0.347	0.347	0.347	0.347
7	0.100	0.100	0.100	0.100
8	3.131	3.131	3.565	3.131
9	0.100	0.347	0.100	0.100
10	4.805	4.805	4.805	4.805
11	0.347	0.539	0.440	0.539
12	0.100	0.347	0.100	0.100
13	5.952	5.952	5.952	5.952
14	0.100	0.100	0.100	0.100
15	6.572	6.572	6.572	6.572
16	0.440	0.954	0.539	0.539
17	0.539	0.440	0.954	0.440
18	7.192	8.525	8.525	8.525
19	0.440	0.100	0.100	0.100
20	8.525	9.300	9.300	9.300
21	0.954	0.954	1.174	0.954
22	1.174	1.081	0.440	0.347
23	10.85	13.33	13.33	13.33
24	0.440	0.539	1.333	0.100
25	10.85	14.29	13.33	13.33
26	1.764	2.142	2.142	1.081
27	8.525	3.813	3.565	5.952
28	13.33	8.525	8.525	10.85
29	13.33	17.17	17.17	14.29
Best (lb)	28075.49	27858.50	27449.25	27125.07
Average (lb)	-	28425.87	27859.42	27575.11
SD (lb)	-	481.59	380.55	221.72
Analyses	-	11644	10000	10000

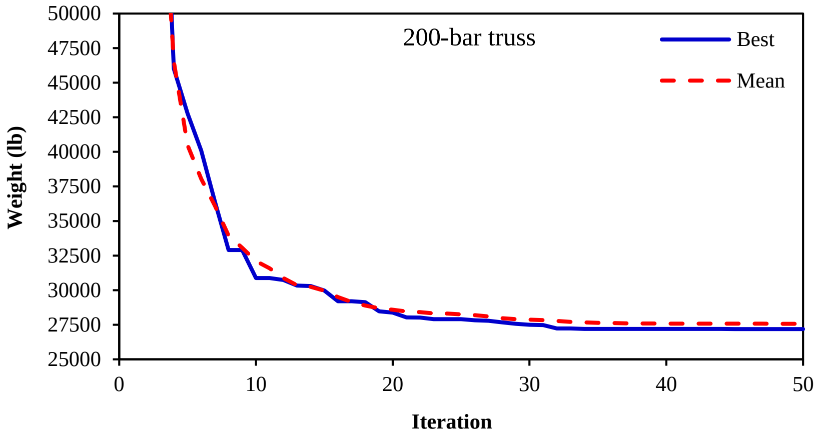


Figure 11. Mean and Best convergence curves for 200-bar truss

5. CONCLUDING REMARKS

The present study focuses on a firstly developed FDA algorithm and proposes a modified FDA (NMA). As the primary version of this meta-heuristic seriously suffers from the slow convergence rate when dealing with the discrete truss optimization problems. The proposed NMA integrates two computational strategies during its search process The first term is derived from the Newton gradient-based method using the approximate derivatives of objective function and constraints. The second term includes the difference between the current solution and the best solution obtained so far.

In order to illustrate the efficiency of the NMA, a sort of well-known discrete benchmark truss optimization problems, including 10-, 25-, 52-,72- and 200-bar trusses, are presented and the results of NMA are compared with those of HPSO, HHS, AEDE, ECBO, and IMBA. The numerical results consistently showed superiority of the proposed algorithm over a number of metaheuristic algorithms in literature. Therefore, this simple and efficient algorithm can be effectively used to deal with discrete optimization of trusses and performance-based design optimization of steel moment frames.

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