THIN WALLED STEEL SECTIONS' FREE SHAPE OPTIMIZATION USING CHARGED SYSTEM SEARCH ALGORITHM

P. Sharafi[1*, †], M. Mortazavi[1], M. Askarian[1], M. E. Uz[2], C. Zhang[3] and J. Zhang[3]

[1]*Centre for Infrastructure Engineering, Western Sydney University, Penrith NSW 2751, Australia*
[2]*Department of Civil Engineering, Adnan Menderes University, Aydin, PK:09100, Turkey*
[3]*School of Civil Engineering, Qingdao University of Technology, Qingdao 266033, China*

ABSTRACT

Graph theory based methods are powerful means for representing structural systems so that their geometry and topology can be understood clearly. The combination of graph theory based methods and some metaheuristics can offer effective solutions for complex engineering optimization problems. This paper presents a Charged System Search (CSS) algorithm for the free shape optimizations of thin-walled steel sections, represented by some popular graph theory based methods. The objective is to find shapes of minimum mass and/or maximum strength for thin-walled steel sections that satisfy design constraints, which results in a general formulation for a bi-objective combinatorial optimization problem. A numerical example involving the shape optimization of thin-walled open and closed steel sections is presented to demonstrate the robustness of the method.

1. INTRODUCTION

Thin walled steel sections, due to their higher strength to weight ratio, are economically advantageous in comparrison with other steel sections. In addition, fast installation, design flexibility and more convenient transportation have made thin-walled structures an attactive choice for steel framing industry in recent years. The manufacturing process allows the cross

---

*Corresponding author: Institute for Infrastructure Engineering, Western Sydney University, Penrith NSW 2751, Australia
†E-mail address: p.sharafi@westernsydney.edu.au (P. Sharafi)

sections to be tailored to suit a vriety of specific applications, by being formed into almost any desired shape. Taking full advantage of this great merit, designers can improve the competitiveness of CFS structures by finding new optimized cross-sectional shapes. Therefore, optimizing the thin walled steel cross section shapes is an area with great research potential, where various optimization methods can compete to demonstrate their robustness. In a thin walled cross section, the section properties are mainly dependent on the topology of the section which is node locations and their connectivity.

There have been a number of different methods applied for shape optimization of thin walled sections in the literature. Simulated annealing algorithm [1] ,Ant Colony [2, 3], gradient-based steepest descent [4], genetic algorithm [4], and some graph theory based methods [5-7] are some of the methods recently used for this purpose. Among them, some methods require an initial guess to begin the optimization process with, while others are independent of the initial guess. The results of the first group of methods are to some extent dependent on the initial guess [4]. This means that different initial guesses can bias the results i.e. the optimum section obtained from the algorithm, which could be a major drawback. Therefore, methods with less sensitivity to initial guess can potentially offer better means for cross-section optimization [4]. There are some fabrication and geometric end-use limitations for optimizing a thin walled section. Depending on the manufacturing process, cold formed or hot rolled, limitations such as the number of bends, the width of coils, and the end-use dimensions or angles can be considered in optimization process [1].

Optimizations of thin-walled steel sections are mostly performed to obtain improvements in strength, serviceability, and vibration characteristics that are performed in the form of shape or sizing optimization of sections. In shape optimization, the vector of design variables represents the form of the boundary of the structural domain, while the design variables in sizing optimization are the dimensions of a predetermined shape. From the differential equations point of view , shape optimization concerns control of the domain of the equation, while sizing concerns control of its parameters.

This paper develops a Charged System Search (CSS) algorithm for optimizing both closed and open thin walled steel sections with the help of some graph theory methods. The optimization criterion is to maximize the section strength under different action effects while minimizing the mass, with no fabrication or end-use constraints. Thin walled section common failure modes such as local and distortional buckling are not considered in this work. However, the same optimization procedure is capable of being applied to the design problems where buckling failure modes of thin walled sections and the fabrication and end-use constraint need to be taken into account. The results obtained from this algorithm are independent of the initial guess, due to a proper primary population selection. To verify the proposed methodology, numerical examples are included.

2. THE SHORTEST PATH AND MINIMUM MEAN CYCLE PROBLEMS

Graph theory based models are powerful means to represent structural systems so that their geometry and topology can be understood clearly. A graph is defined as a set of nodes and a set of edges together with a relation of incidence which associates a pair of nodes with an edge. The pattern of connections and weight or the directions of the edges describe the

characteristics of a graph. In a computational mechanics, graph theory based methods enjoy the advantage of consistency with the finite element discretization. Problems like cross-sectional shape optimization can be directly transformed to a discrete combinatorial optimization problem, arising from a graph theory problem [8]. A graph G (N,M) consists of a set of nodes N and a set of members M, with a relation of incidence that associates each edge with a pair of nodes as its ends. A path P of graph G is a finite sequence whose terms are alternately nodes and edges, in which no edge or node appears more than once. A cycle C is a path for which the starting node and the ending node are the same; i.e. a cycle is a closed path. The length of a path (or cycle) L is taken as the number of its edges.

A practical NP-hard combinatorial optimization problem in the graph theory is the shortest path problem, which is the problem of finding a path from a specified node called the source, to a second specified node, called the destination (or target), such that the sum of the weights (or lengths) of its constituent edges is minimized. Multi-objective shortest path problem can be formulated by Equation (1) through (5) shown in Table 1. Constraints (3) and (4) state that there must be exactly one member leaving the source and entering the target, respectively, that is not on a cycle. Constraint (5) is the ordinary flow conversation constraint and represents that for all the nodes excluding source and target, the number of members entering and leaving is equal.

A Cycle is a path in which the starting and ending nodes are the same. The goal of a minimum mean cycle problem is to find a cycle Γ having a minimum ratio of length to the number of arcs. The multi-objective minimum mean cycle problem is an NP-hard problem that can be formulated by Equations (6) through (8), shown in Table 1. Eq. (8) which is an ordinary flow conversation, guarantees that the selected members are on a cycle.

Table 1: Formulations for Multi-objective shortest path and minimum mean cycle problems

Multi-objective Shortest path problem		Multi-objective minimum mean cycle problem	
$\min f = \left(\sum_{i,j \in V} ce_{ij}^{1}, \sum_{i,j \in V} ce_{ij}^{2}, \dots, \sum_{i,j \in V} ce_{ij}^{r} \right)$	(1)	$\min f = \frac{1}{|\Gamma|} \left(\sum_{i,j \in V} ce_{ij}^{1}, \sum_{i,j \in V} ce_{ij}^{2}, \dots, \sum_{i,j \in V} ce_{ij}^{r} \right)$	(6)
$\forall e_{ij} \in E: e_{ij} = \begin{cases} 1 & if\ Edge\ e_{ij}\ is\ chosen \\ 0 & if\ Edge\ e_{ij}\ is\ not\ chosen \end{cases}$	(2)	$\forall e_{ij} \in E: e_{ij} = \begin{cases} 1 & if\ Edge\ e_{ij}\ is\ chosen \\ 0 & if\ Edge\ e_{ij}\ is\ not\ chosen \end{cases}$	(7)
$\forall i \in N - \{s,t\}: \sum_{j} e_{ij} - \sum_{k} e_{ki} = 0$	(3)	$\forall i \in N - \{s,t\}: \sum_{j} e_{ij} - \sum_{k} e_{ki} = 0$	(8)
$\forall i,j \in N: \sum_{i} e_{si} - \sum_{j} e_{js} = 1$	(4)		
$\forall i,j \in N: \sum_{i} e_{it} - \sum_{j} e_{tj} = 1$	(5)		

Cross section of a thin walled steel section can be defined by a set of nodes connected to each other in x-y plane, which can be represented by a graph. Sharafi et al. [9] developed an innovative graph theory approach for the shape and sizing optimizations of thin-walled steel sections. In this study, it was demonstrated that graph theory based models are a powerful means to represent thin-walled steel sections due to their instinctive clarity. The shape

optimization of open sections can be treated as a multi objective all-pairs shortest path problem, while that of closed sections can be treated as a multi objective minimum mean cycle problem. *Table* 2 shows the different graph theory problems corresponding to the shape and sizing optimization problems of open and closed sections. It also includes the sizing optimization of a predetermined shape or known profile. Detailed discussions on this topic can be found in [9].

Table 2: Graph theory representation of cross-section optimization

Optimization Problem	Corresponding Graph Theory Problem
Shape optimization of open sections	all-pairs shortest path problem
Shape optimization of closed sections	minimum mean cycle problem
Sizing optimization of known profiles	single-pair shortest path problem

3. PROBLEM DEFINITION

Rapid advances in computer technology together with advances in modern metaheuristics have enabled the analyst to deal with problems of large sizes and employ different innovative methods. In most optimization cases in the engineering field, the aim is to find a set of design variables from a given set of feasible solutions that lead to the optimum of the objective function under a given set of constraints. In the present instance, the optimum design entails selecting the best combination among a finite number of design variables for the given objective and some constraints. Such a combinatorial nature, from the mathematical point of view, gives the optimum design procedure a discrete nature.

For a free shape thin-walled cross-section with uniform and invariable wall thickness in the x-y plane, under a general set of actions, as shown in *Fig.* 1, the cross-section can be defined by n nodes and m members connecting the nodes. The cross-sectional optimization problem here is to find the shape resulting in the optimum mass and section strength. The term "section strength" is a generic term that corresponds to the imposed action effects. Minimizing the mass of a section is equivalent to minimizing the cross-section area A, which in turn reduces to the length minimization of the section, as the thickness is constant along the section. The dimensional constraints on the members of a cross-section are defined, considering the effective width (or depth) for the elements [10]. Some fabrication, construction or manufacturing constraints may also be applied to the cross-sectional dimensions. The formulations for finding a free-shape cross-section's area and the second moment of area on a graph, are stated in details in [9].

In the present problem, the variables are the nodal coordinates of the cross-section and their connectivity that turn into discrete and binary variables, respectively, by being mapped onto a graph. The idea is to represent a cross-section design as a mathematical graph, which is made up of members (sub-graphs) having a one-to-one relationship with the physical design. Any changes to the graph reflect same to the cross-section, so the shape optimization of the graph is equivalent to the shape optimization of the cross-section.
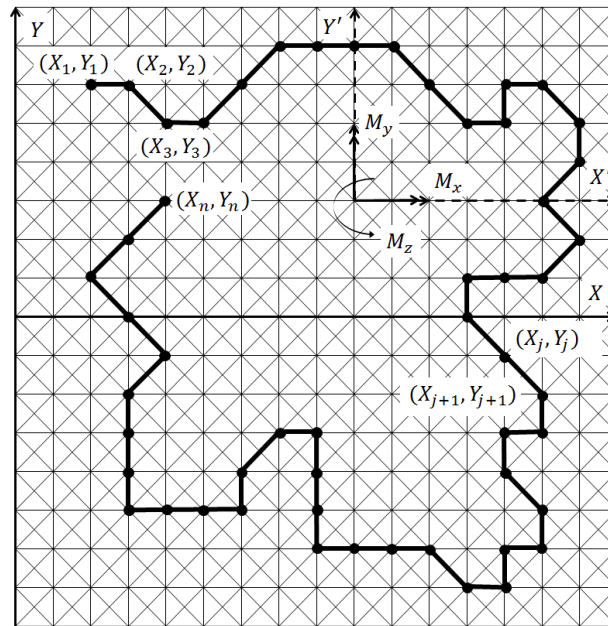
Figure 1. Graph theory for representation of cold formed steel cross-section

This shape optimization method addresses both open and closed thin walled sections. For shape optimization of an open section, all-pairs shortest path problem, represented by Equations (1) through (5) is applicable, while a closed section is considered as a minimum mean cycle problem represented by Equations (6) through (8). Therefore, in regards to an open section, the first cost is equal to the length of each edge (L(P)) and the second one equals to the inverse of section strength ($1/S_G$). On the other hand, for a closed section, the first cost is equal to the square of each edge's length ($L^2(\Gamma)$), and the second cost assigned to the cycle is equal to the product of its length and the inverse of the section strength ($L(\Gamma)/S_G$). The objective functions for open and closed sections are represented by equations (9) and (10) respectively:

$$f_P = \left(L(P_i), \frac{1}{S_{G_i}} \right) \quad i \in All-pairs\ Path \tag{9}$$

$$f_\Gamma = \left(L(\Gamma_i), \frac{1}{S_{G_i}} \right) \quad i \in All\ cycles \tag{10}$$

4. THE FORMULATION OF SHAPE OPTIMIZATION PROBLEM

The optimization problem aiming for a thin-walled section with the minimum mass and maximum strength can therefore, be formulated as follows:

$$\min f = \left(W, \frac{1}{S_G} \right) \quad s.t. \begin{cases} geometric\ constraints \\ strength\ constraints \end{cases} \tag{11}$$

where S_G is the section strength that may represent the cross sectional area, the second moment of area or the torsion constant.

In shape optimization problems, different types of geometric constraints originating from different applications may govern the problem. Constraints such as dimensional limitations of coils, symmetry or anti-symmetry, parallel flanges, section dimension constraints and/or utility pass-through allowance can be considered in the formulation. There are two approaches for determining the optimized solution with maximum possible strength and minimum possible weight. The first approach is to achieve a Pareto-optimal set or a proper approximation of it. The other approach is to assign a weight to each objective before solving the problem and convert the multi-objective optimization problem into a single-objective problem. In the present study, the optimum solution is the one that provides the best compromise between two potentially conflicting objectives of mass minimization and strength maximization. The solution is a Pareto-optimal set or at least a good approximation of it.

Pareto optimality is an economics concept invented by Vilfredo Pareto (1848-1923) that finds applications in engineering [11, 12]. In a Pareto improvement, at least one objective is achieved without sacrificing any other objective. A solution is Pareto optimal when no further Pareto improvements can be made. A Pareto-optimal set is a set of Pareto optimal solutions. Having established a Pareto-optimal set, the ultimate solution may be selected according to the personal intuition of the decision maker. The alternative approach is to formally assign weights or priorities to each objective before solving the problem so that the multi-objective optimization problem is transformed into a single-objective problem (as the various objectives are combined into one through their weighted sum). Obviously, for cases where the minimum required strength of the thin-walled steel section is already determined, the section strength is treated as a state variable and the minimum required strength is treated as a behavioral constraint. In such cases, the problem is instantly reduced to a single objective problem.

5. CHARGED SYSTEM SEARCH ALGORITHM

The two objectives of the problems are mass minimization and strength maximization. Section strength, depending on the instance, can be compressive, flexural, and torsional or shear strength. The objective of mass minimization reduces to finding the paths or cycles of shortest lengths for open and closed sections, respectively. The bi-objective optimization problems, which are also combinatorial optimization problems in the present case, can be dealt with using a CSS algorithm.

The CSS algorithm is developed by Kaveh and Talatahari based on the governing Coulomb and Gauss laws from electrostatics and the Newtonian law of mechanics[13, 14]. This algorithm is a population-based search method and has been proved to be successfully applicable to various optimization problems [14-21]. In the CSS, each agent (CP) is considered as a charged sphere of radius a, which is affected by the electrical forces of other CPs. In case a CP positioned inside the sphere, the force magnitude of the sphere is proportional to the separation distance between the CPs while it is inversely proportional to the square of the separation distance for the ones located outside the sphere. The CPs location is updated considering the resultant forces or acceleration and the laws of motion.

The pseudo-code for the CSS algorithm can be summarized with five steps: initialization, determination of parameters, solution construction, updating and termination.

At the initialization stage, the mechanical properties of the steel, maximum cross-sectional dimensions allowed, and accuracy needed are defined. The grid pattern construction graph is formed in a way that every node/edge on the graph is a potential node/edge of a cross-section. For this purpose, some geometrical, manufacturing and installation constraints, which may determine the shapes' boundaries, are applied to form the construction graph. The initial positions of CPs are determined randomly in the search space, which is the grid graph, and the initial velocities of charged particles are assumed to be zero. Then the CPs fitness function values are determined and the CPs are sorted in an increasing order. A number of the first CPs and their related values of the fitness function are saved in a memory called charged memory (CM). If the number of graph's edges is m and the number of nodes is n, the number of ants for each family is set to be the nearest integer to $m/(n-1)$.

After the initialization stage, the algorithm moves on to the determination of the forces on CPs. The force vector is measured for each CP as follows:

$$F_J = \sum_{i,i \neq j} \left(\frac{q_i}{a^3} \cdot r_{i,j} \cdot i_1 + \frac{q_i}{r_{i,j}^2} \cdot i_2 \cdot \right) a r_{i,j} P_{i,j} (X_i - X_j) \begin{vmatrix} i_1 = 1, i_2 = 0 \leftrightarrow r_{i,j} < a \\ i_1 = 0, i_2 = 1 \leftrightarrow r_{i,j} \geq a \\ j = 1,2, \dots, N \end{vmatrix} \quad (12)$$

where F_J is the resultant force acting on the jth CP and N is the number of CPs. The charged magnitude value of each CP is determined based on the quality of its solutions as follows:

$$q_i = \frac{fit(i) - fitworst}{fitbest - fitworst} \qquad i = 1,2, \dots, N \qquad (13)$$

where *fitbest* and *fitworst* are the best and worst fitness of all the particles, respectively; *fit(i)* presents the ith agent fitness and N is the total number of CPs. The separation distance r_{ij} between two charged particles is defined as follows:

$$r_{ij} = \frac{\|X_i - X_j\|}{\left\| \frac{x_i + x_j}{2} - X_{best} \right\| + \varepsilon} \qquad (14)$$

where X_i, X_j are respectively the positions of the ith and j^{th} CPs; X_{best} is the best current CP location and ε is a small positive number. The probability of moving each CP toward the others, $p_{i,j}$, is determined as follows:

$$p_{i,j} = \begin{cases} 1 & \frac{fit(i) - fitbest}{fit(j) - fit(i)} > rand \wedge fit(j) < fit(i) \\ 0 & else \end{cases} \qquad (15)$$

where *rand* represents a random number.

At the next step of the algorithm, solution construction, the new position and velocity of

each CP is obtained using the following function:

$$X_{j,new} = rand_{j,1}.k_a.F_j + rand_{j,2}.k_v.V_{j,old}.\Delta t + X_{j,old} \tag{16}$$
$$V_{j,new} = X_{j,new} - X_{j,old} \tag{17}$$

where k_a is the acceleration coefficient; k_v is the velocity coefficient to control the influence of the previous velocity; $rand_{j,1}$ and $rand_{j,1}$ are two random numbers uniformly distributed in the range of (0, 1); K_a and K_v are defined as:

$$K_a = 0.5 \times \left(1 + \frac{iter}{iter_{max}}\right), K_v = 0.5 \times \left(1 - \frac{iter}{iter_{max}}\right) \tag{18}$$

where *iter* is the iteration number and *iter$_{max}$* is the maximum number of iterations. If a new CP exits from the allowable search space, a harmony search-based handling approach is applied to correct its position. Also, if some new CP vectors are better than the worst ones in the CM, these are replaced by the worst ones in the CM. This is the updating stage. The algorithm is repeated until a termination criterion is satisfied.

6. NUMERICAL EXAMPLE

In this section, using simple rectangular grid graphs the robustness of the methodology is demonstrated. In this example, an open and a closed thin-walled section under transverse loading are optimized for minimum mass (first objective) and maximum elastic section modulus (second objective). The geometric data and constraints are given in the following:
- Wall thickness: $t = 1$ mm
- Maximum cross-sectional height: 50 mm.
- Maximum cross-sectional width: 50 mm.
- Maximum ratio of flat width to wall thickness, *b/t* : 250
- Minimum resolution (accuracy): $\varepsilon = 1$ mm
- Closed sections are doubly-symmetric and open sections are singly symmetric
  The material properties and the strength constraint are as follows:
- Elastic modulus: 200 GPa
- Yield stress: 300 MPa
- Poisson's ratio: 0.3
- Minimum transverse shear yield strength: 15 kN
- Minimum section flexural strength: 0.25 kN.m
  The present bi-objective optimization problems are summarized in Table 3.
  Since the optimum sections are assumed to be doubly-symmetric for the closed section and mono-symmetric for the open section, only a quarter of the section for the closed cross-section problem and half of the section for the open section are modelled. The construction graph for the closed section problem is 25 mm by 25 mm (including 676 nodes), and for the open section problem is 50 mm by 25 mm (including 1326 nodes).

Table 3: Shape optimization formulation

	Shape Optimization Problem	
	Open Cross-sections	**Closed Cross-sections**
Objective Function	$min\ f_P = \left(L(P_i), 1/S_{fi}\right)$ $\rightarrow min\ f =$ $\left(\sum_{i,j\in V} c^1_{ij}\, e_{ij}, \sum_{i,j\in V} c^2_{ij}\, e_{ij}\right)$	$f_\Gamma = \left(L(\Gamma_i), 1/S_{fi}\right)$ $\rightarrow min\ f =$ $\frac{1}{|\Gamma|}\left(\sum_{i,j\in V} c^1_{ij}\, e_{ij}, \sum_{i,j\in V} c^2_{ij}\, e_{ij}\right)$
Behavioral Constraints	$\forall\ e_{ij} \in E:\quad e_{ij} = \begin{cases}1 & if\ Edge\ e_{ij}\ is\ chosen \\ 0 & if\ Edge\ e_{ij}\ is\ not\ chosen\end{cases}$ $\forall\ i \in N - \{s,t\}:\quad \sum_j e_{ij} - \sum_k e_{ki} = 0$ $\forall\ i,j \in N:\quad \sum_i e_{si} - \sum_j e_{js} = 1$ $\forall\ i,j \in N:\quad \sum_i e_{it} - \sum_j e_{tj} = 1$	$\forall\ i \in N:\quad \sum_j e_{ij} - \sum_k e_{ki} = 0$
Strength Constraints	$S_{v_{min}} \geq 12\ kN$	
Geometric Constraints	$b/t \leq 250$	

The termination criterion for this problem is met when the improvement in the solution quality is less than 0.02% after ten consecutive iterations. The optimum shapes are a Pareto optimal containing the best-so-far solutions. Figs. 2 and 3 show the three best "optimum" shapes (i.e. the first three members of the Pareto-optimal set) for a quarter of the closed section and one half of the open section, respectively.

L = 132 mm
Z_{xx} = 924 mm³

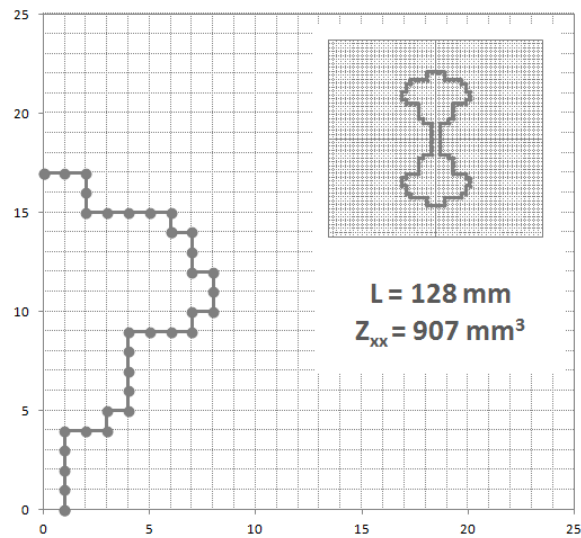L = 136 mm
Z_{xx} = 934 mm³
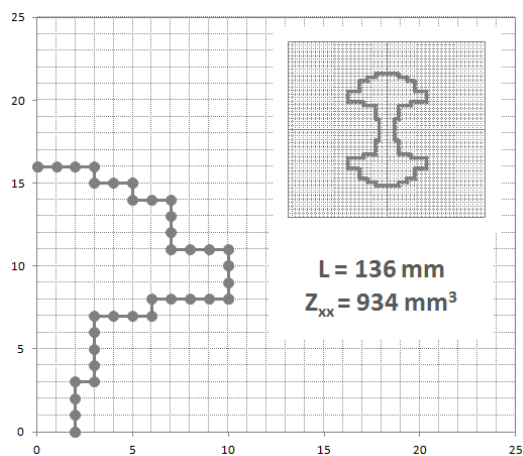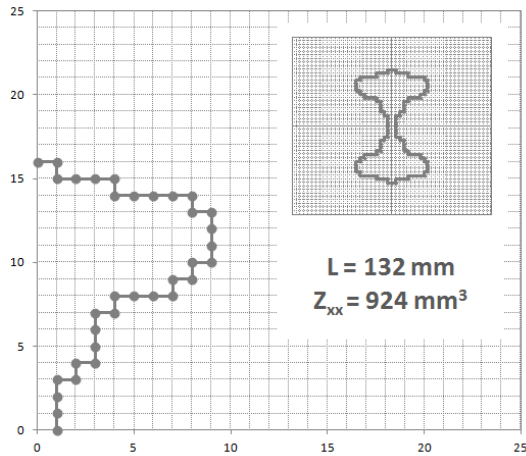
Figure 2. The three best Pareto-optimal solutions for the closed section

In order to accentuate the difference between the obtained open sections, *Fig.* 3 has been drawn by moving all the sections towards the y axis so that the starting and ending points lie on the x and y axes, respectively. The paths are therefore not exactly the paths chosen by CSS particles, but are the equivalent paths having the same characteristics.
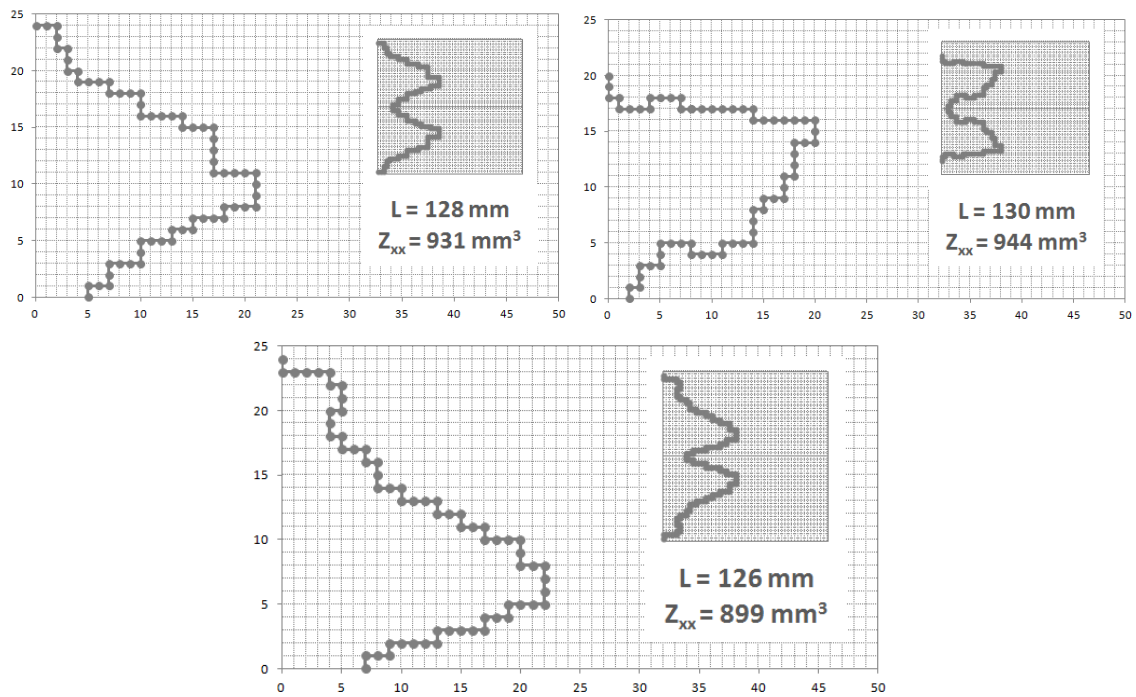

Figure 3. The three best Pareto-optimal solutions for the open section

7. CONCLUSION

Graph theory methods can be easily formulated for a wide range of structural problems, as they have benefitted greatly from interaction with other fields of mathematics. This paper has demonstrated the applications of such algorithms to the shape optimizations of thin-walled steel sections. The shape optimizations of thin-walled steel sections are presented using graph theory approach. The shape optimization of open sections is treated as a multi-objective all-pairs shortest path problem, while that of closed sections is treated as a multi-objective minimum mean cycle problem. CSS algorithm, as a robust meta-heuristics for solving large combinatorial optimization is employed to solve the problems arising from the graph theory approach. The presented method is able to explore the entire solution space of thin-walled steel sections under any loading conditions, without being confined to predetermined cross-sectional shapes. It was demonstrated that the combination of the graph theory methods and the CSS algorithms can offer an effective method for shape optimizations of thin-walled steel sections with multiple conflicting objectives of mass minimization and strength maximization, as well as accounting for the geometric and strength constraints.

REFERENCES

1. Leng J, Li Z, Guest JK, Schafer BW. Shape optimization of cold-formed steel columns with fabrication and geometric end-use constraints, *Thin-Wall Struct* 2014; **85**: 271-90.
2. Kaveh A, Hassani M. Ant colony algorithms for nonlinear analysis and optimal design of structures, *Int J Optim Civil Eng* 2011; **1**(4): 571-95.
3. Hasançebi O, Carbas S. *Ant Colony Search Method in Practical Structural Optimization*, 2011.
4. Leng J, Guest JK, Schafer BW. Shape optimization of cold-formed steel columns, *Thin-Wall Struct* 2011; **49**(12): 1492-1503.
5. Sharafi P, Hadi MNS, Teh LH. Optimum Column layout design of reinforced concrete frames under wind loading, in *Topics on the Dynamics of Civil Structures, Volume 1: Proceedings of the 30th IMAC, A Conference on Structural Dynamics*, Caicedo JM, et al, Editors, Springer New York, New York, NY, 2012; **1**: pp. 327-340.
6. Sharafi P, Hadi MNS, Teh LH. Optimum spans' lengths of multi-span reinforced concrete beams under dynamic loading, in *Topics on the Dynamics of Civil Structures, Proceedings of the 30th IMAC, A Conference on Structural Dynamics,* Caicedo JM, et al, Editors, Springer New York, New York, NY, 2012; **1**: pp. 353-61.
7. Moharrami M, Louhghalam A, Tootkaboni M. Optimal folding of cold formed steel cross sections under compression, *Thin-Wall Struct* 2014; **76**: 145-56.
8. Descamps B, Filomeno Coelho R. Graph theory in evolutionary truss design optimization, *Metaheuristic Applicat Struct Infrastruct* 2013; 241-68.
9. Sharafi P, Teh LH, Hadi MNS. Shape optimization of thin-walled steel sections using graph theory and ACO algorithm, *J Construct Steel Res* 2014; **101**: 331-41.
10. Yu WW, LaBoube RA. *Cold-Formed Steel Design*, Wiley, 2010.

11. Sharafi P, Teh LH, Hadi MNS. Conceptual design optimization of rectilinear building frames: A knapsack problem approach, *Eng Optim* 2015, **47**(10): 1303-23.
12. Sharafi P, Hadi MNS, Teh LH. Geometric design optimization for dynamic response problems of continuous reinforced concrete beams, *J Comput Civil Eng* 2014; **28**(2) 202-9.
13. Kaveh A, Talatahari S. A novel heuristic optimization method: charged system search, *Acta Mech* 2010; **213**(3): 267-89.
14. Talatahari S, Kaveh A, Sheikholeslami R. An efficient charged system search using chaos for global optimization problems*, Int J Optim Civil Eng* 2011; 1(2): 305-25.
15. Kaveh A, Sharafi P. Charged system search algorithm for minimax and minisum facility layout problems, *Asian J Civil Eng* 2011; **12**(6): 703-18.
16. Kaveh A, Sharafi P. Ordering for bandwidth and profile minimization problems via charged system search algorithm, *Iranian J Sci Technol: Transact Civil Eng* 2012; **36**(C1): 39-52.
17. Kaveh A, Mahdavi V. Optimal design of arch dams for frequency limitations using charged system search and particle swarm optimization, *Int J Optim Civil Eng* 2011; **4**: 543-55.
18. Kaveh A, Talatahari S. Optimization of large-scale truss structures using modified charged system search, *Int J Optim Civil Eng* 2011; **1**(1): 15-28.
19. Kaveh A, Zolghadr A. An improved charged system search for structural damage identification in beams and frames using changes in natural frequencies, *Int J Optim Civil Eng* 2012; **2**(3): 321-39.
20. Sheikholeslami R, Kaveh A, Tahershamsi A, Talatahari S. Application of charged system search algorithm to water distrubition networks optimization, *Int J Optim Civil Eng* 2014; **4**(1): 41-58.
21. Sharafi P, Askarian M, Uz ME, Abaci H. Form finding for rectilinear orthogonal buildings through charged system search algorithm, *Int J Optim Civil Eng* 2017; **7**(1): 129-42.