

SEISMIC DESIGN OPTIMIZATION OF STEEL STRUCTURES BY A SEQUENTIAL ECBO ALGORITHM

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ABSTRACT

The objective of the present paper is to propose a sequential enhanced colliding bodies optimization (SECBO) algorithm for implementation of seismic optimization of steel braced frames in the framework of performance-based design (PBD). In order to achieve this purpose, the ECBO is sequentially employed in a multi-stage scheme where in each stage an initial population is generated based on the information derived from the results of previous stages. The required structural seismic responses, at performance levels, are evaluated by performing nonlinear pushover analysis. Two numerical examples are presented to illustrate the efficiency of the proposed SECBO for tackling the seismic performance-based optimization problem. The numerical results demonstrate the computational advantages of the SECBO algorithm.

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1. INTRODUCTION

One of the most important issues in designing a structural system is its sufficient seismic resistance to ensure availability after an earthquake. In recent years, the concepts of performance-based design (PBD) [1] were developed and applied in the framework of powerful and reliable seismic design procedures [2]. In the PBD approach, nonlinear analysis procedures are usually employed to evaluate the nonlinear seismic responses of structures and pushover analysis is one of the popular procedures. This analysis method generally adopts a lumped-plasticity approach that tracks the spread of inelasticity through the formation of nonlinear plastic hinges at the frame element's ends during the incremental loading process [3]. Generally, the number of parameters which affect the structural

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performance in the seismic design process of structures is usually large. In this case, finding cost-efficient solutions satisfying design code requirements is a difficult task. To achieve this purpose, structural optimization methodologies have been developed during the last decades. In the recent decades, many metaheuristics have been developed and each one consists of a group of search agents that explore the feasible region based on randomization and some specified rules inspired the laws of natural phenomena. Metaheuristics have attracted a great deal of attention in recent years, due to their simplicity and flexibility.

Optimization of steel structures using the PBD framework is one of the active research fields and in the recent years a number of researchers have utilized metaheuristics to achieve the PBD optimization task. Kaveh et al. [4] compared the computational performance of ant colony optimization (ACO) and genetic algorithm (GA) for performance-based optimal design of frame structures. Gholizadeh et al. [5] compared the computational performance of GA, ACO, particle swarm optimization (PSO), and harmony search (HS) meta-heuristics for PBD optimization of steel frames. Kaveh and Nasrollahi [6] proposed a methodology for implementation of performance-based seismic design of steel frames utilizing charged system search (CSS) metaheuristic. Gholizadeh [7] proposed an efficient methodology for PBD optimization of steel frames based on application of a modified firefly algorithm (MFA) as an optimizer. One of the recent additions to metaheuristics is enhanced colliding bodies optimization (ECBO) algorithm [8]. Gholizadeh and Milany [9] compared the computational performance ECBO with that of some other recent metaheuristics in tackling the PBD optimization of steel frames. Their obtained results demonstrated the superiority of ECBO over the other algorithms.

In the present work, an efficient version of ECBO, termed as sequential ECBO (SECBO), is proposed to implement the PBD optimization problem of steel braced frames (SBF). For the PBD optimization problem of SBFs, there are some constraints that should be carefully handled. One of the most popular constraint-handling techniques is the penalty function methods and in this study, the exterior penalty function method (EPFM) is employed in the framework of the sequential unconstrained minimization technique (SUMT) [10]. In the framework of SECBO, an initial population is randomly selected and all of the heuristic operations are imposed on the population involving EPFM. For commencing a new optimization process a new population is generated using information derived from the results of previous processes. This procedure is followed until a termination criterion is satisfied.

Two numerical examples of planar SBFs are presented and the numerical results demonstrate the efficiency of the proposed SECBO in comparison with standard ECBO.

2. PBD OPTIMIZATION

In PBD frameworks, a performance objective is defined as a given level of performance for a specific hazard level. To define a performance objective, at first the level of structural performance should be selected and then the corresponding seismic hazard level should be determined. In the present work, immediate occupancy (IO), life safety (LS) and collapse prevention (CP) performance levels are considered according to FEMA-356. Each objective

corresponds to a given probability of being exceeded during 50 years. A usual assumption is that the IO, LS and CP performance levels correspond respectively to a 20%, 10% and 2% probability of exceedance in 50 year period.

In this work, the nonlinear static pushover analysis is utilized to quantify seismic induced nonlinear response of structures. Among various methods of static pushover analyses, the displacement coefficient method [1] procedure is adopted to evaluate the seismic demands on building frameworks under equivalent static earthquake loading. In this method the structure is pushed with a specific distribution of the lateral loads until the target displacement is reached. The target displacement can be obtained as follows:

$$\delta_i = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{4\pi^2} g \quad (1)$$

where C_0 relates the spectral displacement to the likely building roof displacement; C_1 relates the expected maximum inelastic displacements to the displacements calculated for linear elastic response; C_2 represents the effect of the hysteresis shape on the maximum displacement response and C_3 accounts for P-D effects. T_e is the effective fundamental period of the building in the direction under consideration; S_a is the response spectrum acceleration corresponding to the T_e ; and g is ground acceleration.

In this work, the *OpenSees* [11] platform is utilized to conduct the pushover analyses.

In a sizing structural optimization problem, the aim is usually to minimize the weight of the structure under some behavioral constraints. For a steel structure consisting of ne members that are collected in ng design groups, if the variables associated with each design group are selected from a given profile list of steel sections, a discrete optimization problem can be formulated as follows:

$$\text{Find: } X = \{x_1 \ x_2 \ \dots \ x_i \ \dots \ x_{ng}\}^T \quad (2)$$

$$\text{To minimize: } w(X) = \sum_{i=1}^{ng} \rho_i A_i \sum_{j=1}^{nm} L_j \quad (3)$$

$$\text{Subject to: } g_k(X) \leq 0, \ k = 1, 2, \dots, nc \quad (4)$$

where x_i is an integer value expressing the sequence numbers of steel sections assigned to i th group; w represents the weight of the frame, ρ_i and A_i are weight of unit volume and cross-sectional area of the i th group section, respectively; nm is the number of elements collected in the i th group; L_j is the length of the j th element in the i th group; $g_k(X)$ is the k th behavioral constraint. In the present study, design variables are selected from standard sections found in the AISC design manual.

The strength of structural elements is checked for gravity loads to perform serviceability checks based on AISC [12] design code. If the serviceability checks are not satisfied then the candidate design is rejected, else a nonlinear pushover analysis is conducted in order to evaluate the structural responses at performance levels. In order to implement pushover analysis to evaluate the seismic demands of the structures, the target displacement should be determined. To achieve this task, S_a should be calculated for the three performance levels. In this case three acceleration design spectra, which represent three different earthquake

levels corresponding to 20%, 10%, and 2% probability of exceeding in a 50-year period, are taken as the basis for calculating the seismic loading for the three performance levels IO, LS, and CP, respectively. In the present study, S_a for hazard levels is determined according to Table 1. In this table, F_a and F_v are the site coefficient determined from FEMA-356 [1], based on the site class and the values of the response acceleration parameters S_s and S_1 .

Table 1: Site parameters for site class of D

Performance Level	Hazard Level	S_s (g)	S_1 (g)	F_a	F_v
IO	20% / 50-years	0.658	0.198	1.27	2.00
LS	10% / 50-years	0.794	0.237	1.18	1.92
CP	2% / 50-years	1.150	0.346	1.04	1.70

As the lateral drift constraints, the inter-story drifts of all stories at IO, LS, and CP performance levels are limited to 0.5%, 1.5% and 2.0%, respectively [1]. Furthermore, the axial deformation of bracings at IO, LS, and CP performance levels are limited to $0.25\Delta_C$, $5\Delta_C$, and $7\Delta_C$, respectively for braces in compression in which Δ_C is the axial deformation at expected buckling load and to $0.25\Delta_T$, $7\Delta_T$, and $9\Delta_T$, respectively for braces in tension in which Δ_T is the axial deformation at expected tensile yielding load.

In this study, for modeling nonlinear behavior of beams and columns a simple bilinear stress–strain relationship with 3% kinematic hardening is considered. For modeling braces, uniaxial co-rotational truss element is used.

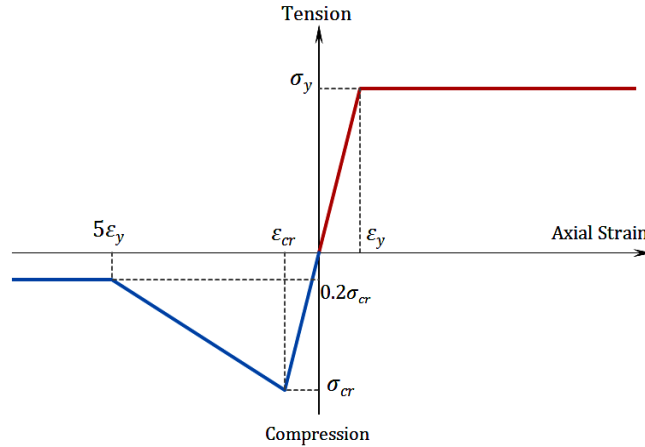


Figure 1. Stress–strain relationship for braces

As shown in Fig. 1 the hardening rule is bi-linear kinematics in tension. In compression, according to FEMA274 [13], it is assumed that the element buckles at its corresponding buckling stress state and its residual stress is about 20% of the buckling stress. In this figure, σ_{cr} and σ_y are buckling and yield stresses, respectively and ϵ_{cr} and ϵ_y are their corresponding strains. Here, the buckling stress of braces is computed based on AISC [12] code.

In this study, the constraints of the optimization problem are handled using the concept of exterior penalty functions method (EPFM). In this case, the pseudo unconstrained objective function, Π , is expressed as follows:

$$\Pi(X, r_p) = w(X) + P(X, r_p) \quad (5)$$

$$P(X, r_p) = r_p \sum_{k=1}^{nc} (\max\{0, g_k\})^2 \quad (6)$$

where P is the penalty function and r_p is positive penalty parameter.

3. ENHANCED COLLIDING BODIES OPTIMIZATION

Kaveh and Mahdavi [14] developed colliding bodies optimization (CBO) algorithm based on one-dimensional collisions between two bodies where they move towards a minimum energy level. CBO is a simple and parameter-free metaheuristic. Kaveh and Ilchi Ghazaan [8] proposed enhanced CBO (ECBO) to improve convergence rate and reliability of CBO by adding a memory to save some of the best solutions during the optimization process and also utilizing a mutation operator to decrease the probability of trapping into local optima. The basic steps of ECBO are summarized as follows [8]:

1. The initial positions of all colliding bodies (CBs) are determined randomly in an m -dimensional search space using Eq. (7).

$$X_i^0 = X_{\min} + R \circ (X_{\max} - X_{\min}), i = 1, 2, \dots, n \quad (7)$$

in which X_i^0 is the initial solution vector of the i th CB. Here, X_{\min} and X_{\max} are respectively the lower and upper bounds of design variables; r is a random vector in the interval $[0, 1]$; n is the number of CBs.

2. The value of mass for each CB is evaluated using Eq. (8).

$$m_i = \frac{1}{F(X_i)} \quad (8)$$

where $F(X_i)$ is the objective function value of the i th CB and.

3. Colliding memory (CM) is utilized to save a number of historically best CB vectors and their related mass and objective function values. Solution vectors which are saved in CM are added to the population and the same numbers of current worst CBs are deleted. Finally, CBs are sorted according to their masses in a decreasing order.
4. CBs are divided into two equal groups:
5. (a) Stationary group; $i_s = 1, 2, \dots, \frac{n}{2}$ and (b) Moving group; $i_M = \frac{n}{2} + 1, \frac{n}{2} + 2, \dots, n$
6. The velocities of stationary and moving bodies before collision are evaluated as follows:

$$V_{i_s} = 0 \quad (9)$$

$$V_{i_M} = X_{i_s} - X_{i_M} \quad (10)$$

7. The velocities of stationary and moving bodies after collision are evaluated as follows:

$$V'_{i_s} = \left(\frac{(1 + \varepsilon) m_{i_M}}{m_{i_s} + m_{i_M}} \right) V_{i_s} \quad (11)$$

$$V'_{i_M} = \left(\frac{(m_{i_M} - \varepsilon m_{i_s})}{m_{i_s} + m_{i_M}} \right) V_{i_M} \quad (12)$$

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \quad (13)$$

where $iter$ and $iter_{\max}$ are the current iteration number and the total number of iteration for optimization process, respectively; ε is the coefficient of restitution (COR).

8. The new position of each CB is calculated as follows:

$$X_{i_s}^{\text{new}} = X_{i_s} + \bar{R}_{i_s} \circ V'_{i_s} \quad (14)$$

$$X_{i_M}^{\text{new}} = X_{i_M} + \bar{R}_{i_M} \circ V'_{i_M} \quad (15)$$

where \bar{R}_{i_s} and \bar{R}_{i_M} are random vectors uniformly distributed in the range of $[-1,1]$.

9. A parameter like pro within $(0, 1)$ is introduced and it is specified whether a component of each CB must be changed or not. For each CB, pro is compared with rni ($i=1, \dots, n$) which is a random number uniformly distributed within $(0, 1)$. If $rni < pro$, one dimension of the i th CB is selected randomly and its value is regenerated in interval $[X_{\min}, X_{\max}]$. In order to protect the structures of CBs, only one dimension is changed.
10. When a stopping criterion is satisfied, the optimization process is terminated.

4. SEQUENTIAL ECBO

In order to increase the probability of finding global or near global solutions in complex optimization problems, such as PBD optimization of SBFs, a computational strategy is proposed in the present study based on ECBO metaheuristic. In order to achieve this purpose, an algorithm based on sequential implementation of ECBO is proposed and therefore the resulted algorithm is termed as sequential ECBO (SECBO). In other words, in the framework of SECBO, the ECBO is applied in a multi-stage fashion to exhaustively search the design space. In the SECBO, the constraints are handled using EPFM in the framework of the sequential unconstrained minimization technique (SUMT) [10]. In the first stage of SECBO, an initial population including n_{CB} colliding bodies (CB) is randomly selected from design space and the ECBO is employed to achieve an optimization process considering a minor value for the penalty parameter, i.e. r_p in Eq. (6). As the value of r_p is small, the algorithm converges to an infeasible solution. In this process the best solution is saved as X_{best} . In the next step, a new population is selected from the neighboring region of the found X_{best} . In this case, X_{best} is directly transformed to the new population and the remaining CBs are randomly selected based on the following equation:

$$X_j = N(X_{\text{best}}, \xi X_{\text{best}}), \quad j = 1, 2, \dots, (n_{cb} - 1) \quad (16)$$

where $N(X_{\text{best}}, \zeta X_{\text{best}})$ represents a random normally distributed vector with the mean X_{best} and the standard deviation ζX_{best} .

According to the SUMT concepts, r_p for the new stage should be increased as follows:

$$r_p^{k+1} = \theta r_p^k \quad (17)$$

where k denotes the optimization process index and θ is a positive constant.

The values of ζ and θ play an important role in convergence behavior of the algorithm and based on the computational experiences of the previous works [15, 16] the best value for this parameter is equal to 0.1 and 10, respectively.

the newly generated population is employed by ECBO to achieve another optimization process and this procedure is repeated for nt times and the best solution found in this manner is reported as the final solution of the algorithm.

5. NUMERICAL EXAMPLES

Two examples including five and ten story SBFs are optimized in the framework of PBD. In these examples, the height of each floor and the length of each span are 3.0 m and 5.0 m, respectively. For beams, columns and bracings the yield stress is 344.7, 344.7 and 317.2 MPa, respectively and the modulus of elasticity and mass density are 200.0 GPa and 76.82 kN/m³, respectively. The dead and live loads of 31.5 kN/m and 9.8 kN/m are respectively applied to the all beams. Moreover, the sections of all members are selected from the available sections listed in Table 2.

Table 2: The available list of standard sections

Beams and Columns						Bracings	
No.	Profile	No.	Profile	No.	Profile	No.	Profile
1	w14×22	16	w14×145	31	w14×550	35	HSS3×3×0.375
2	w14×26	17	w14×159	32	w14×605	36	HSS3-1/2×3-1/2×0.375
3	w14×30	18	w14×176	33	w14×665	37	HSS4×4×0.500
4	w14×34	19	w14×193	34	w14×730	38	HSS4-1/2×4-1/2×0.500
5	w14×38	20	w14×211			39	HSS5×5×0.500
6	w14×43	21	w14×233			40	HSS6×6×0.500
7	w14×48	22	w14×257			41	HSS7×7×0.625
8	w14×53	23	w14×283			42	HSS8×8×0.625
9	w14×61	24	w14×311			43	HSS10×10×0.500
10	w14×68	25	w14×342			44	HSS14×14×0.500
11	w14×74	26	w14×370			45	HSS16×16×0.625
12	w14×82	27	w14×398			46	HSS18×18×0.625
13	w14×109	28	w14×426			47	HSS20×20×0.625
14	w14×120	29	w14×455			48	HSS22×22×0.625
15	w14×132	30	w14×500			49	HSS24×24×0.625

For both examples, the number of CBs is 30 but the maximum number of iterations for first and second examples is 400 and 800, respectively. For SECBO algorithm, 4 stages are considered and in each stage 100 and 200 iterations are carried out for first and second examples, respectively. In addition, r_p^1 is chosen to be 1000.

5.1 Five-story SBF

Two five-bay, five-story SBFs, termed as SBF5-1 and SBF5-2, are respectively depicted in Figs. 2a and 2b together with their element grouping details.

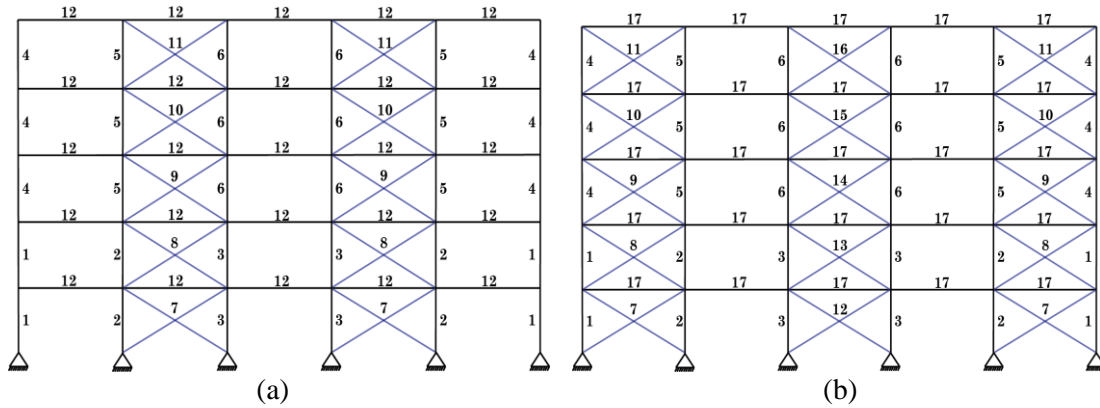


Figure 2. Five-bay, five-story SBFs of (a) SBF5-1 and (b) SBF5-2 and their element groups

For SBF5-1 and SBF5-2 structures 30 independent optimization runs are performed using ECBO and SECBO algorithms and the results are reported in Table 3.

Table 3: The results of 30 independent optimization runs for SBF5-1 and SBF5-2

Metrics	SBF5-1		SBF5-2	
	ECBO	SECBO	ECBO	SECBO
Best	25126	25126	27245	27245
Worst	26209	25935	30693	29569
Mean	25401	25319	28038	27846
Std.	364.79	221.45	1223.20	817.47

The results of PBD optimization of SBF5-1 and SBF5-2 show that for both cases the active constraints are the axial deformations of bracings at IO level. The section numbers of the best solutions found for SBF5-1 and SBF5-2 together with the active constraint values are shown in Figs. 3 and 4, respectively.

The results demonstrate the better computational performance of the proposed SECBO in comparison with the standard ECBO. Both algorithms find the same best solution however, the worst and average structural weights and corresponding standard deviation of SECBO are better than those of the ECBO.

The convergence curves of the best solutions of SBF5-1 and SBF5-2 found by ECBO and SECBO are respectively depicted in Figs. 5 and 6.

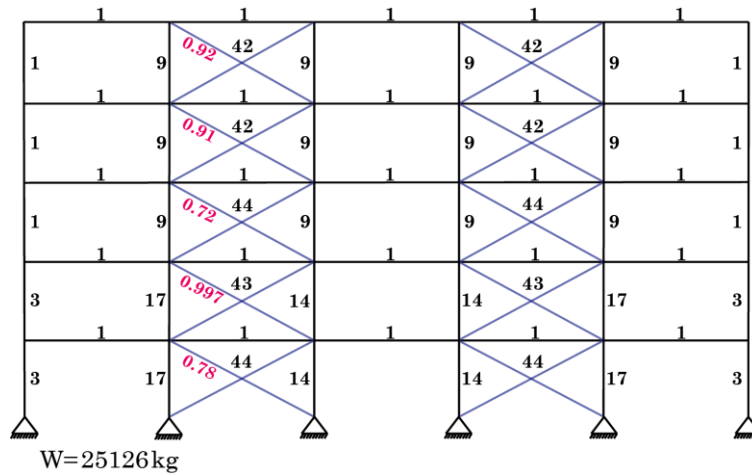


Figure 3. Optimal section numbers and active constraint values for the best design of SBF5-1

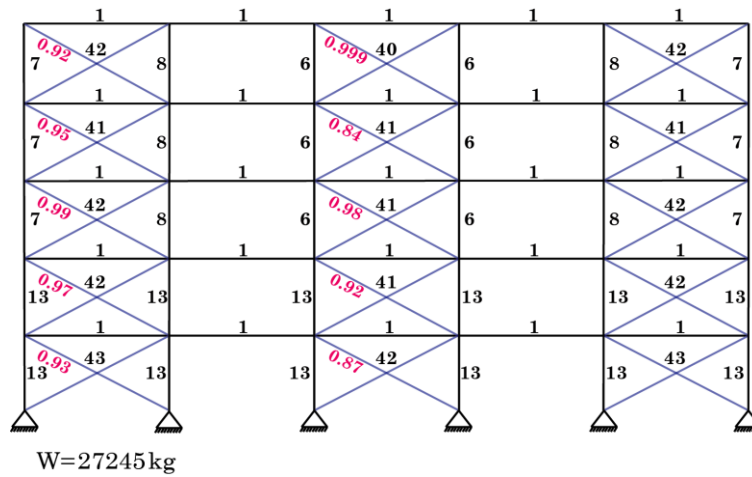


Figure 4. Optimal section numbers and active constraint values for the best design of SBF5-2

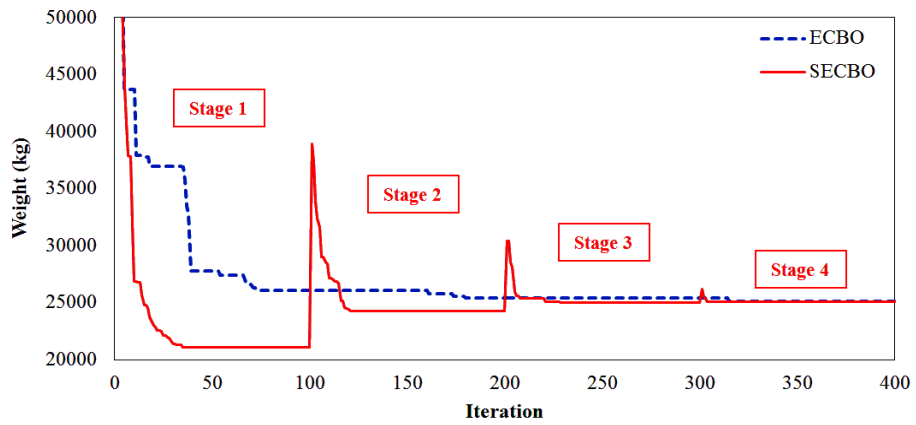


Figure 5. Convergence curve of the best design of SBF5-1 found by ECBO and SECBO

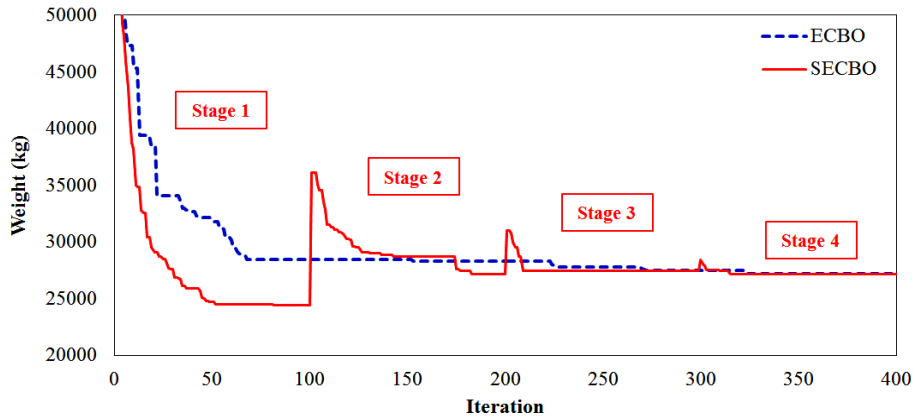


Figure 6. Convergence curve of the best design of SBF5-2 found by ECBO and SECBO

The convergence histories indicate that as in stage 1 the value of r_p^1 is small the SECBO converges to an infeasible solution. In stage 2, by increasing the value of r_p^2 , the algorithm converges to a solution in which the amount of constraints violations is less than that of the stage 1. This improvement is continued in stage 3, and finally an optimal feasible solution is found in stage 4.

5.2 Ten-story SBF

Topology and element groups of SBF10-1 and SBF10-2 as two five-bay, ten-story SBFs are respectively shown in Figs. 7a and 7b.

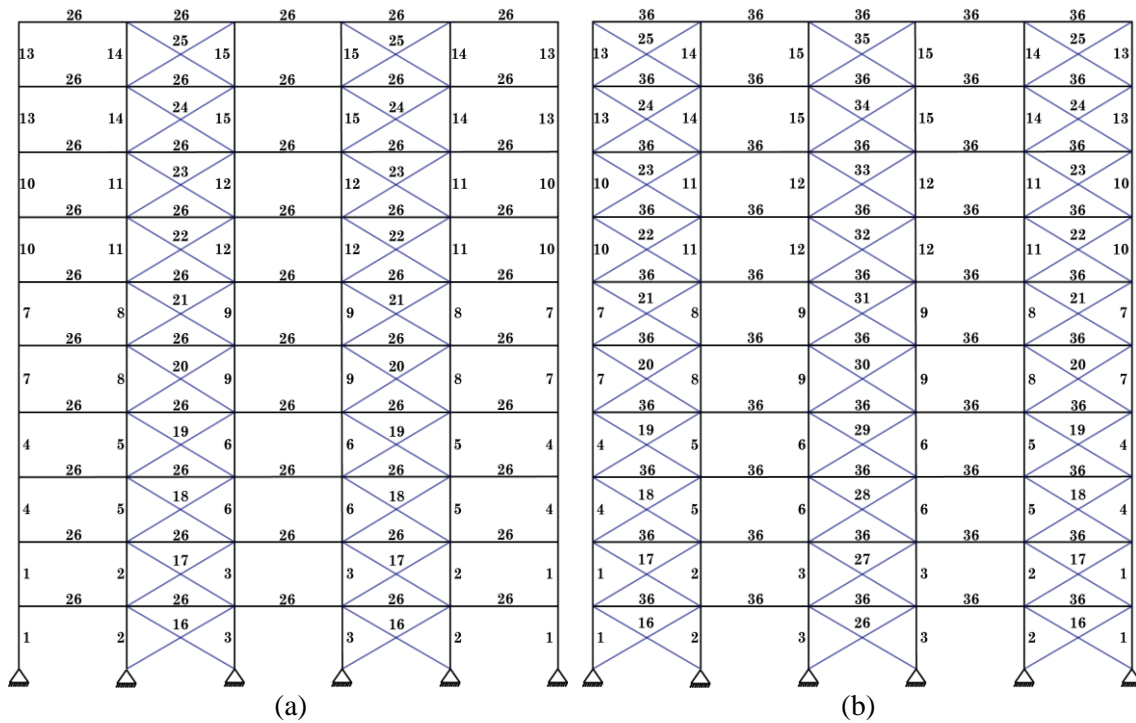


Figure 7. Five-bay, ten-story SBFs of (a) SBF10-1 and (b) SBF10-2 and their element groups

A total number of 30 independent optimization runs are carried out for SBF10-1 and SBF10-2 structures using ECBO and SECBO algorithms and the obtained results are summarized in Table 4.

Table 4: The results of 30 independent optimization runs for SBF10-1 and SBF10-2

Metrics	SBF10-1		SBF10-2	
	ECBO	SECBO	ECBO	SECBO
Best	62719	62355	60603	59897
Worst	71481	66858	72959	71452
Mean	64974	64048	64251	62704
Std.	2591.80	1963.50	3856.70	3658.60

The results of Table 4 indicate that the best, worst and average structural weights and corresponding standard deviation of the solutions found by SECBO are better than those of the ECBO. Therefore, the computational performance of the proposed SECBO is better in comparison with the standard ECBO.

The results of PBD optimization reveal that the axial deformations of bracings at IO level dominate both the optimal designs of SBF10-1 and SBF10-2 structures.

For the best solutions found by SECBO for SBF10-1 and SBF10-2 the numbers of optimal sections and the active constraint values are depicted in Figs. 8 and 9, respectively.

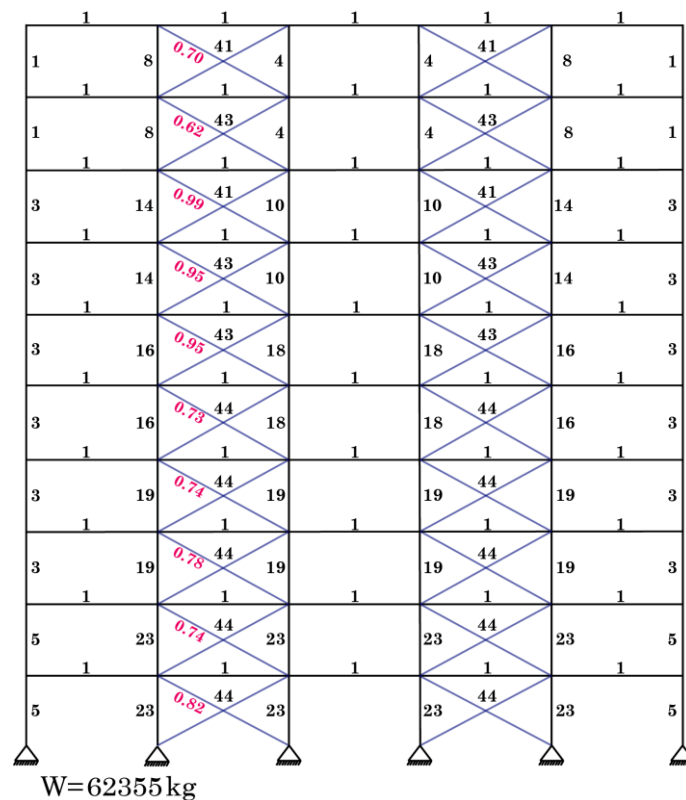


Figure 8. Optimal section numbers and active constraint values for the best design of SBF10-1

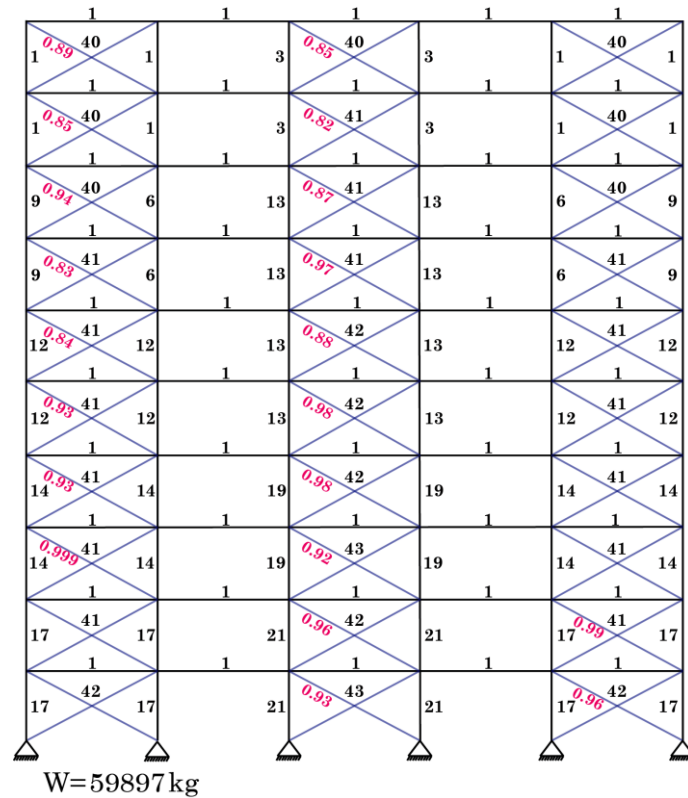


Figure 9. Optimal section numbers and active constraint values for the best design of SBF10-2

Figs. 10 and 11 respectively show the convergence curves of the best solutions of SBF10-1 and SBF10-2 obtained by ECBO and SECBO.

The above convergence curves show that the SECBO finds an infeasible design in stage 1 however in the next stages the algorithm gradually converges to optimal feasible designs.

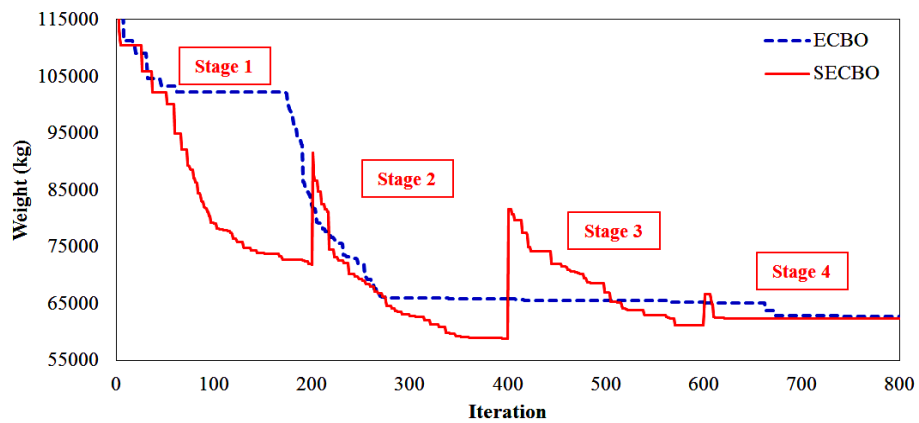


Figure 10. Convergence curve of the best design of SBF10-1 found by ECBO and SECBO

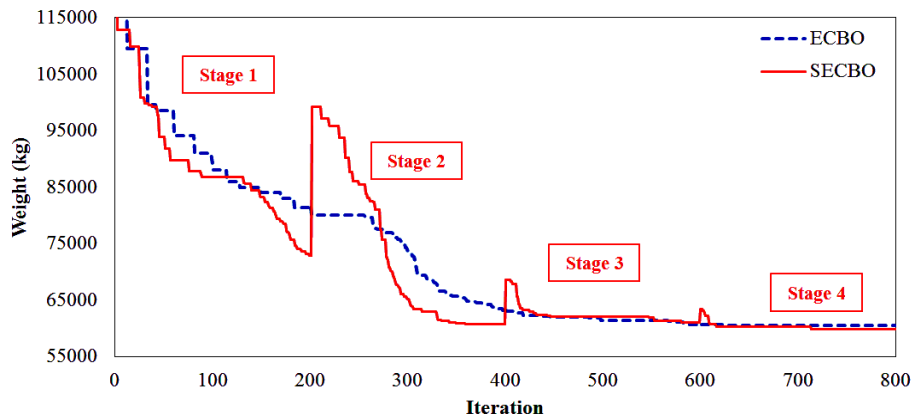


Figure 11. Convergence curve of the best design of SBF10-2 found by ECBO and SECBO

6. CONCLUSION

The present study is devoted to PBD optimization of SBF structures using a sequential version of ECBO metaheuristic algorithm termed here as sequential ECBO (SECBO). The design constraints checked during the optimization process are divided to two groups. As the first group constraints, each structural element is checked to satisfy the AISD constraints for the non-seismic load combinations. As the second group constraints, the check of inter-story drifts and the axial deformation of bracings are achieved at IO, LS and CP performance levels according to the FEMA-356. The discrete design variables of beams, columns and bracings are selected from a list of standard sections. An efficient algorithm based on sequential implementation of ECBO is proposed to deal with the PBD optimization problem. The proposed SECBO algorithm is a multi-stage implementation of ECBO in which the initial population of each stage is generated based on the best solution found in the previous stage. Two numerical examples of five-story and ten-story SBFs are presented and in each example two configurations of bracings are taken into account. For both example, 30 independent optimization runs are performed by employing ECBO and SECBO algorithms and the results are compared. It is observed that in the case of first example and for both configurations of bracings, both ECBO and SECBO algorithms converge to the same best solution however, the worst and average structural weights and corresponding standard deviation of SECBO are better than those of the ECBO. In the second example, the best, worst and average structural weights and corresponding standard deviation of the solutions found by SECBO are better than those of the ECBO. These results demonstrate that the proposed SECBO outperforms ECBO. Therefore, the proposed SECBO can be effectively employed for PBD optimization of SBFs.

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