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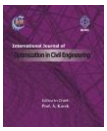

# AN ADAPTIVE IMPORTANCE SAMPLING-BASED ALGORITHM USING THE FIRST-ORDER METHOD FOR STRUCTURAL RELIABILITY

M.A. Shayanfar[*, †], M.A. Barkhordari and M.A. Roudak
*The Centre of Excellence for Fundamental Studies in Structural Engineering, Iran University of Science and Technology, Narmak, Tehran 16846, Iran*

## ABSTRACT

Monte Carlo simulation (MCS) is a useful tool for computation of probability of failure in reliability analysis. However, the large number of samples, often required for acceptable accuracy, makes it time-consuming. Importance sampling is a method on the basis of MCS which has been proposed to reduce the computational time of MCS. In this paper, a new adaptive importance sampling-based algorithm applying the concepts of first-order reliability method (FORM) and using (1) a new simple technique to select an appropriate initial point as the location of design point, (2) a new criterion to update this design point in each iteration and (3) a new sampling density function, is proposed to reduce the number of deterministic analyses. Besides, although this algorithm works with the position of design point, it does not need any extra knowledge and updates this position based on previous generated results. Through illustrative examples, commonly used in the literature to test the performance of new algorithms, it will be shown that the proposed method needs fewer number of limit state function (LSF) evaluations.

**Keywords:** reliability analysis; Monte Carlo simulation; importance sampling; first-order reliability method; design point.

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[*]Corresponding author: The Centre of Excellence for Fundamental Studies in Structural Engineering, Iran University of Science and Technology, Narmak, Tehran 16846, Iran
[†]E-mail address: shayanfar@iust.ac.ir (M.A. Shayanfar)

## 1. INTRODUCTION

In structural reliability analysis different examples of uncertainties are observed in material, loads and geometric properties. In such circumstances and to have a realistic understanding of a structure during its lifetime, uncertainty has to be taken into account. To do so, Structural reliability theory is exploited [1-5]. In the component reliability analysis, the main purpose is to evaluate the probability of failure, $P_f$ , by a multi-dimensional integral on the failure domain as

$$P_f = \int_{g(X)<0} f_X(X)dX \tag{1}$$

where $X = [X_1, X_2, ..., X_n]^T$ represents the vector of random variables and $f_X(X)$ is the joint probability density function (JPDF) of the vector of random variables. $g(X)$ is the LSF such that $g(X) > 0$ defines the safety domain and $g(X) < 0$ defines the failure domain. However, the evaluation of the integration of Eq. (1) is so much difficult and complicated because of involving multiple integral and JPDF of random variables, especially for the large and complex structures with low probability of failure and implicit LSFs. In such a condition, other alternatives such as approximation methods and MCS have attracted more attention [6-14].

Approximation methods such as first- and second-order reliability method (FORM and SORM) are based on first- and second- order approximation of LSF at the design point, respectively. Design point which lies on the limit state surface and has the minimum distance from the origin of standard normal coordinate system (U-space), defined by Hasofer and Lind [15], has been also proved to be the most probable failure point (standard normal coordinate system is reached using the transformation $u_i = (x_i - \mu_i)/\sigma_i$ ) [16]. This minimum distance is called reliability index and is denoted by $\beta$ (it should be noted that LSF in standard normal coordinate system will be denoted by $g_1$, hereafter). Approximation methods can work appropriately in many practical examples but they require a differentiable LSF which is an unattainable condition in many cases.

In simulation methods, such as MCS, random samples are generated based on sampling density function and LSF is evaluated for the sample. Then, probability of failure given by MCS is considered the proportion of the number of samples corresponding to failure, i. e. negative value of LSF, to total sampling number. MCS does not need the mathematical form and derivation of LSF but the large number of simulations, needed to increase the accuracy, has become a big computational burden. That is why many researches have been focused on developing MCS by reducing the number of required random samples to make it more practical. On this way different sampling methods such as Latin Hypercube sampling [17], Antithetic Variates [11,18] and Importance sampling [19] can be named. Since importance sampling has been shown to be effective among different types of sampling, many researches has been focused on improvement of simulation with this technique of sampling.

One useful way to increase the efficiency of importance sampling method is obtained by means of the location of design point in the process of reliability analysis. Many methods with such an approach have been proposed that require the design point or the features of LSF [20-24]. Adaptive importance sampling method is another method which uses design point based on the results of previous deterministic analyses and without any need to the features of LSF and design point [25-27].

In this paper, a new method to improve the effectiveness of MCS has been presented. In this method, which falls into the class of adaptive importance sampling method and uses the concepts of FORM, a simple technique has been proposed to reach an appropriate initial design point from a random point. This initial point is then updated in each iteration, based on a simple proposed criterion. The sampling density function needed to generate random samples is chosen to have the same form as the probability density function (PDF) of random variables, but with different mean and standard deviation in each iteration. The aforementioned technique, criterion and sampling density function as the constituents of the proposed adaptive importance sampling-based algorithm are shown to be effective in improving the performance of MCS through numerical examples.

## 2. IMPORTANCE SAMPLING

The basic MCS computes the value of the multi-dimensional integral of Eq. (1) by generating random samples based on the distribution of random variables. Generating random samples on the whole random variable space without any focus or weight on different regions of the space causes the basic MCS to need a huge number of samples. This makes the basic MCS time-consuming and consequently impractical to many problems. Importance sampling method [12] represents one of the best ideas to decrease the number of random samples and increase the convergence speed. In importance sampling method, the integral of Eq. (1) is rewritten as

$$P_f = \int_{g(X)<0} \frac{f_X(X)}{h_X(X)} h_X(X) dX \tag{2}$$

where $h_X(X)$ is sampling density function. Now random samples are generated based on $h_X(X)$ and then probability of failure is calculated from

$$P_f = \frac{1}{n} \sum_{i=1}^{n} I(X_i) \frac{f_X(X_i)}{h_X(X_i)} \tag{3}$$

where the indicator $I(X_i)$ for safe and failure simulation is defined as

$$I(X_i)=\begin{cases}0 & g(X_i)\geq 0\\ 1 & g(X_i)<0\end{cases} \tag{3}$$

Different choices of $h_X(X)$ can significantly affect the number of required random samples. It has been observed that locating the sampling density function around design point works properly. It was mentioned in the previous section that many methods applying this idea have been proposed, but they need the characteristics of LSF or design point which are both inaccessible in many cases. In such cases, adaptive importance sampling-based methods, which do not need these characteristics and work with knowledge obtained in previous deterministic analyses, can be good choices. In the following section, a new adaptive importance sampling-based algorithm with representation of a simple technique, criterion and sampling density function is proposed.

## 3. PROPOSED ALGORITHM

As was previously mentioned, the proposed importance sampling-based algorithm is made up of (1) a new technique to select an appropriate initial point as the location of initial design point, (2) a new criterion to update the design point in each iteration and (3) a new sampling density function to generate random samples.

### *3.1 A new technique to select an appropriate initial point as the location of initial design point*

Initial design point in simulation methods plays a significant role. In fact, an appropriate initial design point can decrease the number of required random samples. If any random point is selected as the initial design point, it can lead to a lengthy and time-consuming analysis because this point determines the location of sampling density function and directly affects the sample generation procedure. This problem becomes bigger when the initial design point goes further from the LSF [27]. That is why in the proposed algorithm, a specific attention is allocated to the initial design point.

Suppose the random point $P_1$ is the first generated random point (initial random point). If this initial random point is closely located around the LSF, it is accepted as initial design point, otherwise it is moved to $P_2$ which is to be closer to the LSF. This procedure will be continued until the point falls in the neighborhood of LSF with the acceptable distance. In fact, if $|g_1(P_i)|<c$ where $c$ is a predetermined relatively small value, it is considered closely located around LSF and thus selected as the initial design point. However, if $|g_1(P_i)|\geq c$, we move from $P_i$ to $P_{i+1}$ on the line connecting $P_i$ to the origin of coordinate system (Fig. 1). We call this line and the vector connecting the origin of coordinate system, $O$, to $P_i$, direction line and direction vector ($\overrightarrow{OP_i}$), respectively. The represented

explanations mean that the initial point just moves on the direction line (in the positive or negative direction of the direction vector) until the acceptable distance is reached and then this point is selected as the initial design point.

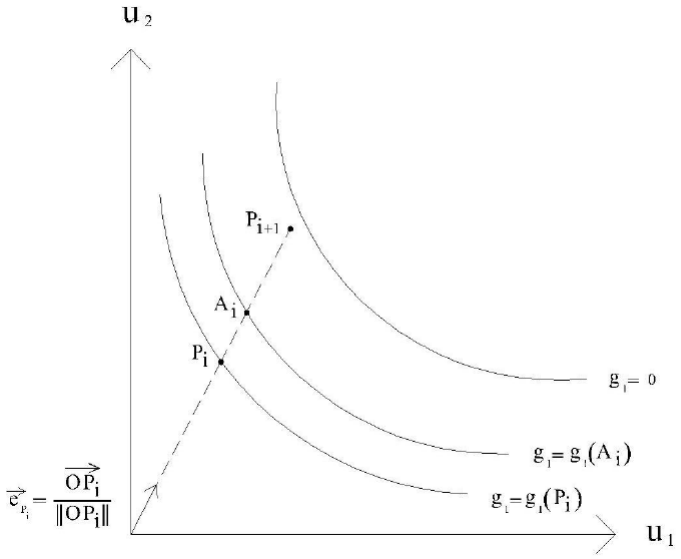


Figure 1. Movement towards limit state surface on the direction line and using the auxiliary point

In order to build a formula by which the next point, $P_{i+1}$, can be obtained from the current point, $P_i$, as it is seen in Fig. 1, we move 1 unit towards positive direction of $\overrightarrow{OP_i}$ to reach the following new point called auxiliary point, $A_i$

$$\overrightarrow{OA_i} = \overrightarrow{OP_i} + 1 \times \frac{\overrightarrow{OP_i}}{\|OP_i\|} \tag{5}$$

where $\|.\|$ denotes norm of the vector. Now by evaluating the function $g_1$ in this point, we can have an understanding of the behavior and rate of change in $g_1$. What we are looking for is the step size $\delta_i$ in

$$\overrightarrow{OP_{i+1}} = \overrightarrow{OP_i} + \delta_i \times \frac{\overrightarrow{OP_i}}{\|OP_i\|} \tag{6}$$

Such that the point $P_{i+1}$ is located on the limit state surface. Since the change of 1 unit in the magnitude of direction vector $\overrightarrow{OP_i}$ results in the change of $g_1(A_i) - g_1(P_i)$, considering

the linear behavior, $\delta_i$ is calculated as follows.

$$\delta_i = \frac{g_1(P_{i+1}) - g_1(P_i)}{g_1(A_i) - g_1(P_i)} \tag{7}$$

Since $P_{i+1}$ is to be located on the limit state surface, i. e. $g_1(P_{i+1}) = 0$, Eq. (7) can be rewritten as

$$\delta_i = \frac{g_1(P_i)}{g_1(P_i) - g_1(A_i)} \tag{8}$$

Eq. (6), the elements of which are calculated from Eqs. (8) and (5), gives the next point $P_{i+1}$ in terms of the current point $P_i$. It should be noted that since the above formulation has been completed by the assumption of linear behavior of the LSF, if it is nonlinear $g_1(P_{i+1})$ will not be zero and therefore the iteration of Eq. (6) should continue until our condition is satisfied and the acceptable distance is reached. This simple technique to bring the initial random point closer to LSF and selecting this point as the initial design point causes the sampling density function to be close to the border of safe and failure region and thus, compared to MCS, more random samples will fall into the failure region and this is one of the reasons the proposed algorithm imposes less computation cost.

### *3.2 A new criterion to update the design point in each iteration*

In the previous part, we discussed that a good initial design point which is closely located around the border of safe and failure region can be a good start. However, since it is just close to this border and is not necessarily close to the real design point, it is not necessarily the best point. Thus, in order to make the algorithm more efficient, the initial design point needs to be updated. One possible criterion to update the design point is to select a point, in the next iterations, with (1) smaller distance to the origin and (2) smaller absolute value of $g_1$ (both compared to the previous design point) as the new design point and not to change the design point if this point does not have these two features [27]. This criterion may work in many problems but it has a big drawback. In fact, if a design point at a specific step of the procedure, falls very close to the LSF such that at this design point $g_1$ is very close to zero, it will be very difficult for other points to replace the previous design point although they are closer to the real design point. In such cases the new point may be closer to the real design point and have the first condition satisfied but since the absolute value of $g_1$ at this point is not smaller than that at the previous design point, although it may be sufficiently small, the second condition is not satisfied and this point fails to become the next design point. Thus, one potentially good design point is missed because of the deficiency and inflexibility of the criterion.

Our suggestion to avoid the above problem and make the criterion more flexible in

acceptance of new potentially good points as design points is to select a point with (1) smaller distance to the origin compared to the previous design point and (2) smaller absolute value of $g_1$ compared to a specific number (we select 1 for this number) as the new design point. As it can be seen, these two conditions are very similar to the previous two conditions. The only difference is in the second condition where the absolute value of $g_1$ is no longer compared to the previous design point. This seemingly small difference lets potentially good candidates, which are sufficiently close and not necessarily closer than the previous design point to the border, be selected. Thus, using our criterion to update the design point, the above mentioned drawback of inflexibility can be removed.

*3.3 A new sampling density function to generate random samples*

After selecting a point as design point (at the beginning or during the analysis), samples should be generated based on sampling density function. We determined the samples to have the same distribution as variables but not necessarily with the same means and standard deviations. At each step of the analysis, we consider the last design point the mean point of the sampling density function. Now, to complete the characteristics of sampling density function and to have a unique function, its standard deviation should be specified, too. Our suggestion for the standard deviation of sampling density function is the following descending linear function

$$\sigma = \left[ \rho_1 + (\rho_n - \rho_1) \frac{i-1}{n-1} \right] \sigma_0 \tag{9}$$

where $n$ is the total number of samples, $i$ is the number of sample and $\sigma_0$ is standard deviation of the random variables. $\rho_1$ and $\rho_n$ are introduced as two coefficients such that $\rho_1 \sigma_0$ and $\rho_n \sigma_0$ are standard deviations of sampling density function at the beginning and end of the analysis, respectively. We determine $\rho_1$ and $\rho_n$ to be $\beta = \min(\beta_i)$ (reliability index until iteration $i$) and 1, respectively. This means for bigger reliability index, standard deviation is selected bigger to cover further space from the mean point. As it is seen, at the end of analysis, standard deviation reduces to $\sigma_0$.

At this spot, all constituents of the proposed algorithm have been prepared. In the following section and through numerical examples, the represented formulation and suggestions are shown to be efficient in reducing the number of required random samples. But before going through numerical examples, the summary of the proposed method based on previous explanations, are expressed as the following step-by-step algorithm:

1. Write the LSF in terms of the basic random variables, i.e. $g(x_1, x_2, ..., x_n) = 0$.
2. Use the transformation $u_i = (x_i - \mu_i)/\sigma_i$ and write the LSF in terms of the variables of standard normal coordinate system, i.e. $g_1(u_1, u_2, ..., u_n) = 0$.
3. Generate a random point ($P_1$) and calculate the value of LSF ($g_1$) and distance to the center of standard normal coordinate system at this point.

4. If $|g_1(P_1)| < c$ (we set $c = 1$ in this paper), $P_1$ is selected as initial design point and if $|g_1(P_1)| \geq c$ use the iterative formula of Eq. (6) until $|g_1(P_i)| < c$. Then $P_i$ is selected as initial design point.
5. Generate a random point according to the random variable PDF with the selected initial design point as its mean point and function of Eq. (9) as its standard deviation.
6. Compute the value of LSF ($g_1$) and distance to the origin of standard normal coordinate system at this point.
7. If the value of LSF is negative, the value $\mathrm{PDF}_{\text{random variables}} / \mathrm{PDF}_{\text{sampling}}$ is added to the failure number ( failure number=failure number+$\mathrm{PDF}_{\text{random variables}} / \mathrm{PDF}_{\text{sampling}}$ ).
8. If the absolute value of LSF is smaller than 1 and the distance to the origin is smaller than this distance for previous points, the new point and its distance to the origin are selected as new design point and new reliability index, respectively.
9. Repeat steps 5 to 8 until all samples are generated.
10. Compute probability of failure by dividing the failure number by the total number of samples.

## 4. NUMERICAL EXAMPLES

In this section, several examples are selected from the literature to demonstrate the strong performance of the proposed method.

**Example 1**

In this example the LSF takes the following form

$$g(x) = x_1 x_2 - 146.14 \tag{10}$$

where $x_1$ and $x_2$ follow normal distribution with statistics $\mu_1 = 78064.4$, $\sigma_1 = 11709.7$, $\mu_2 = 0.0104$ and $\sigma_2 = 0.00156$. The proposed method starts with the random point $z = [0.1841, 0.2652]^T$ with $\beta = 0.3228$ in standard normal coordinate system. Since LSF at this point is too large, i. e. $721.331$, which means the point is not closely located around LSF, the iterative process of Eq. (6) started to work and after $4$ iterations brought the random point to $z = [-3.0789, -4.4349]^T$ where the value of LSF is $0.1278$ and $\beta$ is $5.3988$. Locating the sampling density function at this initial design point, and putting $\rho_1$ of Eq. (9) equal to the reliability index (the minimum distance of all generated points at the spot), the algorithm continued the analysis and after only $175$ iterations probability of failure was calculated $P_f = 1.4691 \times 10^{-7}$, while using MCS, it takes more than $10^9$ samples. This shows the strong performance of the proposed method and also the importance of the location of sampling density function and initial design point.

**Example 2**
The LSF of this example is expressed as

$$g(x) = 0.1(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 2.5 \tag{11}$$

where $x_1$ and $x_2$ both have standard normal distribution. Starting with the initial random point $z = [-0.5083, -0.8911]^T$ with $g_1 = 3.5042$, the proposed algorithm reaches the initial design point $z = [1.2157, 2.1314]^T$ with $g_1 = 0.2171$ after $2$ iterations. By means of this initial design point, the proposed algorithm takes $423$ iterations to give $P_f = 0.0041$, while this number for MCS is $27096$ iterations. This big difference is also a sign of improvement of the proposed algorithm compared to MCS.

**Example 3**
The LSF of this example is defined as follows

$$g(x) = -0.5(x_1 - x_2)^2 - \frac{(x_1 + x_2)}{\sqrt{2}} + 3 \tag{12}$$

where $x_1$ and $x_2$ both follow standard normal distribution. After $3$ iterations, the initial random point $z = [0.1045, -0.9461]^T$ with $g_1 = 3.0432$ was brought to the initial design point $z = [0.3292, -2.981]^T$ with $g_1 = -0.6038$. In the proposed algorithm and using the initial design point to locate the sampling density function, $139$ iterations are required to compute probability of failure $P_f = 0.1057$. However, MCS gives the answer after more iterations, i. e. $942$ iterations.

**Example 4**
The LSF considered in this example has the following form

$$g(x) = 2 - x_2 - 0.1x_1^2 + 0.06x_1^3 \tag{13}$$

with standard normal random variables. In the proposed algorithm after $202$ iterations $P_f = 0.0352$ is reached, while in MCS the number of required iterations is $2734$. This result in the proposed algorithm, is obtained after passing from the initial random point $z = [0.6294, 0.8116]^T$ with $g_1 = 1.1638$ to the initial design point $z = [1.5156, 1.9542]^T$ with $g_1 = 0.025$ by $1$ iteration. Thus, it can be mentioned this is another example of the suitability of our algorithm.

**Example 5**
Consider the following LSF

$$g(x) = 2.5 - 0.2357(x_1 - x_2) + 0.00463(x_1 + x_2 - 20)^4 \quad (14)$$

where $x_1$ and $x_2$ both have normal distribution with the same statistics $\mu = 10$ and $\sigma = 3$. In the proposed algorithm 1633 iterations are required to compute $P_f = 0.0028$. This number compared to the corresponding number in MCS, i. e. 36835, shows the power of the proposed algorithm in reaching the answer. 1 iteration, to bring the initial random point $z = [-0.8403, 0.7059]^T$ with $g_1 = 3.5935$ to the initial design point $z = [1.9174, -1.6106]^T$ with $g_1 = 0.0086$ was a huge contribution in creating this difference.

**Example 6**
The LSF of this example including 6 random variables has the following form

$$g(x) = x_1 + 2x_2 + 2x_3 + x_4 - 5x_5 - 5x_6 + 0.001\sum_{i=1}^{6}\sin(100x_i) \quad (15)$$

where $x_1$ to $x_6$ are random variables with statistics as shown in Table 1. The proposed algorithm started the reliability analysis of the above LSF with the initial random point $z = [-0.7091, 0.2993, -0.6125, 0.8005, 0.9606, 0.0791]^T$ with $g_1 = 186.7901$ and after 1 iteration could bring this point closer to limit state surface and give the initial design point $z = [-2.3011, 0.9712, -1.9875, 2.5973, 3.117, 0.2567]^T$ with $g_1 = -0.0061$. This movement towards the limit state surface and getting closer to this surface caused the proposed algorithm to compute the probability of failure $P_f = 0.0127$ after 3402 iterations, while MCS uses 7655 iterations to give the answer.

Table 1: Statistics of random variables in example 6

| Variable | Mean | Standard deviation | Distribution |
|---|---|---|---|
| $x_1$ | 120 | 12 | Lognormal |
| $x_2$ | 120 | 12 | Lognormal |
| $x_3$ | 120 | 12 | Lognormal |
| $x_4$ | 120 | 12 | Lognormal |
| $x_5$ | 50 | 15 | Lognormal |
| $x_6$ | 40 | 12 | Lognormal |

**Example 7**
The LSF of this example is defined as

$$g(x) = 2 + 0.015\sum_{i=1}^{9} x_i^2 - x_{10} \tag{16}$$

where $x_1$ to $x_{10}$ have standard normal distribution. After 1 iteration, the initial random point $z = [0.6294, 0.8116, -0.746, 0.8268, 0.2647, -0.8049, -0.443, 0.0938, 0.915, 0.9298]^T$ with $g_1 = 1.131$ changed to

$z = [1.5414, 1.9874, -1.8268, 2.0245, 0.6482, -1.9711, -1.0848, 0.2296, 2.2407, 2.2768]^T$

as the initial design point with $g_1 = 0.0879$. This helped the proposed algorithm to reach $P_f = 0.0053$ using 5013 iterations, instead of 17830 iterations used in MCS.

**Example 8**

Consider the conical structure of Fig. 2 subjected to a compressive axial load $P$ and a bending moment $M$ [28].

The geometrical and mechanical features of the structure are presented in Table 2 and are defined as independent normal variables. The main phenomena that can involve the failure of the structure are (1) the loss of strength of the structure and (2) the buckling of the structure due to instability. For the loss of strength, according to the large margin obtained in the analysis, this failure mode shall not be analyzed. It is only the buckling of the structure that will be analyzed under the combined solicitations. According to NASA space vehicles design rules [28] the buckling criterion is

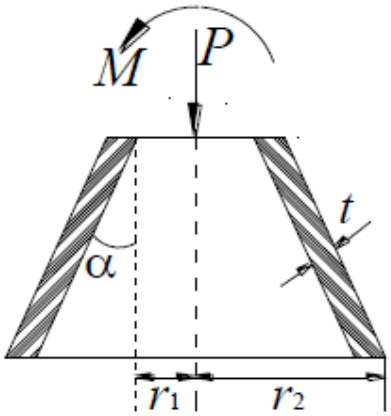


Figure 1. The conical structure of Example 8

$$\frac{P}{P_{cr}} + \frac{M}{M_{cr}} \geq 1 \tag{17}$$

where $P_{cr}$ and $M_{cr}$ are critical axial load and bending moment (for buckling), respectively.

According to NASA [28].

$$P_{cr} = \gamma \frac{2\pi E t^2 \cos^2 \alpha}{\sqrt{3(1-\mu^2)}} \tag{18}$$

$$M_{cr} = \eta \frac{\pi E t^2 r_1 \cos^2 \alpha}{\sqrt{3(1-\mu^2)}} \tag{19}$$

where $\gamma$ and $\eta$ enable to correlate theoretical results with experimental ones ($\gamma = 0.33$, $\eta = 0.41$ according to NASA [28]). According to Eqs. (17)-(19), the LSF of the structure is

$$g(x) = 1 - \frac{\sqrt{3(1-\mu^2)}}{\pi E t^2 \cos^2 \alpha}\left(\frac{P}{2\gamma} + \frac{M}{\eta r_1}\right) \tag{20}$$

MCS needs more than $10^7$ iterations to compute $P_f = 5.232 \times 10^{-7}$, while the proposed algorithm uses $5785$ iterations to give the answer. This is done by passing from the initial random point $z = [0.1376, -0.0612, -0.9762, -0.3258, -0.6756, 0.5886]^T$ with $g_1 = -5.0337 \times 10^5$ to the initial design point $z = [4.9157, -2.1863, -34.8624, -11.6335, -24.1287, 21.0193]^T$ with $g_1 = 0.0254$ after 4 iterations.

Thus, this is another example showing the strength of the proposed algorithm in complicated examples.

Table 2: Statistics of random variables in example 8

| Variable | Mean | Standard deviation | Distribution |
|---|---|---|---|
| $E$ (MPa) | 70000 | 3500 | Normal |
| $t$ (m) | 0.0025 | 0.000125 | Normal |
| $\alpha$ (rd) | 0.524 | 0.01048 | Normal |
| $r_1$ (m) | 0.9 | 0.0225 | Normal |
| $M$ (Nm) | 80000 | 6400 | Normal |
| $P$ (N) | 70000 | 5600 | Normal |

The results of these examples are summarized in Table 3. These results indicate that the proposed algorithm is very efficient and therefore it can be used instead of MCS. As it can be seen in Table 3, the number of required iterations to find probability of failure is smaller than that in MCS. Last column of the table shows a huge reduction in deterministic analyses as a huge success of the proposed algorithm in making the simulation methods more practical. This is generally because the initial point is more appropriately selected and the concentration of sampling is on more important regions, i. e closer to limit state surface and

design point. On this way, the represented technique, criterion and sampling density function of this paper have given a significant contribution to make the proposed algorithm more efficient.

Table 3: Comparison of results in examples 1 to 8

| Example | Probability of failure | Iterations | | |
|---|---|---|---|---|
| | | MCS | Proposed algorithm | Difference |
| 1 | $1.4691\times10^{-7}$ | $>10^{9}$ | 175 | $>10^{9}$ |
| 2 | 0.0041 | 27096 | 423 | 26673 |
| 3 | 0.1057 | 942 | 139 | 803 |
| 4 | 0.0352 | 2734 | 202 | 2532 |
| 5 | 0.0028 | 36835 | 1633 | 35202 |
| 6 | 0.0127 | 7655 | 3402 | 4253 |
| 7 | 0.0053 | 17830 | 5013 | 12817 |
| 8 | $5.232\times10^{-7}$ | $>10^{7}$ | 5785 | $>10^{7}$ |

## 5. CONCLUSION

In this paper, a new reliability algorithm, utilizing the concepts of importance sampling and FORM, has been proposed. In this algorithm, the computational cost is not large and there is no need to derivative of limit state function. This is because of adding a technique, a criterion and a sampling density function which all try to generate samples around the border of safe and failure region. These properties have led to the aforementioned reliability algorithm the power of which in considerable reduction of the number of required samples has been shown in illustrative numerical examples.

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