A CELLULAR AUTOMATA BASED FIREFLY ALGORITHM FOR LAYOUT OPTIMIZAION OF TRUSS STRUCTURES

R. Kamyab Moghadas[1] and S. Gholizadeh[*, †,2]
[1]The Academic Center for Education, Culture and Research, Kerman, Iran
[2]Departement of Civil Engineering, Urmia University, Urmia, Iran

ABSTRACT

In this study an efficient meta-heuristic is proposed for layout optimization of truss structures by combining cellular automata (CA) and firefly algorithm (FA). In the proposed meta-heuristic, called here as cellular automata firefly algorithm (CAFA), a new equation is presented for position updating of fireflies based on the concept of CA. Two benchmark examples of truss structures are presented to illustrate the efficiency of the proposed algorithm. Numerical results reveal that the proposed algorithm is a powerful optimization technique with improved convergence rate in comparison with other existing algorithms.

KEYWORDS: layout optimization; firefly algorithm; cellular automata; truss structure

1. INTRODUCTION

Structural optimization is a critical activity that has received considerable attention in the last four decades. Usually, structural optimization problems involve searching for the minimum of the structural weight. This minimum weight design is subjected to various constraints on performance measures, such as stresses and displacements, and also restricted by practical minimum cross-sectional areas or dimensions of the structural members or components. Due to considering these constraints the possibility of trapping in the local optima will be larger. Optimum layout design of structures is one of the challenging research areas of the structural optimization field. In this class of optimization problems two types of design variables with different natures, including sizing and geometric variables, are involved. The layout optimization problem has been identified as a more difficult but more

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[*] Corresponding author: Department of Civil Engineering, Urmia University, Urmia, Iran
[†] E-mail address: s.gholizadeh@urmia.ac.ir (S. Gholizadeh)

important task than pure sizing optimization, since potential savings in material can be far better improved than by the latter. Most of the engineering optimization algorithms are based on numerical linear and nonlinear programming methods that require substantial gradient information and usually seek to improve the solution in the neighborhood of a starting point. These algorithms, however, reveal a limited approach to complicated real-world optimization problems. If there is more than one local optimum in the problem, the result may depend on the selection of an initial point, and the obtained optimal solution may not necessarily be the global optimum [1].

In the last years, structural optimization has been studied by using different natural phenomena based meta-heuristic algorithms. One of the popular meta-heuristics is firefly algorithm (FA) [2]. The FA is an optimization technique inspired by social behavior of fireflies and the phenomenon of bioluminescent communication. The FA needs much iteration to converge to a good solution and this increases the computational burden of the optimization process. On the other hand, cellular automata (CA) [3] represents simple mathematical idealizations of physical systems in which space and time are discrete and physical quantities are taken from a finite set of discrete values. Models based on CA provide an alternative and more general approach to physical modeling rather than an approximation [4]. In the present study, a computational strategy based on the concept of CA is proposed to modify the computational performance of the FA for layout optimization of truss structures. During the recent years, many researchers have tackled the layout (shape) optimization problem of truss structures. Kaveh and Shahrouzi [5] proposed a modified genetic algorithm (GA) for simultaneous size and shape optimization of structures. Gholizadeh [6] proposed a CA-based particle swarm optimization (PSO) algorithm for layout optimization of truss structures. Kaveh and Ahmadi [7] combined a supervised charged system search meta-heuristic with force method for implementing sizing, geometry and topology optimization of trusses structures. Kaveh and Mahdavi [8] employed colliding bodies optimization (CBO) for size and topology optimization of truss structures. In this study, the so called cellular automata firefly algorithm (CAFA) is proposed for solving the layout optimization of truss structures. In the proposed CAFA meta-heuristic, the fireflies are distributed on a small dimensioned grid and the artificial evolution is evolved by a new position updating equation. In the position updating equation instead of the more attractive firefly in the swarm, the more attractive firefly in the close neighborhood of each firefly is employed to update the position of fireflies. This new equation increases the exploitation ability of the algorithm and prevents quick convergence to local optima in comparison with its standard version.

Two benchmark layout optimization problems of trusses are solved by the proposed CAFA meta-heuristic. The numerical results indicate that the computational performance of the proposed algorithm is better than that of the FA and some other meta-heuristics.

2. OPTIMIZATION PROBLEM

The mathematical formulation of structural optimization problems toward the design variables, the objective and constraint functions depend on the type of the application.

However, all optimization problems can be expressed in standard mathematical terms, which in general form can be stated as follows:

$$\begin{aligned} \text{Minimize} \quad & f(X) \\ \text{Subject to} \quad & g_i(X) \leq 0 \ , \quad i = 1, \ldots, m \\ & X_j^l \leq X_j \leq X_j^u \ , \quad j = 1, \ldots, n \end{aligned} \tag{1}$$

where, X is the vector of design variables including size and layout (shape) variables; $F(X)$ is the objective function to be minimized; $g_i(X)$ is the ith behavioral constraints; X_j^l and X_j^u are the lower and the upper bounds on a typical design variable X_j.

In this study, to transform the constrained structural optimization problem into an unconstrained one the exterior penalty function method (EPFM) is employed. The EPFM transforms the basic optimization problem into an alternative formulation such that numerical solutions are sought by solving a sequence of unconstrained minimization problems. The above mentioned constrained optimization problem can be converted into an unconstrained problem by constructing a function of the following form:

$$\Phi(X, r_p) = f(X) + r_p \sum_{i=1}^{m} [\max\{0, g_i(X)\}]^2 \tag{2}$$

where Φ, and r_p are the pseudo objective function, and positive penalty parameter, respectively.

3. FIREFLY ALGORITHM

The FA is a population-based meta-heuristic optimization algorithm inspired by the flashing behavior of fireflies. Fireflies communicate, search for pray and find mates using bioluminescence with varied flashing patterns [9]. In order to develop the firefly algorithm, natural flashing characteristics of fireflies have been idealized using the following three rules [2]:

a. All of the fireflies are unisex; therefore, one firefly will be attracted to other fireflies regardless of their sex.
b. Attractiveness of each firefly is proportional to its brightness, thus for any two flashing fireflies, the less bright firefly will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
c. The brightness of a firefly is determined according to the nature of the objective function.

The attractiveness of a firefly is determined by its brightness or light intensity which is obtained from the objective function of the optimization problem. However, the attractiveness β, which is related to the judgment of the beholder, varies with the distance between two fireflies. The attractiveness β can be defined by [9]:

$$\beta = \beta_0 e^{-\gamma \cdot r^2} \tag{3}$$

where r is the distance of two fireflies, β_0 is the attractiveness at $r = 0$, and γ is the light absorption coefficient.

The distance between two fireflies i and j at X_i and X_j respectively, is determined using the following equation:

$$r_{ij} = \left\| X_i - X_j \right\| = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \tag{4}$$

where $x_{i,k}$ is the k-th parameter of the spatial coordinate x_i of the i-th firefly.

In the firefly algorithm, the movement of a firefly i towards a more attractive (brighter) firefly j is determined by the following equation [9]:

$$X_i = X_i + \beta_0 e^{-\gamma \cdot r_{ij}^2} (X_j - X_i) + \alpha(\text{rand} - 0.5) \tag{5}$$

where the second term is related to the attraction, while the third term is randomization with α being the randomization parameter. Also rand is a random number generator uniformly distributed in [0, 1].

In this paper, α is changed dynamically according to the following equation:

$$\alpha = \alpha_{max} - \frac{\alpha_{max} - \alpha_{min}}{t_{max}} \cdot t \tag{6}$$

where $\alpha_{max}=1$ and $\alpha_{min}=0.2$. Also, t_{max} and t are the numbers of maximum iterations and present iteration, respectively. It should be noted that, various values are examined for α_{max} and α_{min} and the best results are obtained in the case of reported values.

4. CELLULAR AUTOMATA

Cellular automata (CA) were firstly introduced by Von Neumann [3] and subsequently developed by other researchers in many fields of science. Basically, CA represents simple mathematical idealizations of physical systems in which space and time are discrete, and physical quantities are taken from a finite set of discrete values. Models based on CA provide an alternative and more general approach to physical modeling rather than an approximation. The CA shows a complex behavior analogous to that associated with complex differential equations, but in this case complexity emerges from the interaction of simple entities following simple rules [10].

In its basic form, a cellular automaton consists of a regular uniform grid of sites or cells with a discrete variable in each cell which can take on a finite number of states. The state of the cellular automaton is then completely specified by the values $s_i = s_i(t)$ of the variables at each cell i. During time, cellular automata evolve in discrete time steps according to a

parallel state transition determined by a set of local rules: the variables $s_i^{k+1} = s_i(t_{k+1})$ at each site i at time t_{k+1} are updated synchronously based on the values of the variables $s_{n_c}^k$ in their n_c neighborhood at the preceding time instant t_k. The neighborhood n_c of a cell i is typically taken to be the cell itself and a set of adjacent cells within a given radius r; $i-r \leq n_c \leq i+r$. Thus, the dynamics of a cellular automaton can be formally represented as follows [11]:

$$s_i^{k+1} = \theta(s_i^k, s_{n_c}^k) , \ \ i-r \leq n_c \leq i+r \tag{7}$$

where the function θ is the evolutionary rule of the automaton.

One of the most important features of CA is the neighborhood structure. For updating the value of a cell, its own value and the values of neighboring cells should be considered. Configuration of the neighborhood structure is highly problem dependent and depends on the nature of the physical phenomenon that should be modeled. Clearly, a proper choice of the neighborhood plays a crucial role in determining the effectiveness of such a rule. In this paper, the widely used Moore neighborhood of interaction [10], by $r=1$, is adopted as shown in Fig. 1. In this figure, the Moore neighborhood of the central cell is shown by gray region.
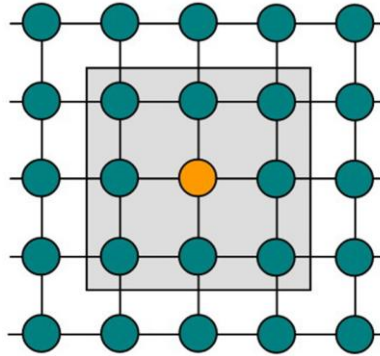

Figure 1. Moore neighborhood [6]

5. CELLULAR AUTOMATA BASED FIREFLY ALGORITHM

In the present paper, CA is employed to modify the performance of the FA for optimal layout design of truss structures. In the so CAFA meta-heuristic, fireflies are distributed on discrete locations of a 2D rectangular grid. The state variables associated with each cell site are simply the design variables of the optimization problem. In the traditional FA, position of fireflies in the search space is updated by applying Eq. (5). In the CAFA meta-heuristic, the updating process is accomplished based on a rule of the automaton. In this case, in the search process, the local information of the Moor neighborhood of each central site is used to efficiently update its position in the design space. When the swarm of fireflies is updated, the evolutionary rules of the automaton are repeated until one of the stopping criteria is met. In the CAFA meta-heuristic, the objective function of the optimization problem is employed to define the fitness of each design vector.

In the proposed CAFA meta-heuristic, a swarm of fireflies is structured on a 2D grid. In this case, each site contains a real-valued vector describing of a design and therefore the state of the cellular automaton in each site is a design vector of design variables as follows:

$$s_i \to X_i = \{x_1, x_2 \dots x_n\}_i^T, i = 1,2,...,n_c \qquad (8)$$

The proposed cellular position updating equation acts on the design variables and employs the information available at the central site and its immediate neighbors as follows:

$$X_i = X_i + \beta_0 e^{-\gamma . r_{ij}^2} (\frac{1}{n_c} \sum_{j=1}^{n_c} (X_{i,j}) - X_i) + \alpha(\text{rand} - 0.5) \qquad (9)$$

where $X_{i,j}$ is the jth firefly in immediate neighbors of the ith central cell.

In each iteration or in each discrete time step, the proposed equation produces a new design at each site.

The values of algorithmic parameters can seriously affect the performance of the CAFA meta-heuristic. A sensitivity analysis is performed and the results reveal that the best values of the parameters are as follows: $\alpha_{max}=1.5$, $\alpha_{min}=0.0$, and $\gamma=0.05$.

6. NUMERICAL EXAMPLES

In order to investigate the computational performance of the proposed CAFA meta-heuristic, two examples are presented. For all examples, the swarm size and maximum number of iterations are 20 and 300, respectively. All of the required computer programs are coded in MATLAB [11] platform.

6.1 15-bar Truss

This problem has been investigated by Wu and Chow [12], Hwang and He [13], Tang et al. [14] and Rahami and Kaveh [15]. The fifteen-bar 2D truss is shown in Fig. 2. The magnitude of the vertical load is P=10 kips. The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi.
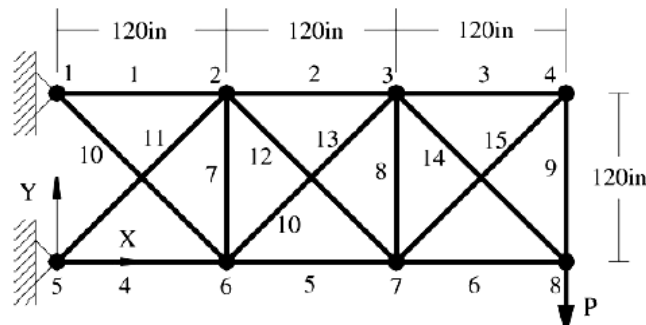

Figure 2. Fifteen-bar truss

In this example there are 23 design variables including two categories: Sizing variables: A_i, $i=1,2,\ldots, 15$ and Geometry variables: $x_2 = x_6$; $x_3 = x_7$; y_2; y_3; y_4; y_6; y_7; y_8. Stress limitation for all elements is $\pm 25\,\text{ksi}$.

The size variables are selected from the following set:

D = { 0.111, 0.141, 0.174, 0.220, 0.270, 0.287, 0.347, 0.440, 0.539, 0.954, 1.081, 1.174, 1.333, 1.488, 1.764, 2.142, 2.697, 2.800, 3.131, 3.565, 3.813, 4.805, 5.952, 6.572, 7.192, 8.525, 9.300, 10.850, 13.330, 14.290, 17.170, 19.180} (in.^{2}). Also side constraints for geometry variables are as follows:

100 in. $\leq x_2 \leq$ 140 in.; 220 in. $\leq x_3 \leq$ 260 in.; 100 in. $\leq y_2 \leq$ 140 in.; 100 in. $\leq y_3 \leq$ 140 in.; 50 in. $\leq y_4 \leq$ 90 in.; 20 in. $\leq y_6 \leq$ 20 in.; -20 in. $\leq y_7 \leq$ 20 in.; 20 in. $\leq y_8 \leq$ 60 in.;

In order to investigate the efficiency of the proposed CAFA meta-heuristic, 20 independent optimization runs are achieved and the best, worst and mean weights of 73.214 lb, 82.148 lb and 77.634 lb are obtained. The best results obtained in this study are compared with those of the other works in Table 1.

Table 1: Optimal layout designs of 15-bar planner truss

Design variables	Wu and Chow [12]		Hwang and He [13]	Tang et al. [14]	Rahami et al. [15]	Present work
					FA	CAFA
A_1	1.174	0.954	1.081	1.081	1.081	0.954
A_2	0.954	1.081	0.539	0.539	0.539	0.539
A_3	0.440	0.440	0.287	0.287	0.141	0.287
A_4	1.333	1.174	0.954	0.954	0.954	0.954
A_5	0.954	1.488	0.954	0.539	0.539	0.539
A_6	0.174	0.270	0.220	0.141	0.347	0.141
A_7	0.440	0.270	0.111	0.111	0.111	0.111
A_8	0.440	0.347	0.111	0.111	0.111	0.111
A_9	1.081	0.220	0.287	0.539	0.141	0.287
A_{10}	1.333	0.440	0.220	0.440	0.220	0.347
A_{11}	0.174	0.220	0.440	0.539	0.539	0.347
A_{12}	0.174	0.440	0.440	0.270	0.287	0.270
A_{13}	0.347	0.347	0.111	0.220	0.270	0.270
A_{14}	0.347	0.270	0.220	0.141	0.270	0.141
A_{15}	0.440	0.220	0.347	0.287	0.220	0.270
x_2	123.189	118.346	133.612	101.5775	126.716	130.221
x_3	231.595	225.209	234.752	227.9112	251.205	255.884
y_2	107.189	119.046	100.449	134.7986	134.609	126.885
y_3	119.175	105.086	104.738	128.2206	123.033	119.339
y_4	60.462	63.375	73.762	54.8630	67.7807	58.9431
y_6	16.728	−20.0	−10.067	−16.4484	3.9830	-3.3529
y_7	15.565	−20.0	−1.339	−16.4484	-1.5743	3.1871
y_8	36.645	57.722	50.402	54.8572	58.9282	59.0057
Weight (lb)	120.52	104.573	79.820	76.6854	78.275	73.214
Analyses	-	-	8000	8000	6000	6000

The geometry of the optimum structure is shown in Fig. 3.
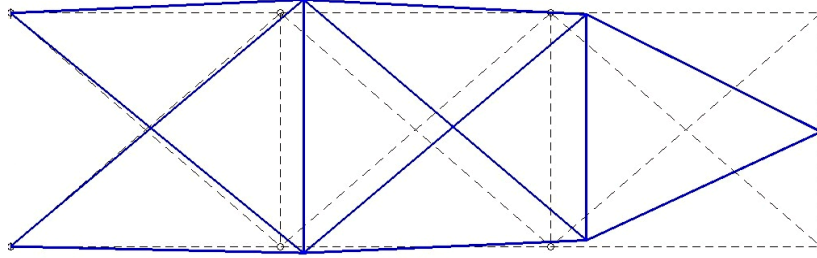

Figure 3. Optimum layout of fifteen-bar planar truss

The results presented in Table 1 indicate that solution found by CAFA meta-heuristic is 6.46%, 4.53%, 8.28%, 29.99% and 39.25% lighter than those of found by FA, Rahami et al. [15], Tang et al. [14], Hwang and He [13], and Wu and Chow [12], respectively all at lower computational cost. This means that the proposed CAFA meta-heuristic has a better convergence behavior in comparison with the other algorithms reported in literature.

4.2 25-bar truss

This problem has been investigated by Wu and Chow [12], Tang et al. [14] and Rahami et al. [15]. The twenty five-bar truss is considered as shown in Fig. 4.
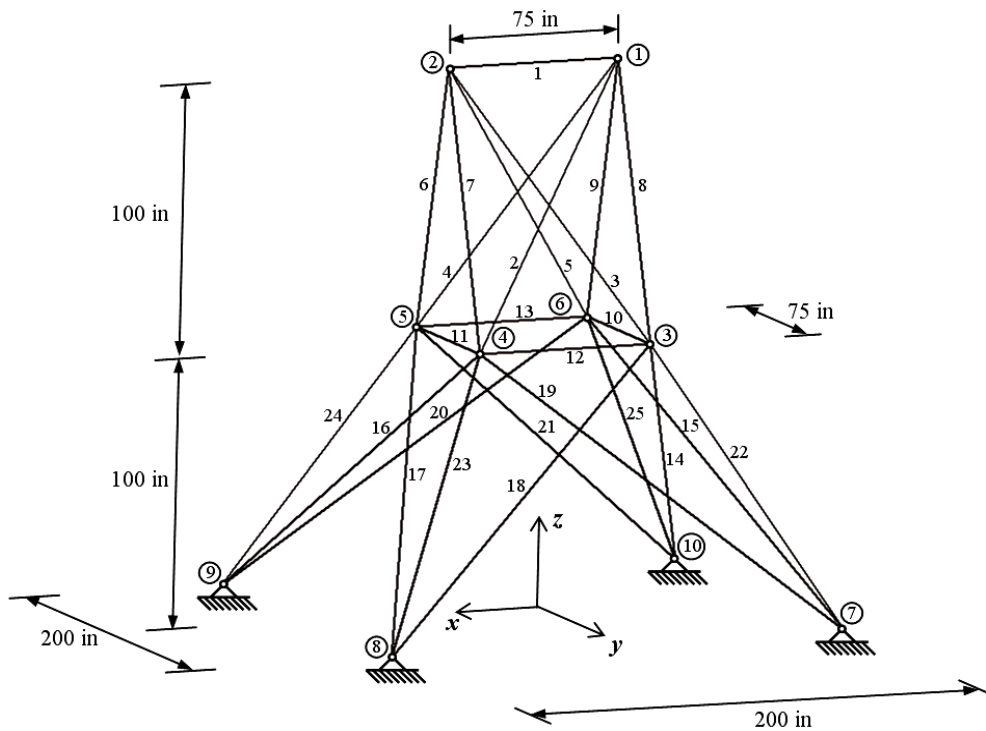

Figure 4. Twenty five-bar space truss

The material density is 0.1 lb/in^3 and the modulus of elasticity is 10^4 ksi. Loading data is given in Table 2.

Table 2: Loading data for twenty five-bar truss

Node	F_x (kips)	F_y (kips)	F_z (kips)
1	1.0	-10.0	-10.0
2	0.0	-10.0	-10.0
3	0.5	0.0	0.0
6	0.6	0.0	0.0

There are 13 design variables including two categories as follows:
Size variables: A_1; $A_2 = A_3 = A_4 = A_5$; $A_6 = A_7 = A_8 = A_9$; $A_{10} = A_{11}$; $A_{12} = A_{13}$; $A_{14} = A_{15} = A_{16} = A_{17}$; $A_{18} = A_{19} = A_{20} = A_{21}$; $A_{22} = A_{23} = A_{24} = A_{25}$
Geometry variables: $x_4 = x_5 = -x_3 = -x_6$; $x_8 = x_9 = -x_7 = -x_{10}$; $y_3 = y_4 = -y_5 = -y_6$; $y_7 = y_8 = -y_9 = -y_{10}$; $z_3 = z_4 = z_5 = z_6$
Stress limitation for elements is ± 40 ksi and displacement constraint is 0.35 in. The size variables are selected from the following set: D={0.1, 0.2, ... , 2.6, 2.8, 3.0, 3.2, 3.4} (in.^{2}).
Also side constraints for geometry variables are as follows:
20 in. $\leq x_4 \leq 60$ in.; 40 in. $\leq x_8 \leq 80$ in.; 40 in. $\leq y_4 \leq 80$ in.; 100 in. $\leq y_8 \leq 140$ in.;
90 in. $\leq z_4 \leq 130$ in.;
In this example, 20 independent optimization runs are implemented by the proposed CAFA meta-heuristic and the results indicate that the best weight of 117.40 lb, the worst weight of 132.94 lb and the mean weight of 118.93 lb are obtained. The best results obtained in this study are compared with those of the others in Table 3.

Table 3: Optimal layout designs of 25-bar space truss

Design variables	Wu and Chow [12]	Tang et al. [14]	Rahami et al. [15]	Present work	
				FA	CAFA
A_1	0.1	0.1	0.1	0.1	0.1
A_2	0.2	0.1	0.1	0.1	0.1
A_3	0.1	1.1	1.1	1.1	0.9
A_4	0.2	0.1	0.1	0.1	0.1
A_5	0.3	0.1	0.1	0.1	0.1
A_6	0.1	0.2	0.1	0.1	0.1
A_7	0.2	0.2	0.2	0.2	0.1
A_8	0.9	0.7	0.8	0.8	1.0
x_4	41.07	35.47	33.0487	32.6284	36.4317
y_4	53.47	60.37	53.5663	53.8209	58.5023
z_4	124.6	129.07	129.9092	129.5824	122.8705
x_8	50.8	45.06	43.7826	43.7827	49.1742
y_8	131.48	137.04	136.8381	136.8782	136.9440
Weight (lb)	136.20	124.94	120.11	120.15	118.81
Analyses	-	6000	8000	6000	6000

The best geometry of the optimum structure found by CAFA meta-heuristic during the layout optimization process is shown in Fig. 5.
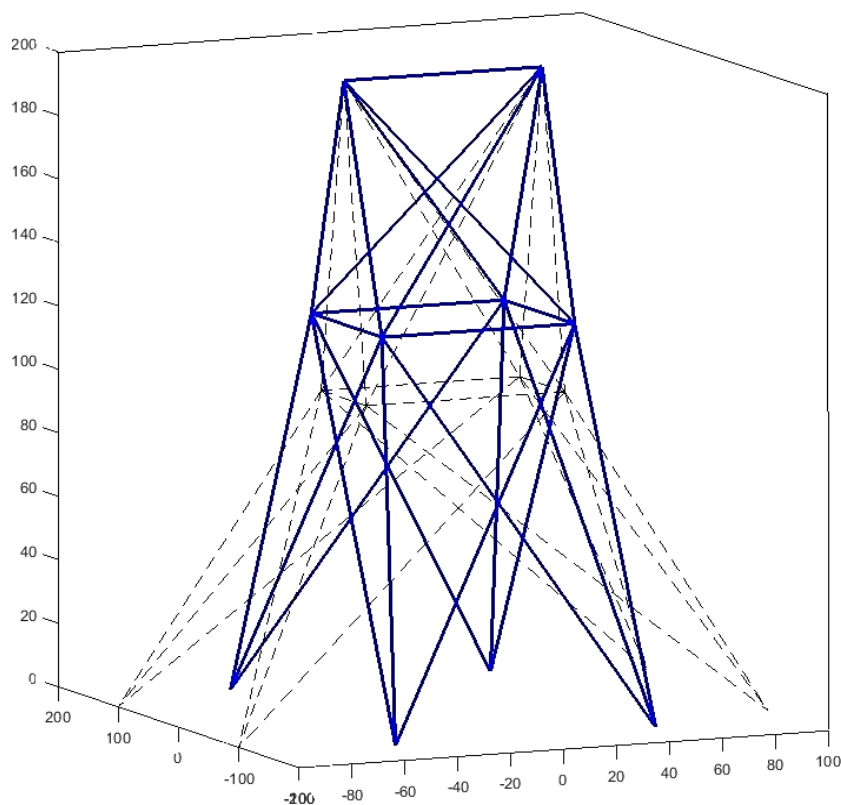

Figure 5. Optimum layout of 25-bar space truss

The numerical results reveal that the CAFA meta-heuristic converges to a solution which is 1.11%, 1.08%, 4.91%, and 12.77% lighter than those of found by FA, Rahami et al. [15], Tang et al. [14], and Wu and Chow [12], respectively spending lower computational cost. Therefore it is demonstrated that the proposed CAFA meta-heuristic is of better computational performance in comparison with other algorithms.

7. CONCLUSION

The main aim of the present study is to propose an efficient optimization algorithm for layout optimization of truss structures. In the present work, FA is selected as the optimizer and its computational performance is improved using the concept of CA. In the original FA the balance between exploration and exploitation cannot be controlled. To eliminate this difficulty and also to reduce the number of required structural analyses during the optimization process, CAFA is proposed in this study. In the proposed CAFA, the fireflies are distributed on a small dimensioned grid and the artificial evolution is evolved by a new position updating equation. In which the position updating rule is defined by employing a

new CA-based term in the conventional equation. In the sequel, an efficient optimization algorithm, denoted as CAFA, is consequently proposed to achieve the difficult layout optimization task. Two benchmark layout optimization problems of truss structures are tackled by FA and CAFA meta-heuristics and the results are compared with those of other existing algorithms. The obtained results demonstrate the superiority of the proposed CAFA meta-heuristic over FA and other algorithms proposed in literature for layout optimization of truss structures in terms of optimal cost and spent computational cost.

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