

A NOVEL META-HEURISTIC ALGORITHM: TUG OF WAR OPTIMIZATION

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ABSTRACT

This paper presents a novel population-based meta-heuristic algorithm inspired by the game of tug of war. Utilizing a sport metaphor the algorithm, denoted as Tug of War Optimization (TWO), considers each candidate solution as a team participating in a series of rope pulling competitions. The teams exert pulling forces on each other based on the quality of the solutions they represent. The competing teams move to their new positions according to Newtonian laws of mechanics. Unlike many other meta-heuristic methods, the algorithm is formulated in such a way that considers the qualities of both of the interacting solutions. TWO is applicable to global optimization of discontinuous, multimodal, non-smooth, and non-convex functions. Viability of the proposed method is examined using some benchmark mathematical functions and engineering design problems. The numerical results indicate the efficiency of the proposed algorithm compared to some other methods available in literature.

Keywords: tug of war optimization; meta-heuristic algorithm; mathematical functions; optimal design; truss structures.

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1. INTRODUCTION

Nature-inspired meta-heuristic algorithms have been celebrated as powerful global optimization techniques in the last few decades. These methods do not require any gradient information of the involved functions and are generally independent of the starting points. Due to these characteristics, meta-heuristic optimizers are favorable choices when dealing with discontinuous, multimodal, non-smooth, and non-convex functions. This is especially the case when near-global optimum solutions are sought and the intended computational effort is limited.

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Many different meta-heuristic optimization techniques have been presented and successfully applied to different problems. Examples include Genetic Algorithms (GA) [1], Particle Swarm Optimization (PSO) [2], Ant Colony Optimization (ACO) [3], Harmony Search (HS) [4], Big Bang-Big Crunch (BB-BC) [5], Charged System Search (CSS) [6], Magnetic Charged System Search (MCSS) [7], Ray Optimization (RO) [8], Democratic PSO (DPSO) [9], Dolphin Echolocation (DE) [10], Colliding Bodies Optimization (CBO) [11], Water Cycle, Mine Blast and Improved Mine Blast algorithms (WC-MB-IMB) [12], Search Group Algorithm (SGA) [13] and the Ant Lion Optimizer (ALO) [14]. Some applications of meta-heuristics on optimization of structural engineering problems could be found in [15-19].

The aim of the present paper is to introduce a new meta-heuristic algorithm based on the physical phenomenon associated with the game of tug of war. The algorithm, which is called Tug of War Optimization (TWO), is then compared to some of the documented optimization techniques using benchmark problems. The remainder of the paper is organized as follows. The new algorithm together with its physical background is presented in Section 2. In Section 3 some mathematical and engineering design benchmark problems are studied in order to demonstrate the efficiency of the proposed algorithm. The concluding remarks are presented in Section 4.

2. TUG OF WAR OPTIMIZATION

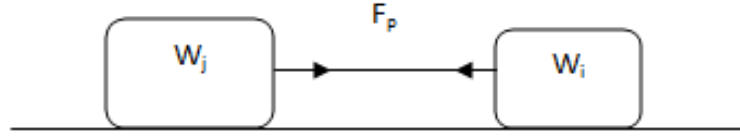
2.1 Idealized tug of war framework

Tug of war or rope pulling is a strength contest in which two competing teams pull on the opposite ends of a rope in an attempt to bring the rope in their direction against the pulling force of the opposing team. The activity dates back to ancient times and has continued to exist in different forms ever since. There has been a wide variety of rules and regulations for the game but the essential part has remained almost unaltered. Naturally, as far as both teams sustain their grips of the rope, movement of the rope corresponds to the displacement of the losing team. Fig. 1 shows one of the competing teams in a tug of war.



Figure 1. A competing team in a tug of war

Triumph in a real game of tug of war generally depends on many factors and could be difficult to analyze. However, an idealized framework is utilized in this paper where the two teams having weights W_i and W_j are considered as two objects lying on a smooth surface as shown in Fig. 2.



μ_s : Static coefficient of friction

μ_k : Kinematic coefficient of friction

Figure 2. An idealized tug of war framework

As a result of pulling the rope, the teams experience two equal and opposite forces (F_p) according to Newton's third law. For object i , as far as the pulling force is smaller than the maximum static friction force ($W_i\mu_s$) the object rests in its place. Otherwise the non-zero resultant force can be calculated as:

$$F_r = F_p - W_i\mu_k \quad (1)$$

As a result, the object i accelerates towards the object j according to the Newton's second law:

$$a = \frac{F_r}{\left(\frac{W_i}{g}\right)} \quad (2)$$

Since the object i starts from zero velocity, its new position can be determined as:

$$X_i^{new} = \frac{1}{2}at^2 + X_i^{old} \quad (3)$$

2.2 Tug of war optimization algorithm

TWO is a population-based meta-heuristic algorithm, which considers each candidate solution $X_i = \{x_{i,j}\}$ as a team engaged in a series of tug of war competitions. The weight of the teams is determined based on the quality of the corresponding solutions, and the amount of pulling force that a team can exert on the rope is assumed to be proportional to its weight. Naturally, the opposing team will have to maintain at least the same amount of force in order to sustain its grip of the rope. The lighter team accelerates toward the heavier team and this forms the convergence operator of the TWO. The algorithm improves the quality of the solutions iteratively by maintaining a proper exploration/exploitation balance using the described convergence operator. The steps of TWO can be stated as follows:

Step 1: Initialization

A population of N initial solutions is generated randomly:

$$x_{ij}^0 = x_{j,\min} + rand(x_{j,\max} - x_{j,\min}) \quad j = 1, 2, \dots, n \quad (4)$$

where x_{ij}^0 is the initial value of the j th variable of the i th candidate solution; $x_{j,\max}$ and $x_{j,\min}$ are the maximum and minimum permissible values for the j th variable, respectively; $rand$ is a random number from a uniform distribution in the interval $[0, 1]$; n is the number of optimization variables.

Step 2: Evaluation of candidate designs and weight assignment

The objective function values for the candidate solutions are evaluated. All of the initial solutions are sorted and recorded in a memory denoted as the league. Each solution is considered as a team with the following weight:

$$W_i = \left(\frac{fit(i) - fit_{worst}}{fit_{best} - fit_{worst}} \right) + 1 \quad i = 1, 2, \dots, N \quad (5)$$

where $fit(i)$ is the fitness value for the i th particle; The fitness value of the i th team, evaluated as as the penalized objective function value for constrained problems; fit_{best} and fit_{worst} are the fitness values for the best and worst candidate solutions of the current iteration. According to Eq. (5) the weights of the teams range between 1 and 2.

Step 3: Competition and displacement

In TWO each of the teams of the league competes against all the others one at a time to move to its new position in each iteration. The pulling force exerted by a team is assumed to be equal to its static friction force ($W\mu_s$). Hence the pulling force between teams i and j ($F_{p,ij}$) can be determined as $\max\{W_i\mu_s, W_j\mu_s\}$. Such a definition keeps the position of the heavier team unaltered.

The resultant force affecting team i due to its interaction with heavier team j in the k th iteration can then be calculated as follows:

$$F_{r,ij}^k = F_{p,ij}^k - W_i^k \mu_k \quad (6)$$

where $F_{p,ij}^k$ is the pulling force between teams i and j in the k th iteration and μ_k is coefficient of kinematic friction. Consequently, team i accelerates towards team j :

$$a_{ij}^k = \left(\frac{F_{r,ij}^k}{W_i^k \mu_k} \right) g_{ij}^k \quad (7)$$

where a_{ij}^k is the acceleration of team i towards team j in the k th iteration; g_{ij}^k is the gravitational acceleration constant defined as:

$$g_{ij}^k = X_j^k - X_i^k \tag{8}$$

where X_j^k and X_i^k are the position vectors for candidate solutions j and i in the k th iteration. Finally, the displacement of the team i after competing with team j can be derived as:

$$\Delta X_{ij}^k = \frac{1}{2} a_{ij}^k \Delta t^2 + \alpha^k \beta (X_{\max} - X_{\min}) \circ randn(1, n) \tag{9}$$

The second term of Eq. (9) introduces randomness into the algorithm. This term can be interpreted as the random portion of the search space traveled by team i before it stops after the applied force is removed. The role of α^k is to gradually decrease the random portion of the team's movement. For most of the applications α could be considered as a constant chosen from the interval [0.9, 0.99]; bigger values of α decrease the convergence speed of the algorithm and help the candidate solutions explore the search space more thoroughly. β is a scaling factor which can be chosen from the interval (0,1]. This parameter controls the steps of the candidate solutions when moving in the search space. When the search space is supposed to be searched more accurately with smaller steps, smaller values should be chosen for this parameter. For our numerical examples values between 0.01 and 0.05 seem to be appropriate for this parameter; X_{\max} and X_{\min} are the vectors containing the upper and lower bounds of the permissible ranges of the design variables, respectively; \circ denotes element by element multiplication; $randn(1, n)$ is a vector of random numbers drawn from a standard normal distribution.

It should be noted that when team j is lighter than team i , the corresponding displacement of team i will be equal to zero (i.e. ΔX_{ij}^k). Finally, the total displacement of team i in iteration k is equal to (*i not equal j*):

$$\Delta X_i^k = \sum_{j=1}^N \Delta X_{ij}^k \tag{10}$$

The new position of the team i at the end of the k th iteration is then calculated as:

$$X_i^{k+1} = X_i^k + \Delta X_i^k \tag{11}$$

Step 4: Updating the league

Once the teams of the league compete against each other for a complete round, the league should be updated. This is done by comparing the new candidate solutions (the new positions of the teams) to the current teams of the league. That is to say, if the new candidate solution i is better than the N th team of the league in terms of objective function value, the N th team is removed from the league and the new solution takes its place.

Step 5: Handling the side constraints

It is possible for the candidate solutions to leave the search space and it is important to deal with such solutions properly. This is especially the case for the solutions corresponding to lighter teams for which the values of ΔX is usually bigger. Different strategies might be used in order to solve this problem. For example, such candidate solutions can be simply brought back to their previous feasible position (flyback strategy) or they can be regenerated randomly. In this paper a new strategy is introduced and incorporated using the global best solution. The new value of the j th optimization variable of the i th team that violated side constraints in the k th iteration is defined as:

$$x_{ij}^k = GB_j + \left(\frac{randn}{k}\right)(GB_j - x_{ij}^{k-1}) \quad (12)$$

where GB_j is the j th variable of the global best solution (i.e. the best solution so far); $randn$ is a random number drawn from a standard normal distribution. There is a very slight possibility for the newly generated variable to be still outside the search space. In such cases a flyback strategy is used.

The abovementioned strategy is utilized with a certain probability (0.5 in this paper). For the rest of cases the violated limit is taken as the new value of the j th optimization variable.

Step 6: Termination

Steps 2 through 5 are repeated until a termination criterion is satisfied. The pseudo code of TWO is presented in Table 1.

Table 1: Pseudo-code of the TWO algorithm developed in this study

```

procedure Tug of War Optimization
begin
  Initialize parameters;
  Initialize a population of  $N$  random candidate solutions;
  Initialize the league by recording all random candidate solution;
  while (termination condition not met) do
    Evaluate the objective function values for the candidate solutions
    Sort the new solutions and update the league
    Define the weights of the teams of the league  $W_i$  based on  $fit(X_i)$ 
    for each team  $i$ 
      for each team  $j$ 
        if ( $W_i < W_j$ )
          Move team  $i$  towards team  $j$  using Eq. (9);
        end if
      end for
    Determine the total displacement of team  $i$  using Eq. (10)
    Determine the final position of team  $i$  using Eq.(11)
    Use the side constraint handling technique to regenerate violating variables
  end for
end while
end

```

3. TEST PROBLEMS AND OPTIMIZATION RESULTS

In order to evaluate the efficiency of the proposed algorithm, some benchmark test problems are considered from the literature. A set of uni-modal and multi-modal mathematical optimization problems are studied in Section 3.1. In addition, six well-studied engineering design problems are investigated in Section 3.2. Except for the last three examples a population of 20 agents and a maximum number of permitted iterations of 200 are used for all test problems. For the fourth, fifth, and sixth structural design problems the numbers of agents are taken as 30, 40, and 40, respectively while 400 iterations are used for these examples. The coefficient of static friction (μ_s) is taken as unity, while the coefficient of kinematic friction (μ_k) varies linearly from 1 to 0.1. Since smaller values of μ_k let the teams slide more easily towards each other and vice versa, such a parameter selection helps the algorithm's agents to explore the search space at early iterations without being severely affected by each other. As the optimization process proceeds, the values of μ_k gradually decrease allowing for convergence. It was found that using the same value of kinematic friction coefficient for all teams yield the best optimization results for TWO.

3.1 Mathematical optimization problems

In this section the efficiency of the TWO is evaluated by solving the mathematical benchmark problems summarized in Table 2. These benchmark problems are taken from Ref. [20], where some variants of GA were used as the optimization algorithm. The results obtained by TWO are presented in Table 3 along with those of GA variants. Each objective function is optimized 50 times independently starting from different initial populations and the average number of function evaluations required by each algorithm is presented. The numbers in the parentheses indicate the ratio of the successful runs in which the algorithm has located the global minimum with predefined accuracy, which is taken as $\varepsilon = f_{\min} - f_{\text{final}} = 10^{-4}$. The absence of the parentheses means that the algorithm has been successful in all independent runs.

Table 2: Details of the benchmark mathematical problems solved in this study

| Function name | Side constraints | Function | Global minimum |
|-----------------|---|--|----------------|
| Aluffi-Pentiny | $X \in [-10,10]^2$ | $f(X) = \frac{1}{4}x_1^4 - \frac{1}{2}x_1^2 + \frac{1}{10}x_1 + \frac{1}{2}x_2^2$ | -0.352386 |
| Bohachevsky 1 | $X \in [-100,100]^2$ | $f(X) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1) - \frac{4}{10}\cos(4\pi x_2) + \frac{7}{10}$ | 0.0 |
| Bohachevsky 2 | $X \in [-50,50]^2$ | $f(X) = x_1^2 + 2x_2^2 - \frac{3}{10}\cos(3\pi x_1)\cos(4\pi x_2) + \frac{3}{10}$ | 0.0 |
| Becker and Lago | $X \in [-10,10]^2$ | $f(X) = (x_1 - 5)^2 + (x_2 - 5)^2$ | 0.0 |
| Branin | $0 \leq x_2 \leq 15$ $-5 \leq x_1 \leq 10$ | $f(X) = (x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1)^2 + 10(1 - \frac{1}{8\pi})\cos(x_1) + 10$ | 0.397887 |
| Camel | $X \in [-5,5]^2$ | $f(X) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$ | -1.0316 |

| | | | |
|---------------------|--------------------------------|--|-----------|
| Cb3 | $X \in [-5,5]^2$ | $f(X) = 2x_1^2 - 1.05x_1^4 + \frac{1}{6}x_1^6 + x_1x_2 + x_2^2$ | 0.0 |
| Cosine mixture | $n=4,$ $X \in [-1,1]^n$ | $f(X) = \sum_{i=1}^n x_i^2 - \frac{1}{10} \sum_{i=1}^n \cos(5\pi x_i)$ | -0.4 |
| DeJong | $X \in [-5.12,5.12]^3$ | $f(X) = x_1^2 + x_2^2 + x_3^2$ | 0.0 |
| Exponential | $n=2,4,8,$ $X \in [-1,1]^n$ | $f(X) = -\exp\left(-0.5 \sum_{i=1}^n x_i^2\right)$ | -1 |
| Goldstein and price | $X \in [-2,2]^2$ | $f(\mathbf{X}) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)]$ | 3.0 |
| Griewank | $X \in [-100,100]^2$ | $f(\mathbf{X}) = 1 + \frac{1}{200} \sum_{i=1}^2 x_i^2 - \prod_{i=1}^2 \cos\left(\frac{x_i}{\sqrt{i}}\right)$ | 0.0 |
| Hartman 3 | $X \in [0,1]^3$ | $f(X) = -\sum_{i=1}^4 c_i \exp\left(-\sum_{j=1}^3 a_{ij}(x_j - p_{ij})^2\right)$ $a = \begin{bmatrix} 3 & 10 & 30 \\ 0.1 & 10 & 35 \\ 3 & 10 & 30 \\ 0.1 & 10 & 35 \end{bmatrix}, c = \begin{bmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{bmatrix}$ and $p = \begin{bmatrix} 0.3689 & 0.117 & 0.2673 \\ 0.4699 & 0.4387 & 0.747 \\ 0.1091 & 0.8732 & 0.5547 \\ 0.03815 & 0.5743 & 0.8828 \end{bmatrix}$ | -3.862782 |
| Hartman 6 | $X \in [0,1]^6$ | $f(\mathbf{X}) = -\sum_{i=1}^4 c_i \exp\left(-\sum_{j=1}^6 a_{ij}(x_j - p_{ij})^2\right)$ $a = \begin{bmatrix} 10 & 3 & 17 & 3.5 & 1.7 & 8 \\ 0.05 & 10 & 17 & 0.1 & 8 & 14 \\ 3 & 3.5 & 17 & 10 & 17 & 8 \\ 17 & 8 & 0.05 & 10 & 0.1 & 14 \end{bmatrix}, c = \begin{bmatrix} 1 \\ 1.2 \\ 3 \\ 3.2 \end{bmatrix}$ and $p = \begin{bmatrix} 0.1312 & 0.1696 & 0.5569 & 0.0124 & 0.8283 & 0.5886 \\ 0.2329 & 0.4135 & 0.8307 & 0.3736 & 0.1004 & 0.9991 \\ 0.2348 & 0.1451 & 0.3522 & 0.2883 & 0.3047 & 0.6650 \\ 0.4047 & 0.8828 & 0.8732 & 0.5743 & 0.1091 & 0.0381 \end{bmatrix}$ | -3.322368 |

Table 3: Performance comparison of TWO and GA variants in the mathematical optimization problems

| Function name | GEN | GEN-S | GEN-S-M | GEN-S-M-LS | TWO |
|---------------|--------------|-------|---------|------------|------|
| AP | 1,360 (0.99) | 1,360 | 1,277 | 1,253 | 1092 |
| Bf1 | 3,992 | 3,356 | 1,640 | 1615 | 1404 |
| Bf2 | 20,234 | 3,373 | 1,676 | 1636 | 1232 |
| BL | 19,596 | 2412 | 2,439 | 1,436 | 1216 |
| Branin | 1,442 | 1,418 | 1,404 | 1,257 | 1189 |
| Camel | 1,358 | 1,358 | 1,336 | 1,300 | 1212 |
| Cb3 | 9,771 | 2,045 | 1,163 | 1,118 | 992 |
| CM | 2,105 | 2,105 | 1,743 | 1,539 | 1420 |

| | | | | | |
|---------------------|---------------|--------------|--------------|--------------|-------------|
| DeJoung | 9,900 | 3,040 | 1,462 | 1,281 | 1346 |
| Exp2 | 938 | 936 | 817 | 807 | 314 |
| Exp4 | 3,237 | 3,237 | 2,054 | 1,496 | 815 |
| Exp8 | 3,237 | 3,237 | 2,054 | 1,496 | 1257 |
| Goldstein and Price | 1,478 | 1,478 | 1,408 | 1,325 | 1690 |
| Griewank | 18,838 (0.91) | 3,111 (0.91) | 1,764 | 1,652 (0.99) | 1766 |
| Hartman3 | 1,350 | 1,350 | 1,332 | 1,274 | 1026 |
| Harman6 | 2,562 (0.54) | 2,562 (0.54) | 2,530 (0.67) | 1,865 (0.68) | 1601 (0.68) |

As it can be seen from Table 3, TWO generally performs better than GA and its variants in the mathematical optimization problems considered in this study.

3.2 Structural design problems

In order to further investigate the efficiency of the TWO, three engineering design problems are considered in this section. These problems have been previously studied using different optimization algorithms. These constrained optimization problems are turned into unconstrained ones using a penalty approach. If the constraints are satisfied, then the amount of penalty will be zero; otherwise its value can be calculated as the ratio of violated constraint to the corresponding allowable limit.

3.2.1 Design of a tension/compression spring

Weight minimization of the tension/compression spring shown in Fig. 3, subject to constraints on shear stress, surge frequency, and minimum deflection is considered as the first engineering design example. This problem was first described by Belegundu [21] and Arora [22]. The design variables are the mean coil diameter $D(x_1)$, the wire diameter $d(x_2)$, and the number of active coils $N(x_3)$.

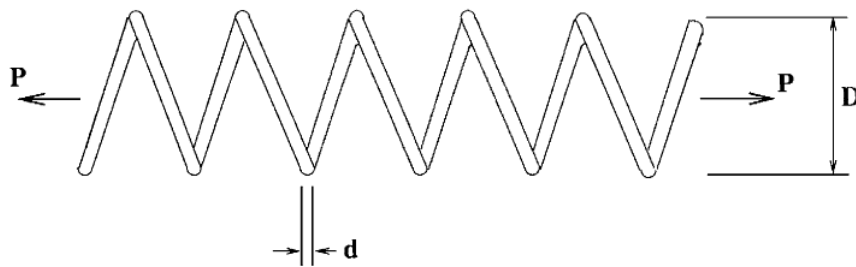


Figure 3. Schematic of the tension/compression spring

The cost function can be stated as:

$$f_{cost}(X) = (x_3 + 2)x_2x_1^2 \tag{13}$$

while the constraints are:

$$\begin{aligned}
g_1(\mathbf{X}) &= 1 - \frac{x_2^3 x_3}{71785x_1^4} \leq 0 \\
g_2(\mathbf{X}) &= \frac{4x_2^2 - x_1 x_2}{1256(x_2 x_1^3 - x_1^4)} + \frac{1}{5108x_1^2} - 1 \leq 0 \\
g_3(\mathbf{X}) &= 1 - \frac{140.45x_1}{x_2^2 x_3} \leq 0 \\
g_4(\mathbf{X}) &= \frac{x_1 + x_2}{1.5} - 1 \leq 0
\end{aligned} \tag{14}$$

The side constraints are defined as follows:

$$\begin{aligned}
0.05 &\leq x_1 \leq 2 \\
0.25 &\leq x_2 \leq 1.3 \\
2 &\leq x_3 \leq 15
\end{aligned} \tag{15}$$

This problem has been solved by Belegundu [21] using eight different mathematical optimization techniques. Arora [22] utilized a numerical optimization technique called a constraint correction at the constant cost to investigate the problem. Coello [23] and Coello and Montes [24] used a GA-based algorithm to solve the problem. He and Wang [25] used a co-evolutionary particle swarm optimization (CPSO). Montes and Coello [26] used evolution strategies. Kaveh and Talatahari used improved ant colony optimization [27] and charged system search CSS [6]. Recently, the problem has been solved by Kaveh and Mahdavi [11] using Colliding bodies optimization (CBO).

The optimization results obtained by TWO are presented in Table 4 along with those of other methods. It can be seen that TWO found the best design overall. It should be noted that some constraints are slightly violated by the designs quoted by Kaveh & Talatahari [6, 26]. Table 5 shows the statistical results obtained for 30 independent optimization runs.

Table 4: Comparison of the optimization results obtained in the tension/compression spring problem

| Optimal design variables | | | | |
|--------------------------|----------|-----------|-----------|-------------------|
| Methods | $x_1(d)$ | $x_2(D)$ | $x_3(N)$ | f_{cost} |
| Belegundu [21] | 0.050000 | 0.315900 | 14.250000 | 0.0128334 |
| Arora [22] | 0.053396 | 0.399180 | 9.185400 | 0.0127303 |
| Coello [23] | 0.051480 | 0.351661 | 11.632201 | 0.0127048 |
| Coello & Montes [24] | 0.051989 | 0.363965 | 10.890522 | 0.0126810 |
| He and Wang [25] | 0.051728 | 0.357644 | 11.244543 | 0.0126747 |
| Montes and Coello [26] | 0.051643 | 0.355360 | 11.397926 | 0.012698 |
| Kaveh & Talatahari [27] | 0.051865 | 0.361500 | 11.000000 | 0.0126432 |
| Kaveh & Talatahari [6] | 0.051744 | 0.358532 | 11.165704 | 0.0126384 |
| Kaveh & Mahdavi [11] | 0.051894 | 0.3616740 | 11.007846 | 0.0126697 |
| Present work | 0.051592 | 0.354379 | 11.428784 | 0.0126671 |

Table 5: Comparison of statistical optimization results obtained in the tension/compression spring design problem

| Methods | Best | Mean | Worst | Std dev |
|-------------------------|-----------|-----------|-----------|------------|
| Belegundu [21] | 0.0128334 | N/A | N/A | N/A |
| Arora [22] | 0.0127303 | N/A | N/A | N/A |
| Coello [23] | 0.0127048 | 0.012769 | 0.012822 | 3.9390e-5 |
| Coello & Montes [24] | 0.0126810 | 0.0127420 | 0.012973 | 5.9000e-5 |
| He and Wang [25] | 0.0126747 | 0.012730 | 0.012924 | 5.1985e-5 |
| Montes and Coello [26] | 0.012698 | 0.013461 | 0.016485 | 9.6600e-4 |
| Kaveh & Talatahari [27] | 0.0126432 | 0.012720 | 0.012884 | 3.4888e-5 |
| Kaveh & Talatahari [6] | 0.0126384 | 0.012852 | 0.013626 | 8.3564e-5 |
| Kaveh & Mahdavi [11] | 0.0126697 | 0.0127296 | 0.012881 | 5.00376e-5 |
| Present work | 0.0126671 | 0.0129709 | 0.0135213 | 2.6125e-4 |

3.2.2 Design of a welded beam

The second test problem regards the design optimization of the welded beam shown in Fig.4. This problem has been used for testing different optimization methods. The aim is to minimize the total manufacturing cost subject to constraints on shear stress (τ), bending stress (σ), buckling load (P_c), and deflection (δ). The four design variables, namely h (x_1), l (x_2), t (x_3), and b (x_4), are also shown in the figure.

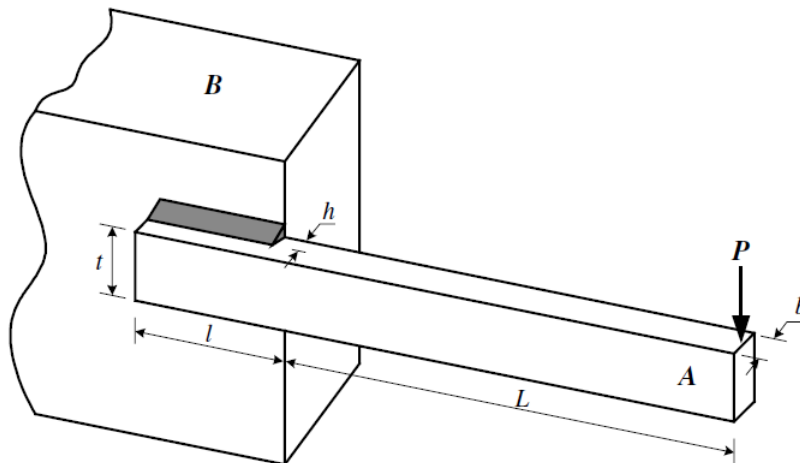


Figure 4. Schematic of the welded beam

The objective function can be mathematically stated as:

$$f_{cost}(X) = 1.10471x_1^2x_2 + 0.04811x_3x_4(14.0 + x_2) \tag{16}$$

The optimization constraints are

$$g_1(X) = \tau(\{x\}) - \tau_{max} \leq 0 \tag{17}$$

$$\begin{aligned}
g_2(\mathbf{X}) &= \sigma(\{x\}) - \sigma_{\max} \leq 0 \\
g_3(\mathbf{X}) &= x_1 - x_4 \leq 0 \\
g_4(\mathbf{X}) &= 0.10471x_1^2 + 0.04811x_3x_4(14.0 + x_2) - 5.0 \leq 0 \\
g_5(\mathbf{X}) &= 0.125 - x_1 \leq 0 \\
g_6(\mathbf{X}) &= \delta(\{x\}) - \delta_{\max} \leq 0 \\
g_7(\mathbf{X}) &= P - P_c(\{x\}) \leq 0
\end{aligned}$$

where

$$\begin{aligned}
\tau(\mathbf{X}) &= \sqrt{(\tau')^2 + 2\tau'\tau''\frac{x_2}{2R} + (\tau'')^2} \\
\tau' &= \frac{P}{\sqrt{2x_1x_2}}, \quad \tau'' = \frac{MR}{J} \\
M &= P(L + \frac{x_2}{2}), \quad R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2} \\
J &= 2 \left\{ \sqrt{2x_1x_2} \left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2 \right] \right\} \\
\sigma(\mathbf{X}) &= \frac{6PL}{x_4x_3^2}, \quad \delta(\mathbf{X}) = \frac{4PL^3}{Ex_3^3x_4} \\
P_c(\mathbf{X}) &= \frac{4.013E\sqrt{\frac{x_3^2x_4^6}{36}}}{L^2} \left(1 - \frac{x_3}{2L} \sqrt{\frac{E}{4G}} \right) \\
P &= 6000b, \quad L = 14in, \\
E &= 30 \times 10^6 \text{ psi}, \quad G = 12 \times 10^6 \text{ psi} \\
\tau_{\max} &= 13.6 \times 10^3 \text{ psi}, \quad \sigma_{\max} = 30 \times 10^3 \text{ psi} \\
\delta_{\max} &= 0.25in
\end{aligned} \tag{18}$$

The side constraints can be stated as:

$$0.1 \leq x_1 \leq 2, \quad 0.1 \leq x_2 \leq 10, \quad 0.1 \leq x_3 \leq 10, \quad 0.1 \leq x_4 \leq 2 \tag{19}$$

Radgsdell and Phillips [28] utilized different optimization methods mainly based on mathematical programming to solve the problem and compared the results. GA-based methods are used by Deb [29], Coello [23], Coello and Montes [24]. This test problem was also solved by He and Wang [25] using CPSO and Montes and Coello [26] using evolution strategies. Kaveh and Talatahari employed ant colony optimization [27] and charged system search [6]. Kaveh and Mahdavi [11] solved the problem using the colliding bodies

optimization method.

Table 6 compares the best results obtained by the different optimization algorithms considered in this study. The statistical results for 30 independent runs are provided in Table 7. It can be seen from Table 6 that optimum design found by TWO is about 0.01 percent heavier than that found by CBO which was the best design quoted so far in literature.

Table 6: Comparison of the optimization results obtained in the welded beam problem

| Methods | Optimal design variables | | | | |
|-----------------------------|--------------------------|----------|----------|----------|------------|
| | $x_1(h)$ | $x_2(l)$ | $x_3(t)$ | $x_4(b)$ | f_{cost} |
| Radgsdell and Phillips [28] | 0.2444 | 6.2189 | 8.2915 | 0.2444 | 2.3815 |
| Deb [29] | 0.248900 | 6.173000 | 8.178900 | 0.253300 | 2.433116 |
| Coello [23] | 0.208800 | 3.420500 | 8.997500 | 0.210000 | 1.748309 |
| Coello and Montes [24] | 0.205986 | 3.471328 | 9.020224 | 0.206480 | 1.728226 |
| He and Wang [25] | 0.202369 | 3.544214 | 9.048210 | 0.205723 | 1.728024 |
| Montes and Coello [26] | 0.199742 | 3.612060 | 9.037500 | 0.206082 | 1.737300 |
| Kaveh & Talatahari [27] | 0.205700 | 3.471131 | 9.036683 | 0.205731 | 1.724918 |
| Kaveh & Talatahari [6] | 0.205820 | 3.468109 | 9.038024 | 0.205723 | 1.724866 |
| Kaveh & Mahdavi [11] | 0.205722 | 3.47041 | 9.037276 | 0.205735 | 1.724663 |
| Present work | 0.205728 | 3.47052 | 9.036631 | 0.205730 | 1.724855 |

Table 7: Comparison of statistical optimization results obtained in the welded beam problem

| Methods | Best | Mean | Worst | Std Dev |
|-----------------------------|----------|----------|----------|-----------|
| Radgsdell and Phillips [28] | 2.3815 | N/A | N/A | N/A |
| Deb [29] | 2.433116 | N/A | N/A | N/A |
| Coello [23] | 1.748309 | 1.771973 | 1.785835 | 0.011220 |
| Coello and Montes [24] | 1.728226 | 1.792654 | 1.993408 | 0.074713 |
| He and Wang [25] | 1.728024 | 1.748831 | 1.782143 | 0.012926 |
| Montes and Coello [26] | 1.737300 | 1.813290 | 1.994651 | 0.070500 |
| Kaveh & Talatahari [27] | 1.724918 | 1.729752 | 1.775961 | 0.009200 |
| Kaveh & Talatahari [6] | 1.724866 | 1.739654 | 1.759479 | 0.008064 |
| Kaveh & Mahdavi [11] | 1.724662 | 1.725707 | 1.725059 | 0.0002437 |
| Present work | 1.724855 | 1.726016 | 1.729970 | 0.0009951 |

3.2.3 Design of a planar 10-bar truss structure subject to frequency constraints

The sizing optimization of a planar 10-bar truss subject to frequency constraints is selected as the third test case. The configuration of the structure is depicted in Fig. 5. This is a well-known benchmark problem in the field of frequency constraint structural optimization. Each of the members' cross-sectional area is assumed to be an independent variable. A non-structural mass of 454.0 kg is attached to all free nodes. Table 8 summarizes the material properties, variable bounds, and frequency constraints for this example. This problem has been investigated by different researchers: Grandhi and Venkayya [30] using an optimality algorithm, Sedaghati et al. [31] using a sequential quadratic programming and finite element force method, Wang et al. [32] using an evolutionary node shift method, Lingyun et al. [33] utilizing a niche hybrid genetic algorithm, Gomes [34] employing the standard particle

swarm optimization algorithm and Kaveh and Zolghadr utilizing democratic PSO [9], and a hybridized PSRO algorithm [35] among others.

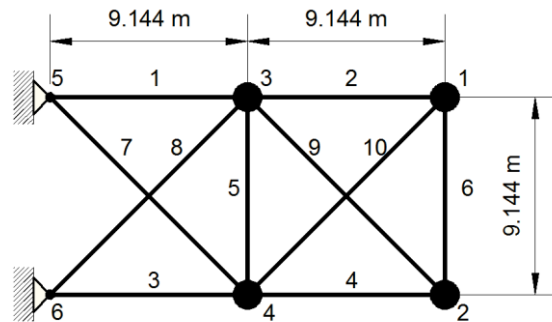


Figure 5. Schematic of the planar 10-bar truss structure

Table 8: Material properties, variable bounds and frequency constraints for the 10-bar truss structure

| Property/unit | Value |
|--|---|
| E (Modulus of elasticity)/ N/m ² | 6.89×10^{10} |
| ρ (Material density)/ kg/m ³ | 2770.0 |
| Added mass/kg | 454.0 |
| Design variable lower bound/m ² | 0.645×10^{-4} |
| Design variable upper bound/m ² | 50×10^{-4} |
| L (Main bar's dimension)/m | 9.144 |
| Constraints on first three frequencies/Hz | $\omega_1 \geq 7, \omega_2 \geq 15, \omega_3 \geq 20$ |

Table 9 summarizes the design vectors of the optimal structures found by different methods.

Table 9: Optimized designs (cm²) obtained for the planar 10-bar truss problem (the optimized weight does not include the added masses)

| Element number | Grandhi & Venkayya [30] | Sedaghati et al. [31] | Wang et al. [32] | Lingyun et al. [33] | Gomes [34] | Kaveh & Zolghadr | | |
|----------------|-------------------------|-----------------------|------------------|---------------------|------------|------------------|-----------|--------------|
| | | | | | | DPSO [9] | PSRO [35] | Present work |
| 1 | 36.584 | 38.245 | 32.456 | 42.23 | 37.712 | 35.944 | 37.075 | 35.198 |
| 2 | 24.658 | 9.916 | 16.577 | 18.555 | 9.959 | 15.530 | 15.334 | 14.311 |
| 3 | 36.584 | 38.619 | 32.456 | 38.851 | 40.265 | 35.285 | 33.665 | 35.305 |
| 4 | 24.658 | 18.232 | 16.577 | 11.222 | 16.788 | 15.385 | 14.849 | 14.833 |
| 5 | 4.167 | 4.419 | 2.115 | 4.783 | 11.576 | 0.648 | 0.645 | 0.645 |
| 6 | 2.070 | 4.419 | 4.467 | 4.451 | 3.955 | 4.583 | 4.643 | 4.671 |
| 7 | 27.032 | 20.097 | 22.810 | 21.049 | 25.308 | 23.610 | 24.528 | 23.806 |
| 8 | 27.032 | 24.097 | 22.810 | 20.949 | 21.613 | 23.599 | 23.188 | 24.894 |
| 9 | 10.346 | 13.890 | 17.490 | 10.257 | 11.576 | 13.135 | 12.436 | 12.843 |
| 10 | 10.346 | 11.452 | 17.490 | 14.342 | 11.186 | 12.357 | 13.500 | 12.803 |
| Weight (kg) | 594.0 | 537.01 | 553.8 | 542.75 | 537.98 | 532.39 | 532.85 | 532.17 |

According to Table 9, the weight of the optimal structure found by TWO is 532.17 kg which is slightly better than that of DPSO, the best design quoted so far in the literature. The mean value and the standard deviation of 30 independent runs of TWO on this problem are 537.43 and 3.43 kg, respectively. Table 10 shows the natural frequencies of the optimized structures obtained by different methods. It can be seen that all of the constraints are satisfied.

3.2.4 Design of a spatial 25-bar truss structure

Weight minimization of the spatial 25-bar truss schematized in Fig. 6 is considered as the fourth design example. The material density and modulus of elasticity are 0.1 lb/in³ and 10000 ksi, respectively. Table 11 shows the two independent loading conditions applied to the structure. The twenty five bars of the truss are classified into eight groups, as follows:

- (1) A₁, (2) A₂-A₅, (3) A₆-A₉, (4) A₁₀-A₁₁, (5) A₁₂-A₁₃, (6) A₁₄-A₁₇, (7) A₁₈-A₂₁, and (8) A₂₂-A₂₅.

Table 10: Natural frequencies (Hz) evaluated at the optimized designs for the planar 10-bar truss

| Frequency number | Grandhi & Venkaya [30] | Sedaghati et al. [31] | Wang et al. [32] | Lingyun et al. [33] | Gomes [34] | Kaveh and Zolghadr | | |
|------------------|------------------------|-----------------------|------------------|---------------------|------------|--------------------|-----------|--------------|
| | | | | | | DPSO [9] | PSRO [35] | Present work |
| 1 | 7.059 | 6.992 | 7.011 | 7.008 | 7.000 | 7.000 | 7.000 | 7.000 |
| 2 | 15.895 | 17.599 | 17.302 | 18.148 | 17.786 | 16.187 | 16.143 | 16.127 |
| 3 | 20.425 | 19.973 | 20.001 | 20.000 | 20.000 | 20.000 | 20.000 | 20.000 |
| 4 | 21.528 | 19.977 | 20.100 | 20.508 | 20.063 | 20.021 | 20.032 | 20.002 |
| 5 | 28.978 | 28.173 | 30.869 | 27.797 | 27.776 | 28.470 | 28.469 | 28.699 |
| 6 | 30.189 | 31.029 | 32.666 | 31.281 | 30.939 | 29.243 | 29.485 | 29.068 |
| 7 | 54.286 | 47.628 | 48.282 | 48.304 | 47.297 | 48.769 | 48.440 | 48.280 |
| 8 | 56.546 | 52.292 | 52.306 | 53.306 | 52.286 | 51.389 | 51.257 | 50.822 |

Table 11: Independent loading conditions acting on the spatial 25-bar truss

| Node | Case 1 | | | Case 2 | | |
|------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| | P _x kips | P _y kips | P _z kips | P _x kips | P _y kips | P _z kips |
| 1 | 0.0 | 20.0 | -5.0 | 1.0 | 10.0 | -5.0 |
| 2 | 0.0 | -20.0 | -5.0 | 0.0 | 10.0 | -5.0 |
| 3 | 0.0 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 |
| 6 | 0.0 | 0.0 | 0.0 | 0.5 | 0.0 | 0.0 |

Maximum displacement limitations of 0.35 in are imposed on all nodes in all directions. The axial stress constraints, which are different for each group, are shown in Table 12. The cross-sectional areas vary continuously from 0.01 to 3.4 in² for all members.

Table 12: Member stress limits for the 25-bar spatial truss

| Element group | Compressive stress limits ksi (MPa) | Tensile stress limit ksi (MPa) |
|---------------|-------------------------------------|--------------------------------|
| 1 | 35.092 (241.96) | 40.0 (275.80) |
| 2 | 11.590 (79.913) | 40.0 (275.80) |
| 3 | 17.305 (119.31) | 40.0 (275.80) |
| 4 | 35.092 (241.96) | 40.0 (275.80) |
| 5 | 35.092 (241.96) | 40.0 (275.80) |
| 6 | 6.759 (46.603) | 40.0 (275.80) |
| 7 | 6.959 (47.982) | 40.0 (275.80) |
| 8 | 11.082 (76.410) | 40.0 (275.80) |

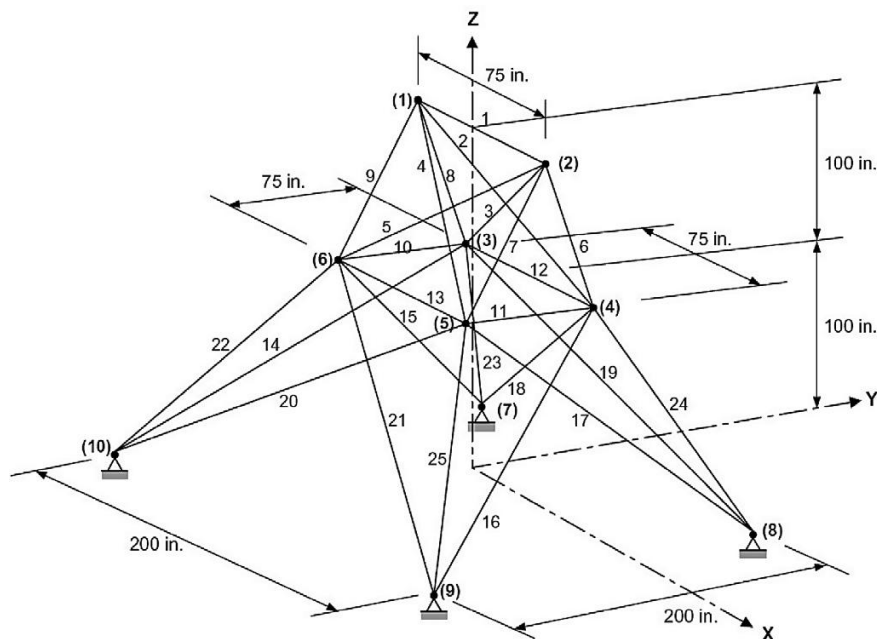


Figure 6. Schematic of the spatial 25-bar truss structure

Table 13 shows that the different optimization methods converged almost to the same structural weight. It seems that the results obtained by different methods are very close together for this example. The best result obtained by TWO is 544.42 kg which is only slightly heavier than that of HS. However, it should be noted that according to our codes the optimum design of HS slightly violates displacement constraints. Moreover TWO requires 12000 analyses to complete the optimization process, while HS has used 15000 analyses. The mean value and standard deviation of 30 independent optimization runs by TWO are 544.53 and 0.211 lb, respectively. Small differences in weight may originate from the level of precision in the implementation of the optimization problem. For example, the displacement limit of 0.350in set for TWO resulted in feasible optimized design quoted in Table 13 yielding a maximum displacement of 0.3504in which can be rounded to the above mentioned limit.

3.2.5 Design of a spatial 72-bar truss structure

A spatial 72-bar truss structure shown in Fig. 7 is considered as the fifth design example. The elements are grouped to form 16 design variables according to Table 15. The material density and the modulus of elasticity are taken as 0.1 lb/in³ and 10,000 ksi, respectively. All members are subjected to a stress limitations of ±25 ksi. The displacements of the uppermost nodes along x and y axes are limited to ±0.25 in. Cross-sectional areas of bars can vary between 0.10 and 4.00 in². The two loading conditions acting on the structure are summarized in Table 14.

This problem has been studied by Erbatur et al. [39] using Genetic Algorithms, Camp and Bichon [40] using Ant Colony Optimization, Perez and Behdinan [41] using Particle Swarm Optimization, Camp [42] using Big Bang-Big Crunch algorithm, and Kaveh and Khayatazad [8] using Ray Optimization among others.

Table 13: Comparison of the optimization results obtained in the spatial 25-bar truss problem

| Element group | | Optimal cross-sectional areas (in ²) | | | |
|---------------|----------------------------------|--|---------------------------------|-----------------------|--------------|
| | | Rajeev and Krishnamoorthy, GA [36] | Schutte and Groenwold, PSO [37] | Lee and Geem, HS [38] | Present work |
| 1 | A ₁ | 0.10 | 0.010 | 0.047 | 0.010 |
| 2 | A ₂ -A ₅ | 1.80 | 2.121 | 2.022 | 1.979 |
| 3 | A ₆ -A ₉ | 2.30 | 2.893 | 2.950 | 2.993 |
| 4 | A ₁₀ -A ₁₁ | 0.20 | 0.010 | 0.010 | 0.010 |
| 5 | A ₁₂ -A ₁₃ | 0.10 | 0.010 | 0.014 | 0.010 |
| 6 | A ₁₄ -A ₁₇ | 0.80 | 0.671 | 0.688 | 0.684 |
| 7 | A ₁₈ -A ₂₁ | 1.80 | 1.611 | 1.657 | 1.678 |
| 8 | A ₂₂ -A ₂₅ | 3.0 | 2.717 | 2.663 | 2.656 |
| | Best weight (lb) | 546 | 545.21 | 544.38 | 544.42 |
| | Average weight (lb) | N/A | 546.84 | N/A | 544.53 |
| | Std Dev (lb) | N/A | 1.478 | N/A | 0.211 |
| | No. of analyses | N/A | 9596 | 15000 | 12000 |

Table 14: Independent loading conditions acting on the spatial 72-bar truss

| node | Case 1 | | | Case 2 | | |
|------|--------------------------|--------------------------|--------------------------|-------------------------|-------------------------|--------------------------|
| | P _x kips (kN) | P _y kips (kN) | P _z kips (kN) | P _x kips(kN) | P _y kips(kN) | P _z kips (kN) |
| 1 | 5 | 5 | -5 | - | - | -5 |
| 2 | - | - | - | - | - | -5 |
| 3 | - | - | - | - | - | -5 |
| 4 | - | - | - | - | - | -5 |

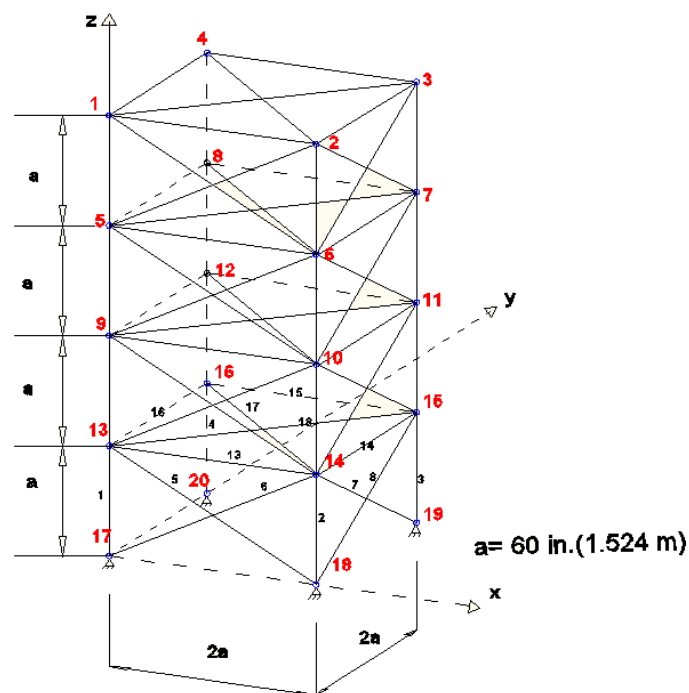


Figure 7. Schematic of the spatial 72-bar truss structure

Table 15 compares the results obtained by the present method to those previously reported in the literature. The weight of the best result obtained by TWO is 379.846 lb which is the best among the compared methods. Moreover, the mean weight of the results for 20 independent optimization runs of TWO is 381.98 lb which is less than all other methods. Also, TWO requires only 16000 structural analyses while ACO, BB-BC and RO require 18,500, 19,621, and 19,084, respectively.

Table 15: Comparison of the optimization results obtained in the spatial 72-bar truss problem

| Element group | Optimal cross-sectional areas (in ²) | | | | | |
|---------------|--|----------------------|-------------------------|-----------|--------------------------|--------------|
| | Erbatur et al. [39] | Camp and Bichon [40] | Perez and Behdinan [41] | Camp [42] | Kaveh and Khayatazad [8] | Present work |
| 1–4 | 1.755 | 1.948 | 1.7427 | 1.8577 | 1.8365 | 1.9961 |
| 5–12 | 0.505 | 0.508 | 0.5185 | 0.5059 | 0.5021 | 0.5100 |
| 13–16 | 0.105 | 0.101 | 0.1000 | 0.1000 | 0.1000 | 0.1000 |
| 17–18 | 0.155 | 0.102 | 0.1000 | 0.1000 | 0.1004 | 0.1000 |
| 19–22 | 1.155 | 1.303 | 1.3079 | 1.2476 | 1.2522 | 1.2434 |
| 23–30 | 0.585 | 0.511 | 0.5193 | 0.5269 | 0.5033 | 0.5184 |
| 31–34 | 0.100 | 0.101 | 0.1000 | 0.1000 | 0.1002 | 0.1000 |
| 35–36 | 0.100 | 0.100 | 0.1000 | 0.1012 | 0.1001 | 0.1001 |
| 37–40 | 0.460 | 0.561 | 0.5142 | 0.5209 | 0.5730 | 0.5211 |
| 41–48 | 0.530 | 0.492 | 0.5464 | 0.5172 | 0.5499 | 0.5098 |
| 49–52 | 0.120 | 0.100 | 0.1000 | 0.1004 | 0.1004 | 0.1000 |

| | | | | | | |
|-------------------------|--------|--------|--------|--------|---------|---------|
| 53–54 | 0.165 | 0.107 | 0.1095 | 0.1005 | 0.1001 | 0.1000 |
| 55–58 | 0.155 | 0.156 | 0.1615 | 0.1565 | 0.1576 | 0.1569 |
| 59–66 | 0.535 | 0.550 | 0.5092 | 0.5507 | 0.5222 | 0.5346 |
| 67–70 | 0.480 | 0.390 | 0.4967 | 0.3922 | 0.4356 | 0.3959 |
| 71–72 | 0.520 | 0.592 | 0.5619 | 0.5922 | 0.5971 | 0.5821 |
| Best weight (lb) | 385.76 | 380.24 | 381.91 | 379.85 | 380.458 | 379.846 |
| Mean weight (lb) | N/A | 383.16 | N/A | 382.08 | 382.553 | 381.976 |
| Standard deviation (lb) | N/A | 3.66 | N/A | 1.912 | 1.221 | 3.161 |
| Number of analyses | N/A | 18,500 | N/A | 19621 | 19,084 | 16000 |

3.2.6 Design of a 120-bar dome truss

The last test problem is the weight minimization of a 120-bar dome truss shown in Fig. 8. This structure was considered by Soh and Yang [43] as a configuration optimization problem. It has been solved by Lee and Geem [38], Kaveh and Talatahari [44], Kaveh et al. [45], Kaveh and Khayatizad [8], and Kaveh and Mahdavi [11] as a sizing optimization problem. The members of the structure are divided into seven groups as shown in Fig. 8.

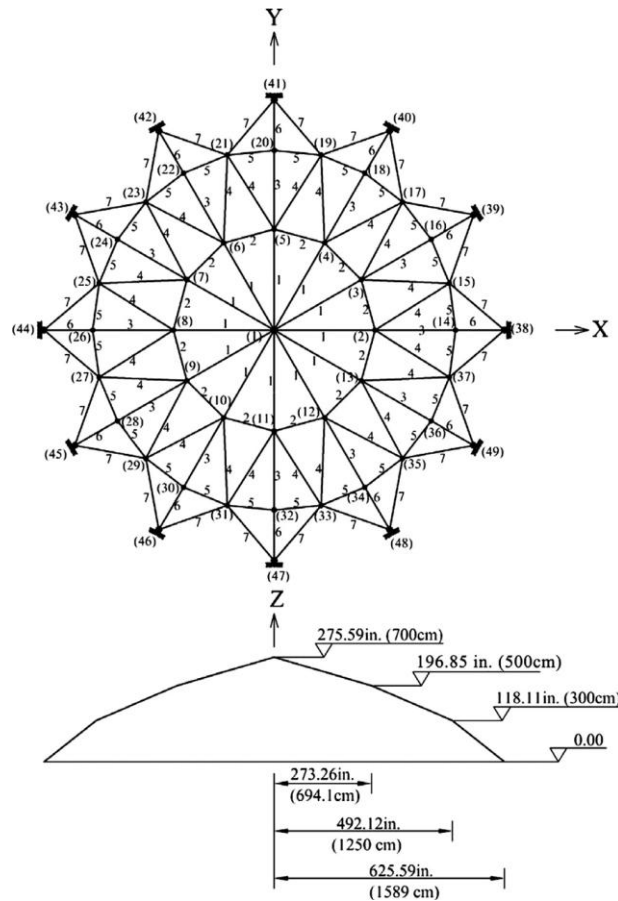


Figure 8. Schematic of the 120-bar dome truss structure

The allowable tensile and compressive stresses are set according to the AISC-ASD [46] provisions as follows:

$$\begin{cases} \sigma_i^+ = 0.6F_y & \text{for } \sigma_i \geq 0 \\ \sigma_i^- & \text{for } \sigma_i \leq 0 \end{cases} \quad (20)$$

where σ_i^- is the compressive allowable stress and depends on the elements slenderness ratio

$$\sigma_i^- = \begin{cases} \left[\left(1 - \frac{\lambda_i^2}{2C_c^2} \right) F_y \right] / \left(\frac{5}{3} + \frac{3\lambda_i}{8C_c} - \frac{\lambda_i^3}{8C_c^3} \right) & \text{for } \lambda_i \leq C_c \\ \frac{12\pi^2 E}{23\lambda_i^2} & \text{for } \lambda_i \geq C_c \end{cases} \quad (21)$$

where E is the modulus of elasticity, F_y is the yield stress of the material, λ_i is the slenderness ratio ($\lambda_i = \frac{K_i L_i}{r_i}$), K_i is the effective length factor, L_i is the member length and r_i is the radius of gyration. C_c is the critical slenderness ratio separating elastic and inelastic buckling regions ($C_c = \sqrt{2\pi^2 E / F_y}$).

The modulus of elasticity and the material density are taken as 30,450 ksi (210 GPa) and 0.288 lb/in³, respectively. The yield stress is taken as 58.0 ksi (400 MPa). The radius of gyration is expressed in terms of cross-sectional areas of the members as $r_i = aA_i^b$ [47]. Constants a and b depend on the types of sections adopted for the members such as pipes, angles, etc. In this example pipe sections are used for the bars for which $a=0.4993$ and $b=0.6777$. The dome is considered to be subjected to vertical loads at all unsupported nodes. These vertical loads are taken as -13.49 kips (60 kN) at node 1, -6.744 kips (30 kN) at nodes 2 through 14, and -2.248 kips (10 kN) at the other nodes. Four different problem variants are considered for this structure: with stress constraints and no displacement constraints (Case 1), with stress constraints and displacement limitations of ± 0.1969 in (5 mm) imposed on all nodes in x and y directions (Case 2), no stress constraints and displacement limitations of ± 0.1969 in (5 mm) imposed on all nodes in z direction (Case 3), and all the abovementioned constraints imposed together (Case 4). For cases 1 and 2 the maximum cross-sectional area is taken as 5.0 in² (32.26 cm²) while for cases 3 and 4 it is taken as 20 in² (129.03 cm²). The minimum cross-sectional is taken as 0.775 in² (5 cm²) for all cases.

Table 16 compares the results obtained by different optimization techniques for this example. It can be seen that the results found by TWO are comparable to those of other methods. In case 1 the best result obtained by TWO is the same as that of CBO which is the best result so far. In cases 3 and 4 the results obtained by TWO are better than those of RO, IRO, and CBO and are only slightly heavier than that of HPSACO (0.01 and 0.004 percent in cases 3 and 4, respectively). The average number of structural analyses required by the RO and IRO algorithms were reported as 19,900 and 18,300, respectively, while TWO uses

40 candidate solutions and 400 iterations like CBO resulting in a maximum number of analyses of 16000. HPSACO and HS, respectively, performed 10000 and 35000 structural analyses to obtain their optimal results. It should be noted that HPSACO is a hybrid method which combines good features of PSO, ACO and HS.

Table 16: Comparison of the optimization results obtained in the 120-bar dome problem

| Element group | Optimal cross-sectional areas (in ²) | | | | | | | | | |
|---------------------|--|-------------|-----------|--------------|--------------|-------------|-------------|----------|----------|--------------|
| | Case 1 | | | | | Case 2 | | | | |
| | HS [38] | HPSACO [44] | RO [8] | CBO [11] | Present work | HS [38] | HPSACO [44] | RO [8] | CBO [11] | Present work |
| 1 | 3.295 | 3.311 | 3.128 | 3.1229 | 3.1229 | 3.296 | 3.779 | 3.084 | 3.0832 | 3.0831 |
| 2 | 3.396 | 3.438 | 3.357 | 3.3538 | 3.3538 | 2.789 | 3.377 | 3.360 | 3.3526 | 3.3526 |
| 3 | 3.874 | 4.147 | 4.114 | 4.1120 | 4.1120 | 3.872 | 4.125 | 4.093 | 4.0928 | 4.0928 |
| 4 | 2.571 | 2.831 | 2.783 | 2.7822 | 2.7822 | 2.570 | 2.734 | 2.762 | 2.7613 | 2.7613 |
| 5 | 1.150 | 0.775 | 0.775 | 0.7750 | 0.7750 | 1.149 | 1.609 | 1.593 | 1.5918 | 1.5923 |
| 6 | 3.331 | 3.474 | 3.302 | 3.3005 | 3.3005 | 3.331 | 3.533 | 3.294 | 3.2927 | 3.2927 |
| 7 | 2.784 | 2.551 | 2.453 | 2.4458 | 2.4458 | 2.781 | 2.539 | 2.434 | 2.4336 | 2.4336 |
| Best weight (lb) | 19707.77 | 19491.3 | 19476.193 | 19454.7 | 19454.67 | 19893.34 | 20078.0 | 20071.9 | 20064.5 | 20064.86 |
| Average weight (lb) | - | - | - | 19466.0 | 19454.98 | - | - | - | 20098.3 | 20106.85 |
| Std (lb) | - | - | 33.966 | 7.02 | 1.17 | - | - | 112.135 | 26.17 | 116.15 |
| | Case 3 | | | | | Case 4 | | | | |
| | HPSACO [44] | RO [8] | CBO [11] | Present work | | HPSACO [44] | RO [8] | IRO [45] | CBO [11] | Present work |
| 1 | 2.034 | 2.044 | 2.0660 | 1.9667 | | 3.095 | 3.030 | 3.0252 | 3.0273 | 3.0247 |
| 2 | 15.151 | 15.665 | 15.9200 | 15.3920 | | 14.405 | 14.806 | 14.8354 | 15.1724 | 14.7261 |
| 3 | 5.901 | 5.848 | 5.6785 | 5.7127 | | 5.020 | 5.440 | 5.1139 | 5.2342 | 5.1338 |
| 4 | 2.254 | 2.290 | 2.2987 | 2.1960 | | 3.352 | 3.124 | 3.1305 | 3.119 | 3.1369 |
| 5 | 9.369 | 9.001 | 9.0581 | 9.5439 | | 8.631 | 8.021 | 8.4037 | 8.1038 | 8.4545 |
| 6 | 3.744 | 3.673 | 3.6365 | 3.6688 | | 3.432 | 3.614 | 3.3315 | 3.4166 | 3.2946 |
| 7 | 2.104 | 1.971 | 1.9320 | 1.9351 | | 2.499 | 2.487 | 2.4968 | 2.4918 | 2.4956 |
| Best weight (lb) | 31670.0 | 31733.2 | 31724.1 | 31673.62 | | 33248.9 | 33317.8 | 33256.48 | 33286.3 | 33250.31 |
| Average weight (lb) | - | - | 32162.4 | 31680.34 | | - | - | - | 33398.5 | 33282.64 |
| Std (lb) | - | 274.991 | 240.22 | 6.15 | | - | 354.333 | - | 67.09 | 25.38 |

4. CONCLUSIONS

In this paper a novel population-based meta-heuristic algorithm based on a game of tug of war is introduced. The algorithm, denoted as Tug of War Optimization (TWO), considers each candidate solution as a team participating in a series of tug of war competitions. Like other meta-heuristic optimization algorithms TWO uses a combination of randomness and exploitation of previously obtained favorable results to perform global optimization. The quality of the candidate solutions improves iteratively as the optimization process proceeds. Unlike many other meta-heuristic methods, the algorithm is formulated in such a way that considers the qualities of both of the interacting teams. In other words, since the weights of the teams are involved in the convergence operator, two losing teams are not treated the same by a particular winning team. The team with less weight would probably move more drastically.

As a meta-heuristic optimization algorithm, TWO does not require information on the derivatives of the objective function and the constraints. This makes the algorithm applicable to a wide range of optimization problems from different fields. Some classical mathematical and structural optimization problems are solved to evaluate the efficiency of the proposed algorithm. Results indicate the superiority of TWO compared to some other state-of-the-art optimization algorithms.

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