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OPTIMIZATION OF ENDURANCE TIME ACCELERATION FUNCTIONS FOR SEISMIC ASSESSMENT OF STRUCTURES

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ABSTRACT

Numerical simulation of structural response is a challenging issue in earthquake engineering and there has been remarkable progress in this area in the last decade. Endurance Time (ET) method is a new response history based analysis procedure for seismic assessment and structural design in which structures are subjected to a gradually intensifying dynamic excitation and their seismic performance is evaluated based on their responses at different excitation levels. Generating appropriate artificial dynamic excitation is essential in this type of analysis. In this paper, an optimization procedure is presented for computation of the intensifying acceleration functions utilized in the ET method and the results of this procedure are discussed. A set of the ET acceleration functions (ETAFs) is considered which has been produced utilizing numerical optimization considering 2048 acceleration points as optimization variables by an unconstrained optimization procedure. The ET formulation is then modified from the continuous time condition into the discrete time state; thus the optimization problem is reformulated as a nonlinear least squares problem. In this way, a second set of the ETAFs is generated which better satisfies the proposed objective function. Subsequently, acceleration points are increased to 4096, for 40 seconds duration, and the third set of the ETAFs is produced using a multi level optimization procedure. Improvement of the ETAFs is demonstrated by analyzing several SDOF systems.

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KEY WORDS: response history analysis, Endurance Time method, Intensifying acceleration functions, Numerical optimization, Nonlinear least squares

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1. INTRODUCTION

One of the major challenges in Earthquake Engineering is the numerical simulation of seismic structural response and there has been remarkable progress in developing appropriate software for this purpose in the last decade [1-3]. Providing appropriate safety margin against structural failure in destructive earthquakes is one of the major objectives in the seismic design. There are different methods to analyze seismic performance of structures which can be selected according to design conditions and requirements. Equivalent static analysis, modal analysis and dynamic time-history analysis are all common methods utilized in the seismic analysis and design of structures. Obvious shortages in the traditional seismic design methods and recent developments in the computational technology and analysis instruments lead researchers to more advanced and qualified methods such as performance based design method [4, 5]. Generally, dynamic behavior of structures and their performance under seismic loads is considered as the earthquake engineering necessity and to determine the expected structural damage can be an important step to improve such approach [6]. However, the great amount of computational demand is still considered a limitation in practical applicability of many of realistic dynamic analysis procedures [7, 8].

Since 2004, the Endurance Time (ET) method has been introduced as an alternative response history based method for the seismic analysis and design of structures [9]. In this method, the computational demand is considerably reduced by subjecting the structure to an intensifying acceleration function (AF) and monitoring the objective performance indexes through time. Afterwards, structural performance can be evaluated based on the response of system at each excitation level [10, 11]. Generating appropriate artificial dynamic input is essential for the ET method's success. With respect to this issue, an input function can be considered as appropriate if the results estimated in the ET analysis are consistent with the performance of different structures under real earthquakes. The acceleration functions currently applied in the ET method have two specific properties: (I) these functions are intensifying as their amplitude increase with time, (II) these functions are optimized such that the response spectrum of any window from t=0 to t=t1 is proportional to a template response spectrum with a scale factor that linearly varies with time [10]. As will be explained, to generate the AFs with these properties is a formidably complicated problem from analytical viewpoint; consequently, numerical optimization turns out to be the only practical approach in order to tackle this issue.

The general procedure for generating the ET acceleration functions (ETAFs) is illustrated in Figure 1. To generate these functions, the template response spectrum matching with either a required design spectrum or a spectrum resulted from a set of ground motions should be considered. Currently, different template spectrums have been considered. These include the design spectrum of Iranian National Building Code (INBC), ASCE design spectrum and average of response spectra from sets of ground motions [12, 13]. Several sets of the AFs have been produced for each target response. A set of the ET acceleration functions (ETA20d01-3) was optimized utilizing the INBC design spectrum as the target response spectrum. In this study, the optimization process of this set of the AFs is considered as a basis and different approaches are proposed to improve these functions and to achieve more consistent results with the template design spectrum.

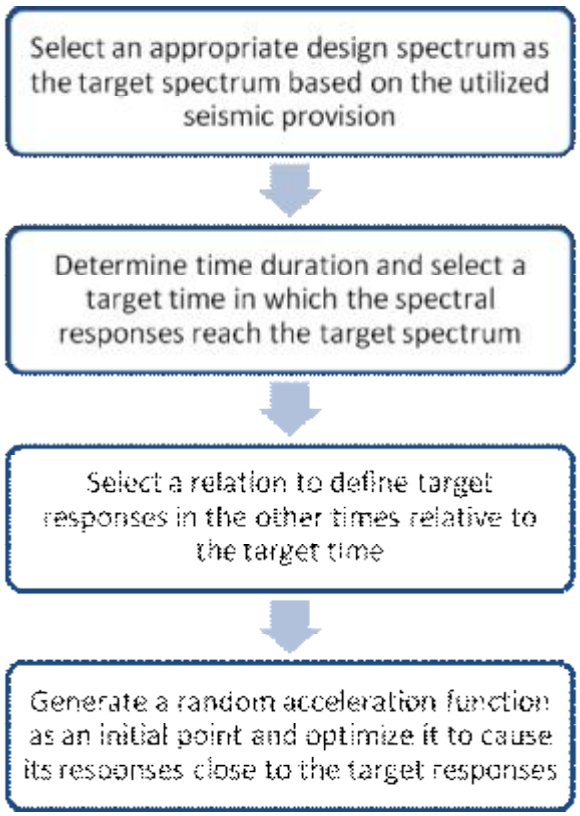


Figure 1. General procedure of generating ET acceleration functions

2. GENERATING ET ACCELERATION FUNCTIONS

The ETA20d series of the ET acceleration functions were generated utilizing design spectrum of standard No. 2800 of INBC for soil type (II) as the target response [10, 12]. Duration of these AFs is 20.47 seconds which consist of 2048 acceleration points in 0.01s time steps. The target time of the functions is 10^{th} second when the response of a SDOF system with a damping ratio of 5% equals the codified template design spectrum with a scale factor of unity. Objective response in all other times is defined by a linear function of time based on the target response as follows:

$$S_{aT}(T_i, t_j) = \frac{t_j}{t_{Target}} S_{aC}(T_i)$$
$$S_{i,j}^{aT} = S_{aT}(T_i, t_j) \quad (1)$$

Where, T is the fundamental period of structure, t_{Target} is the target time, S_{aT} is the target acceleration response of structure and S_{aC} is the codified spectral acceleration which can be obtained from equation 2 below:

$$\begin{cases} B = 1 + 1.5(\dfrac{T}{0.1}) & T < 0.1s \\ B = 2.5 & 0.1s \le T < 0.5s \\ B = 2.5(\dfrac{0.5}{T})^{\frac{2}{3}} & 0.5s \le T \end{cases}$$

$$S_{aC} = \frac{0.35BI}{R} \tag{2}$$

Where I is the importance factor of under design building considered to be 1.0 and R is the response reduction factor that has not been applied (i.e. assumed equal to 1.0) [12].

Similarly, the displacement target response can be obtained from the codified spectral acceleration as follows:

$$\begin{aligned} S_{uT}(T_i, t_j) &= \frac{t_j}{t_{Target}} S_{aC}(T_i) \times \frac{T_i^2}{4p^2} \\ S_{i,j}^{uT} &= S_{uT}(T_i, t_j) \end{aligned} \tag{3}$$

Since the 10th second is selected as the target time, it is obvious that the target response for example at 5th second is half of the codified value and at 20th second is twice the codified value. The objective response will be in m*n matrix form which the number of rows (m) is equal to the number of period points and the number of columns (n) is equal to the number of time steps. Thus t_j can be formulated as follows:

$$t_j = j \times dt \quad , \quad j = 1, 2, ..., n \tag{4}$$

For calculation of time history responses due to a dynamic input, we can consider the differential equation of motion for an SDOF system under an earthquake excitation:

$$\ddot{u}(t) + 2x w_n \dot{u}(t) + w_n^2 u(t) = -\ddot{u}_g(t) \tag{5}$$

Where, x is the damping ratio, w_n is the natural circular frequency which corresponds to the natural period of vibration with $2p/T$ and $\ddot{u}_g(t)$ is the ground excitation time history. The acceleration response function can be calculated from absolute acceleration responses as follows:

$$\begin{aligned} S_a(T_i, t_j) &= \max\left(\left|\ddot{u}(t) + \ddot{u}_g(t)\right|\right) \qquad 0 \le t \le t_j \\ S_{i,j}^a &= S_a(T_i, t_j) \end{aligned} \tag{6}$$

Further the displacement response function can be obtained from relative displacement

responses as:

$$S_u\left(T_i,t_j\right)=\max\left(\left|u\left(t\right)\right|\right) \qquad 0\le t\le t_j$$
$$S_{i,j}^{u}=S_u\left(T_i,t_j\right) \tag{7}$$

Now, the problem is approached by formulating it as an unconstrained optimization problem in the time domain with the following objective function which can be minimized:

$$F(a_g)=\sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left\{\left[S_{i,j}^{a}-S_{i,j}^{aT}\right]^2+a\left[S_{i,j}^{u}-S_{i,j}^{uT}\right]^2\right\}}{n\times m}} \tag{8}$$

Where $a_g(t)$ is the acceleration function as the optimization variable. It should be noted that either acceleration or displacement response or combination of them can be utilized as the target response. However, since the acceleration and displacement responses are closely correlated for a SDOF system, only one of them can be considered as the target response [10, 14]. Therefore, acceleration response is merely selected for the objective function of optimization (i.e. the weight parameter α assumed equal to 0).

For optimization process, an unconstrained optimization procedure, which applies quasi-Newton algorithm, is applied [15]. Two hundred period points are distributed logarithmically in the range of 0 and 5 seconds and twenty long period points are used to control displacements. In addition, damping ratio is assumed to be 5% for all of the SDOF systems. Since the structural responses are calculated in all the time steps, the response function is produced in a 220*2048 matrix form which needs two hundred twenty time-history analyses for its calculation in each cycle.

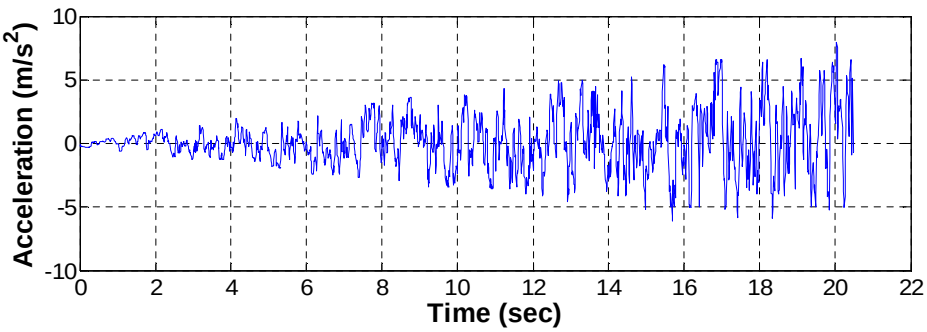
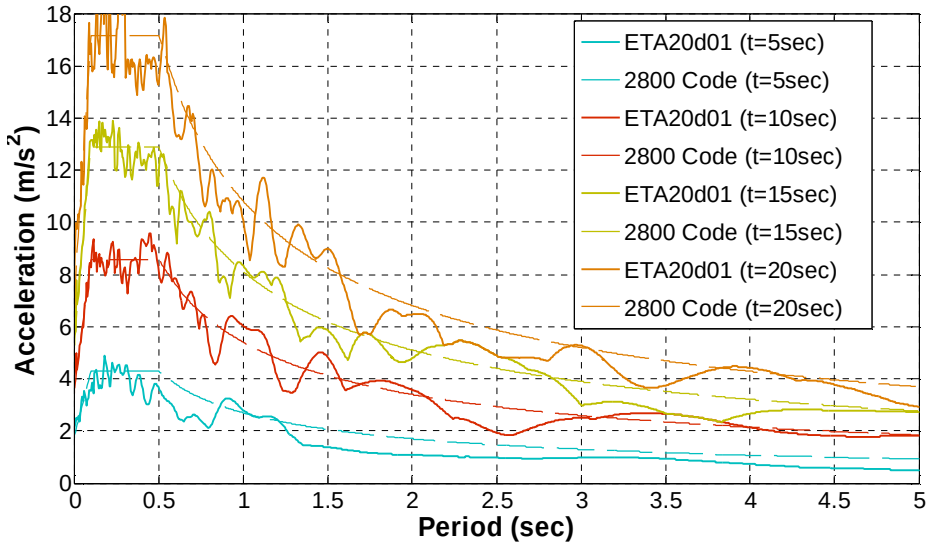


Figure 2. ETA20d01 acceleration function

A typical ET acceleration function generated utilizing this approach is shown in Figure 2. The acceleration and displacement response spectra of these functions at 5th, 10th, 15th and 20th seconds are illustrated in Figure 3. In Figure 4, the acceleration and displacement response spectra and average response spectra of the three AFs can be seen at the target time matching the template spectrum with a scale factor of unity. As can be seen, the optimization procedure

has been successful in producing the AFs that are matched with the specified target with reasonable accuracy. These acceleration functions are available online [16] as well. Dynamic properties of the AFs produced employing this procedure, are investigated by Valamanesh et al. [17].



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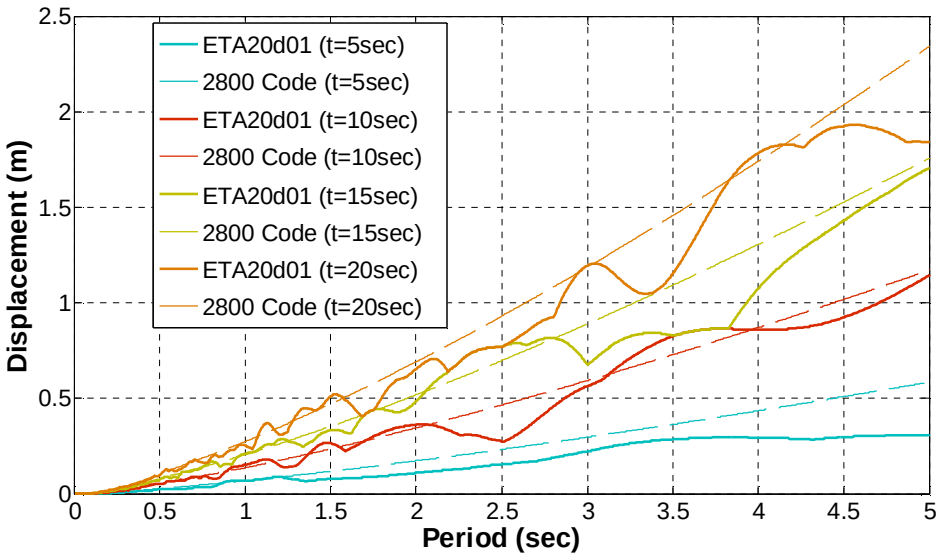
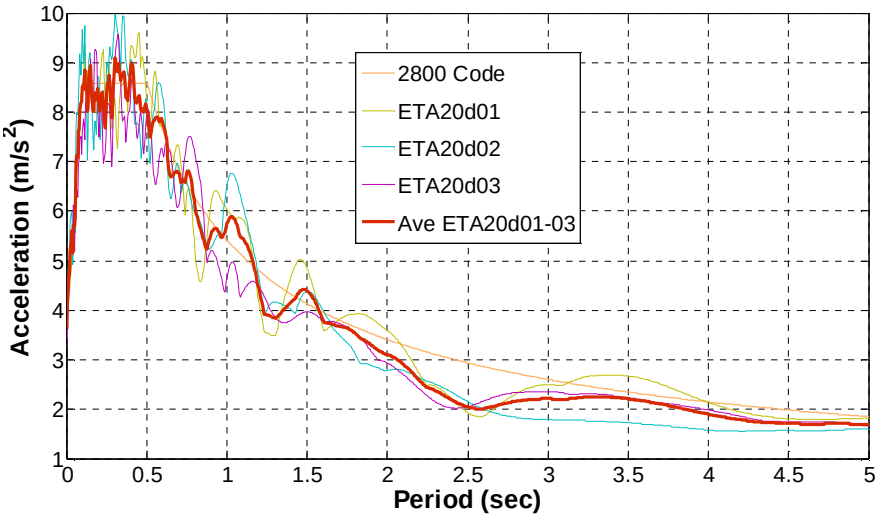


(b)

Figure 3. Response spectra of ETA20d01 at 5th, 10th, 15th and 20th seconds, (a) acceleration, (b) displacement



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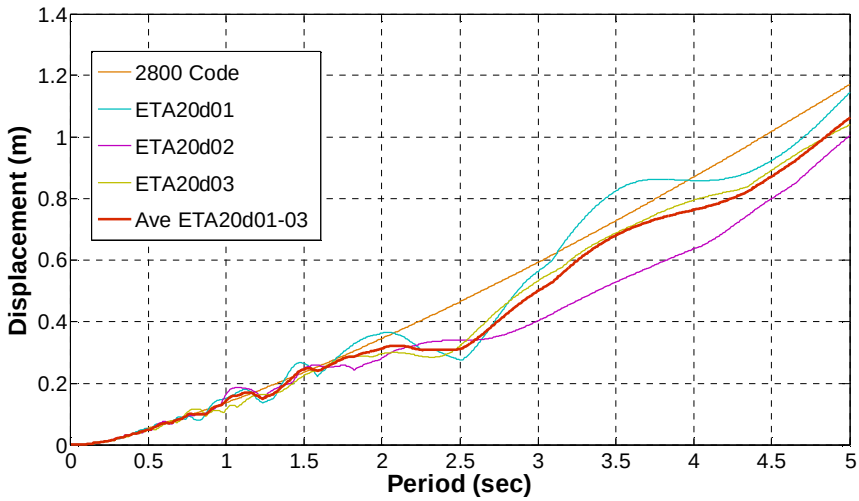


(b)

Figure 4. Response spectra of ETA20d01-03 at the target time (the 10th second), (a) acceleration, (b) displacement

The calculated errors for the each AF are given in Table 1. Two approaches can be applied for the error calculation: the first approach is the same as the objective function for the optimization process which is characterized by relation 8 and the second one the so called base error is similar to the first approach; however in definition, its purpose is to negate the effect of period points distribution and the optimization time steps in calculation of errors. To calculate the base error, period points between 0 to 5 seconds with uniform distribution with a step of 0.005s, and all of the time steps (2048 steps) are utilized. Therefore, the function for calculating the base error will be a 1001×2048 matrix. The base error is applied in order to compare the convergence of the different AFs to target of perfect match with the optimization objective.

Table 1. Errors of acceleration responses of the first series ETAFs

Acceleration function	Absolute error (m/s^2)	
	Optimized points	Base points
ETA20d01	0.5094	0.5378
ETA20d02	0.7126	0.7929
ETA20d03	0.5321	0.5545
Average	0.5847	0.6284
Ave ETA20d01-03	0.4141	0.4564

As can be seen, the average of base errors for the three acceleration functions of the ETA20d series is 0.6284 m/s^2 and the error of average response of these acceleration functions is 0.4564 m/s^2. Thus, by averaging the results from the three records, the amount of deviation is reduced about 27 percent. Preliminary studies have revealed that the applying the three AFs is effective in decreasing the average response error, and the advantage of accuracy diminishes because of the required additional computations with increasing the number of AFs to more than three [10, 15]. Thus averaging the results of the three ET analyses is recommended as an optimal solution to minimize calculations while reducing the effect of random scattering in the results and obtaining a sense of the expected level of dispersion in the estimates.

The Optimization of ETA20d series of the ET acceleration functions using the unconstrained optimization procedure, considering the high volume of computations, is a time consuming process, e.g. to produce each AF, more than 120 hours was required by utilizing Pentium IV CPU with a frequency of 2800 GHz. Therefore, improving the optimization procedure is essential to make the process practically appealing.

3. NONLINEAR LEAST SQUARES FORMULATION

As was shown, to produce the ETA20d series of the ET acceleration functions, the objective function of optimization is formulated as the square root of sum of squares. However, the objective function of optimization can further be defined in the form of the least squares and special algorithms for the optimization of nonlinear least squares problems can be applied. By employing this method, a computer code was developed which takes the objective function of in a matrix form and proceeds to minimize every element in the matrix, and utilizes the two different algorithms for two functional states:

1. If the number of elements in the objective matrix of optimization is fewer than the number of optimization variables, the program will use quasi Newton algorithm similar to the procedure used in previous unconstrained optimization [15].
2. If the number of elements in the objective function is equal to or higher than the number of optimization variables, the program will utilize the Trust Region algorithm based on Interior Reflective Newton method, which is optimized for nonlinear least squares problems [18, 19].

In fact, it is highly considerable to notice the higher power of the Trust Region algorithm in the second state in order to optimize the least squares problems, and what is important is the transformation of the objective function of into a appropriate form for this function. Hence, the objective function of optimization is expressed as follows:

$$M_{i,j}(a_g) = \left\{ \left[S_{i,j}^{a} - S_{i,j}^{aT} \right] + a \left[S_{i,j}^{u} - S_{i,j}^{uT} \right] \right\} \tag{9}$$

After performing a number of primary experiments, it was concluded that available memory of the program was not sufficient to define the complete ET objective function in a way that for a problem with 2048 variables, an objective function in a matrix form with at most 9000 elements could be defined [15]. Therefore, in order to utilize the program, the objective function needs to be compressed in such a way that it conforms to the requirements of the usable memory. The idea is to apply a discrete time definition for the objective function, such that a limited number of identical times are chosen for the optimization:

$$t_k = p \times k \times dt \quad , \quad k = 1, 2, ..., l \quad , \quad l = [\frac{n}{p}] \tag{10}$$

Where, t_k is the discrete times included in the objective function, p is an integer parameter to determine the discrete time intervals and dt is the time history analysis time step (assumed equal to 0.01s).

As a result, a smaller sized matrix can be produced from the initial objective matrix. It should be mentioned that, considering the concept of the ET acceleration functions and the definition of spectral responses, responses at the times between two consecutive discrete times are restricted within the responses at those discrete times:

$$t_k \le t_j \le t_{k+1} \quad \Rightarrow \quad \begin{cases} S_{i,k}^{u} \le S_{i,j}^{u} \le S_{i,k+1}^{u} \\ S_{i,k}^{a} \le S_{i,j}^{a} \le S_{i,k+1}^{a} \end{cases} \tag{11}$$

In fact, by utilizing this procedure, we can avoid changing the time step of time history analysis, and retain the time step of 0.01 seconds and the results accuracy is maintained; while the size of the objective function is reduced. Therefore, similar to what was performed to produce ETA20d series of the ET acceleration functions, the objective function is formed; however it retains its matrix form and chooses specific time steps (a number of rows in the objective matrix) with all the considered period points (all the columns of the objective matrix), which an objective function with smaller size is produced.

4. IMPROVED ET ACCELERATION FUNCTIONS

As a result of utilizing the new formulation of discrete ET objective function, a new series of the ET acceleration functions are produced. For the optimization of these AFs, the dynamic properties of former functions, presented in section 2, are utilized. To consider the

computational limitations explained above, after performing the primary experiments, selective times are considered to begin at 0.5 seconds with equal intervals of 0.5 seconds ending at the 20.00 seconds duration (i.e. p assumed equal to 50 so l equals to 40). Therefore, taking the 220 points of the period into account, the initial 220×2048 objective matrix is compressed into a 220×40 matrix and as was explained, the acceleration response can be individually considered in the objective function as follows:

$$M_{i,j}(a_g) \xrightarrow{Compressed} N_{i,k}(a_g) = \begin{bmatrix} S^{a}_{1,1} - S^{aT}_{1,1} & \mathbf{L} & S^{a}_{1,l} - S^{aT}_{1,l} \\ \mathbb{M} & \mathbf{O} & \mathbb{M} \\ S^{a}_{m,1} - S^{aT}_{m,1} & \mathbf{L} & S^{a}_{m,l} - S^{aT}_{m,l} \end{bmatrix} \tag{12}$$

It should be noted that, if the size of the compressed matrix exceeds the limitations, the Out of Memory error will occur [15]. In order to compare the convergence of the current procedure (least squares optimization) with the previous procedure (unconstrained optimization), the optimization is performed utilizing identical initial points for both procedures and the results of 200 iterations are considered. As was explained, the optimization is a time consuming process and 200 iterations take more than 120 hours in the old procedure. In figure 5, the results of both procedures for the production of the first acceleration function of the new series, are indicated. As can be seen, not only the convergence rate of the new procedure is about 10 times higher than the previous method, but also the accuracy of the new procedure is improved.

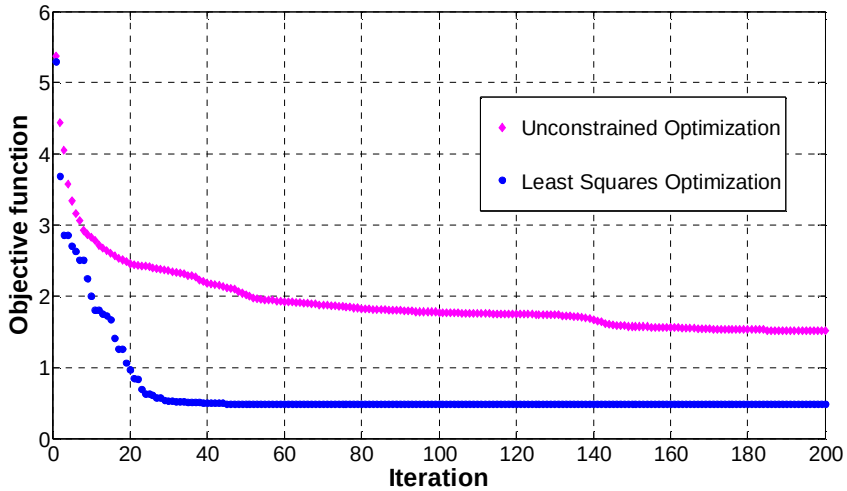


Figure 5. Comparison of convergence of least squares procedure (this work) with unconstrained procedure in optimization of ETA20d-TR01 acceleration function

The produced acceleration functions are named as ETA20d-TR01-03. The acceleration response spectra of the three AFs along with the average response spectra of the AFs, at the target time (the tenth second), are presented in figure 6. Similar convergence exists at all other times.

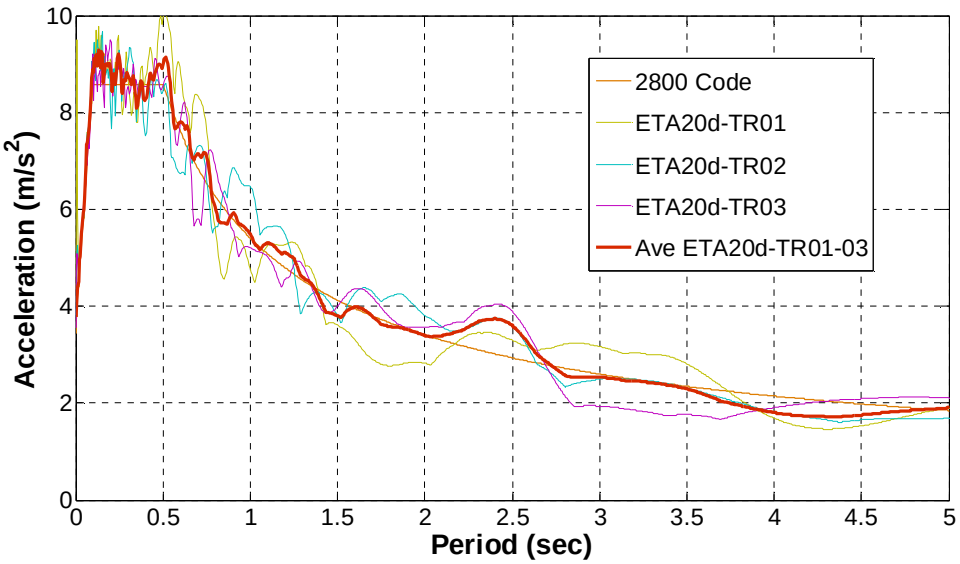


Figure 6. Acceleration response spectra of ETA20d-TR01-03 at the target time (the 10th second)

The absolute errors of responses of the new AFs, for both methods, the error in optimized points and the base error, are listed in Table 2. The absolute error of the average response of the three AFs is computed as well.

Table 2. Errors of acceleration responses of the second series ETAFs

Acceleration functions	**Absolute error (m/s^2)**	
	Optimized points	**Base points**
ETA20d-TR01	0.5132	0.5095
ETA20d-TR02	0.4896	0.4993
ETA20d-TR03	0.4999	0.4941
Average	0.5008	0.5009
Ave ETA20d-TR01-03	0.3041	0.3098

Comparing the results of the ETA20d and ETA20d-TR series of the acceleration functions, it can be seen that the average error of the first series AFs is 0.6284 m/s^2. While this numerical value for the second series AFs is 0.5009 m/s^2, which is reduced about 20 percent. In addition, the error of average response of the first series AFs is 0.4564 m/s^2. While the same amount for the new AFs is 0.3098 m/s^2, which is improved about 27 percent. It can be concluded that the utilization of the three AFs reduces the error by more than 38 percent as well.

5. OPTIMIZATION OF LONG DURATION ET ACCELERATION FUNCTIONS

Duration of the ET acceleration functions produced previously was 20 seconds. If the duration of the AFs is increased from 20 seconds to 40 seconds, the number of acceleration points as the optimization variables is increased from 2048 points to 4096 points. Consequently, the efficiency of the optimization procedure seriously declines. On the other hand, by increasing the size of the objective matrix, the utilization of generated code for least squares optimization is nearly impossible. Since for transforming the initial objective matrix into a compressed matrix, a few objective times could be subjected to the optimization, which increase the errors and lead to degraded results. Furthermore, due to the increase of number of variables, the computational demand is significantly increased; as a result, the convergence rate is decreased. Therefore, to produce AFs with longer duration, other techniques are required to be applied. The proposed idea for this issue is utilizing the same number of 2048 acceleration points with 0.02 seconds time steps for production of 40 seconds AFs; subsequently, transforming them into the AFs with 4096 acceleration points with 0.01 seconds time steps. After the primary experiments were conducted, a multi level optimization procedure was adopted for production of the 40 seconds ET acceleration functions:

In the first step, a 20 seconds AF is produced by 2048 acceleration points with a time step of 0.01 seconds. The process of the optimization is similar to the process applied for the optimization of the second series of the ETAFs. Subsequently, this AF is chosen as the initial point and the optimization is performed by the time step of 0.02 seconds. Hence, a 40 seconds AF is produced with 2048 acceleration points. In the next step, this AF is transformed into a 40 seconds one with 4096 acceleration points. In this regard, the average of two acceleration points is considered as the acceleration numerical value for the time step between the two primary time steps. For example, the average of two acceleration values in 0.02^{th} and 0.04^{th} seconds is assumed as the 0.03^{th} second acceleration value. Finally, 2048 acceleration points resulted from the average of the primary optimized acceleration points are considered as the variables of the optimization. Next, the objective function is assumed to keep the primary optimized points unchanged and the optimization process is conducted on the acceleration points between them, 2048 points, in a way that, these points take place between the primary acceleration points. Afterwards, the objective function with 4096 acceleration points and 0.01 seconds time step is calculated. The explained procedure is briefly illustrated in Figure 7.

In fact, by utilizing this procedure, a 40 seconds AF can be produced via two times optimization with 2048 variables, without dealing with the 4096 variables optimization process. It should be noted that only the duration of these AFs have been enlarged respect to 20 seconds ones and both have the same response range; thus the target time of 40 seconds AFs is the 20^{th} second and their responses reach twice the codified values at the 40^{th} second. The results of the optimization steps of the first 40 seconds ETAF by utilizing the aforementioned procedure are presented in Table 3. For each step, the numerical values of the base errors and the errors in optimized points have been calculated.

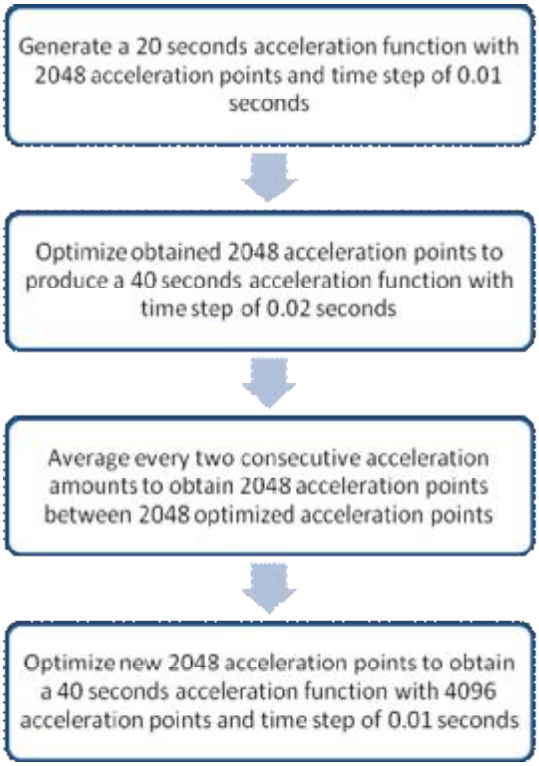


Figure 7. Optimization procedure of 40 seconds ET acceleration functions

Table 3. Errors in different steps of optimization of ETA40d01 acceleration function

Optimization step	Error in optimized points (m/s^2)	Error in base points (m/s^2)
1	0.5094	0.5378
2	0.3301	0.3647
3	0.5356	0.3741
4	0.3527	0.3705

The improvement of the results can be observed in different steps of the procedure. It should be noted that, in the third step, the numerical value of base error is acceptable; however the error for optimized periods has been increased. Due to the fact that these periods are chosen with the logarithmic distribution and the number of short periods is much more than long periods and considering the sensitivity of the time-history analysis to the time steps in short periods, the amount of error is increased [20]. Hence, the results of the third step are not satisfactory and performing another step of optimization is recommended in order the both errors to reach acceptable levels.

By applying this method, three 40 seconds AFs are produced, which are assumed to be the third series of the ET acceleration functions and are called ETA40d01-03. A sample of these AFs and its acceleration and displacement response spectra at 10^{th}, 20^{th}, 30^{th} and 40^{th} seconds

are presented respectively in Figures 8, 9. Moreover, the acceleration response spectra of the third series of the AFs and the average response spectrum of the three AFs, at the target time (the twentieth second) are depicted in Figure 10.

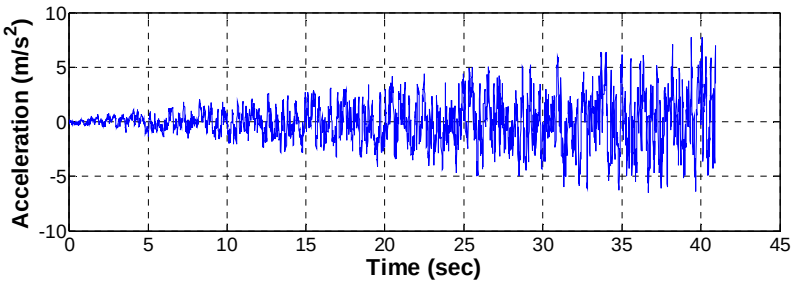
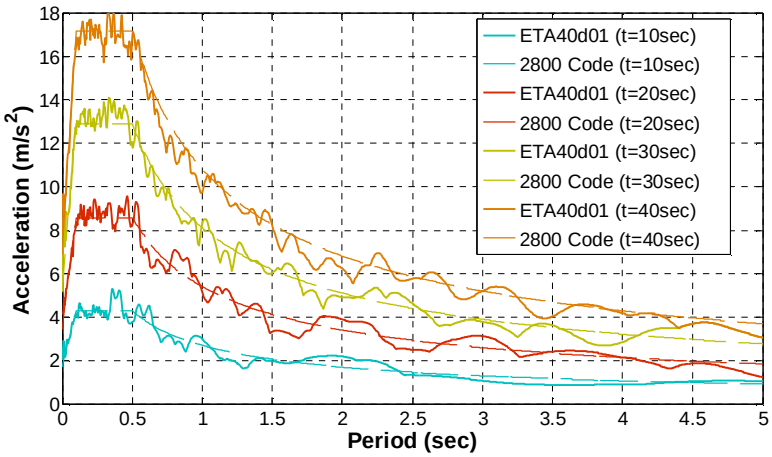


Figure 8. ETA40d01 acceleration function



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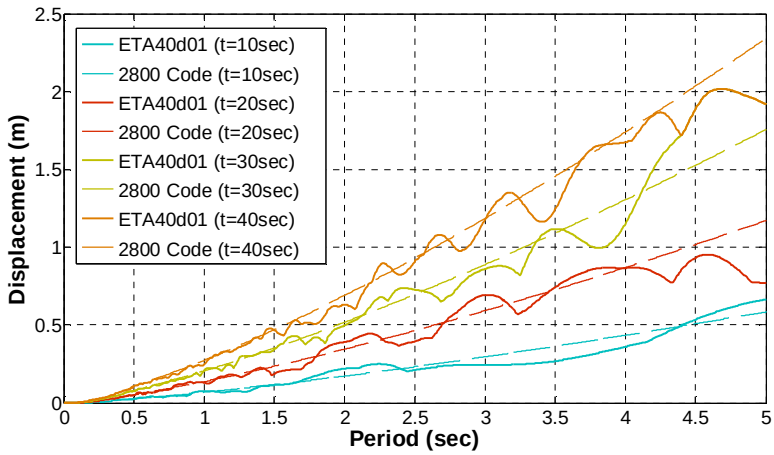

(b)

Figure 9. Response spectra of ETA40d01 at 10th, 20th, 30th and 40th seconds, (a) acceleration, (b) displacement

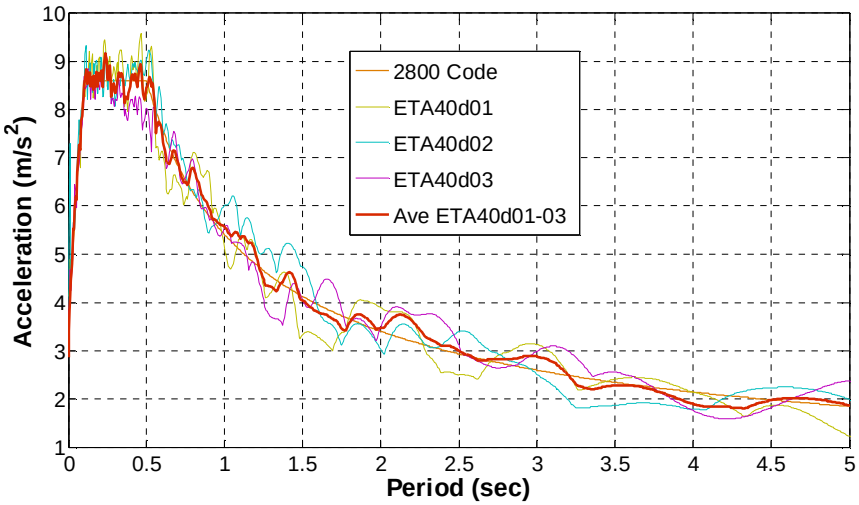

Figure 10. Acceleration response spectra of ETA40d01-03 at the target time (the 20th second)

In Table 4, the absolute errors for the 40 seconds AFs are listed in two states: (I) error in optimized points, (II) base error. In addition, the absolute error for the average response of the three AFs is calculated. In Figure 11, the errors of the first, second and third series of the ETAFs are compared, and the trend of error reduction from the first series to the third series could be clearly identified. In this figure, the error reduction by utilizing the three AFs for each series is further evident.

Table 4. Errors of acceleration responses of the third series ETAFs

Acceleration function	**Absolute Error (m/s^2)**	
	Optimized points	**Base points**
ETA40d01	0.3527	0.3705
ETA40d02	0.3965	0.4142
ETA40d03	0.4193	0.4173
Average	0.3895	0.4007
Ave ETA40d01-03	0.2217	0.2263

In order to study the effect of expanding the duration of the AFs in their results accuracy, the errors of the ETA20d-TR and ETA40d series of acceleration functions were compared. As can be observed, the average error of three second series AFs is 0.5009 m/s^2, while the same numerical value for the third series AFs is 0.4006 m/s^2, which is reduced about 20 percent. Furthermore, the average response error of three second series AFs is 0.3098 m/s^2, while the same numerical value for the third series AFs is 0.2263 m/s^2, which has improved by more than 26 percent.

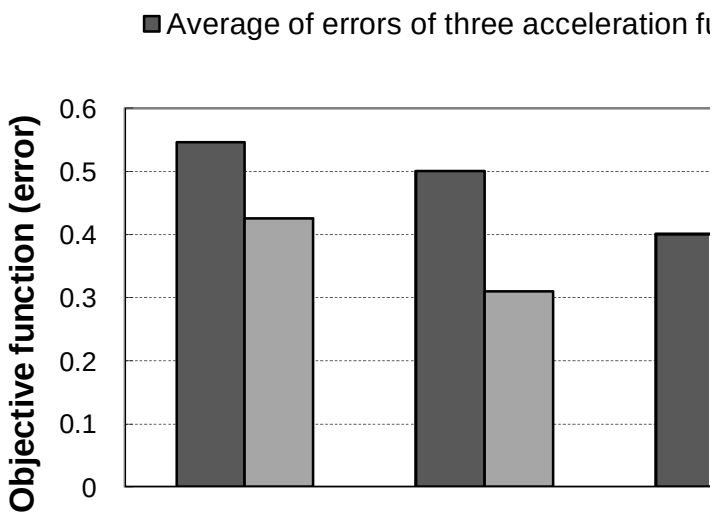

Figure 11. Comparison of three series of ET acceleration functions

By comparing the errors of the first series and the third series AFs, it can be observed that the average error of the first series AFs is 0.6284 m/s^2, while the same numerical value for the third series AFs is 0.4007 m/s^2, which is reduced about 36 percent. In addition, the average response error of three first series AFs is 0.4564 m/s^2, while the same numerical value for the third series AFs is 0.2263 m/s^2, which is improved about 50 percent. It can further be observed that the utilization of three AFs reduces the error by more than 43 percent.

6. COMPARISON OF ETAFS IN THE ANALYSIS OF SDOF SYSTEMS

In this section, four SDOF systems with natural periods of 0.5, 1, 2 and 4 seconds are studied, respectively. The damping ratio of these systems is assumed to be 5 percent, which the same value has been utilized to produce the ET acceleration functions. These systems are analyzed with the ETAFs and the time-history responses are compared with the target time-history response calculated using the definition of the Endurance Time method based on the spectral response associated with the standard 2800.

The ETA20d series of the ET acceleration functions are applied and the SDOF systems are analyzed with the three AFs (ETA20d01-03). The acceleration and displacement responses of each system are calculated and the average of the three responses is assumed as the final response of the system and is contrasted versus the target response. In Figure 12, the average acceleration and displacement responses of four SDOF systems are shown. Similarly, the SDOF systems are analyzed with the second series AFs (ETA20d-TR01-03) and the third series AFs (ETA40d01-03) and the results are compared with the target responses. Figures 13, 14 indicate the average acceleration and displacement responses for the second and the third series AFs. As can be observed, the results obtained from the ET analysis with the second series of AFs are more consistent with the target response compared with the results of the first series AFs. Moreover, the consistency of results for the third series AFs (40 seconds

AFs), is more reasonable than the results obtained from the both former series.

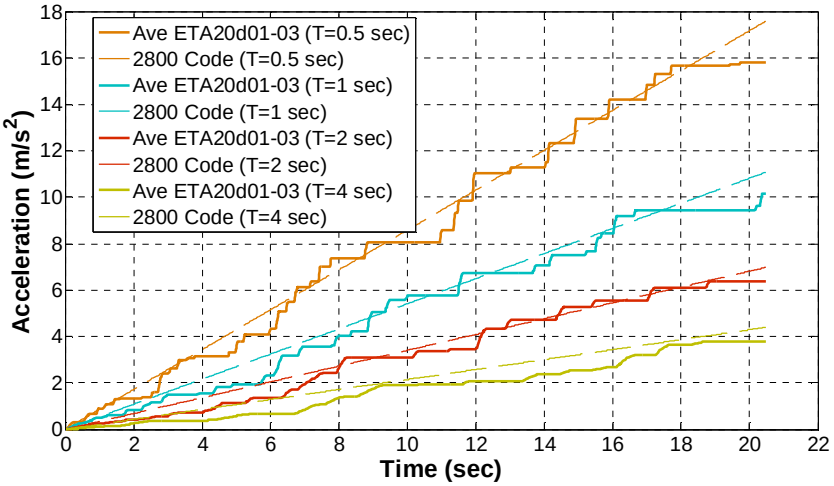

Figure 12. Acceleration response time-history of four SDOF systems for the first series ETAFs

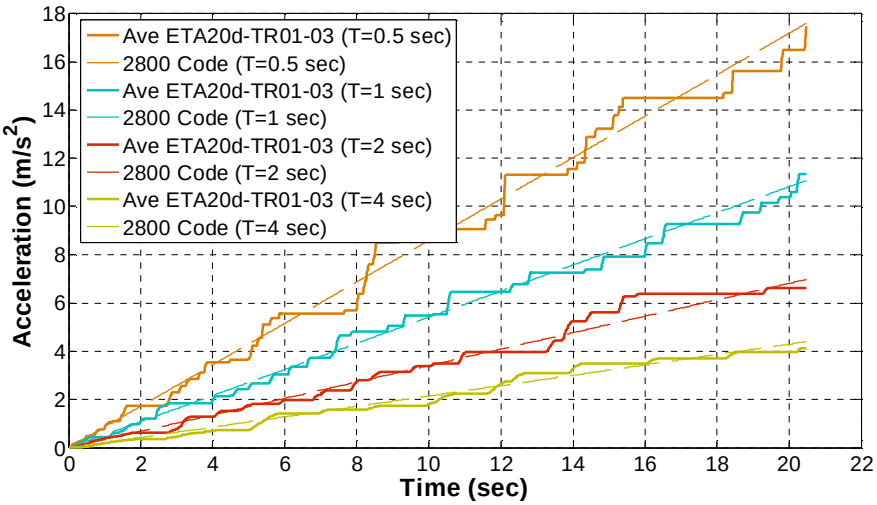

Figure 13. Acceleration response time-history of four SDOF systems for the second series ETAFs

In Table 5, the acceleration responses errors of each SDOF system under the ET acceleration functions are presented. Despite a number of exceptional cases, the descending trend of the error numerical values from the first series to the third series of the ETAFs can be clearly identified. Specifically, the reduction of error for the third series AFs is more significant. For instance, as for the SDOF system with the period of 0.5 seconds, the average acceleration response error of the first and the second series AFs are close to each other, about 0.55m/s^2; while the same numerical value for the third series AFs is about 0.30m/s^2, which is improved by 45 percent.

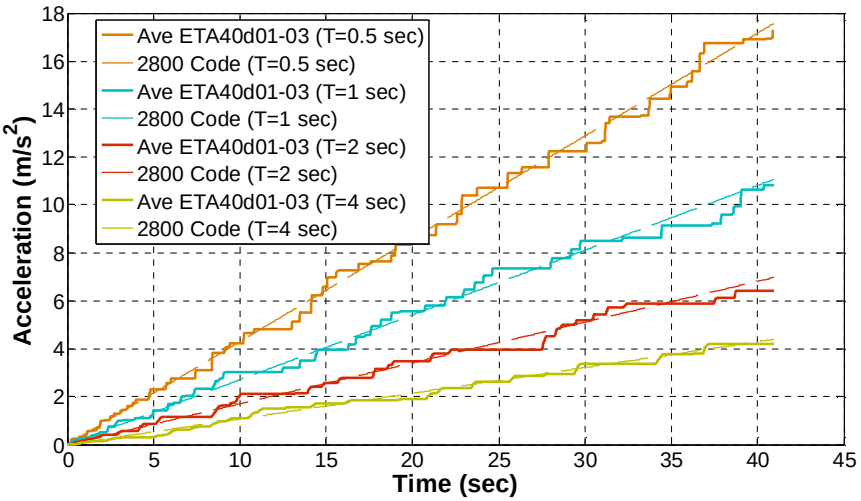

Figure 14. Acceleration response time-history of four SDOF systems for the third series ETAFs

In addition, for all the cases, the average response error of three AFs is reduced in comparison with the average of errors of three AFs responses. For example, as for the SDOF system with 4 seconds period, the average error of responses resulted from the three first series ETAFs is about 0.60 m/s^2, while the error of average response of the first series ETAFs equals to 0.48 m/s^2, which indicates 20 percent of error reduction. This error reduction for the second and the third series of the ETAFs is about 42 and 55 percent respectively.

Table 5. Acceleration responses errors for four SDOF systems under three series of the ETAFs

Acceleration function	**Absolute Errors of SDOF systems responses (m/s^2)**			
	T=0.5 sec	**T=1 sec**	**T=2 sec**	**T=4 sec**
ETA20d01	0.9076	0.6134	0.4235	0.4447
ETA20d02	0.8258	1.0034	0.6346	0.8646
ETA20d03	0.8993	0.7540	0.4803	0.5000
Average	0.8776	0.7903	0.5128	0.6031
Ave ETA20d01-03	0.5531	0.5278	0.3340	0.4783
ETA20d-TR01	1.0318	0.7001	0.3675	0.2767
ETA20d-TR02	0.7544	0.6256	0.5022	0.3362
ETA20d-TR03	0.7893	0.6298	0.4391	0.3773
Average	0.8585	0.6518	0.4363	0.3301
Ave ETA20d-TR01-03	0.5681	0.3033	0.3283	0.1887
ETA40d01	0.4913	0.4135	0.3633	0.2707
ETA40d02	0.6306	0.4664	0.3768	0.2986
ETA40d03	0.5520	0.7400	0.4076	0.2998
Average	0.5580	0.5400	0.3826	0.2897
Ave ETA40d01-03	0.2997	0.3222	0.2330	0.1226

7. SUMMARY AND CONCLUSIONS

Numerical simulation of seismic structural response is among the major challenges in earthquake engineering. There has been a remarkable progress in developing advanced software for this purpose in recent years. The intensive computational demand is still a considerable issue in practical applicability of many realistic simulation procedures capable of including complicated structural responses such as material and geometric nonlinearity. The Endurance Time (ET) method is a new tool for the seismic design of structures in which structures are subjected to a gradually intensifying dynamic excitation and their seismic performance is evaluated based on its response at different excitation levels. Consequently, substantial reductions in computational demand can be achieved when structural performance at various excitation intensity levels is to be predicted. Generating appropriate artificial dynamic excitations is essential for the ET method's success.

In this paper the basic numerical procedure for generating the ET acceleration functions and its formulation as a numerical optimization problem was presented. The Trust Region algorithm utilized in the developed optimization program exhibits high convergence rate in the optimization of the ETAFs. By the discrete time formulation of the ET method and defining the objective function of optimization in the matrix form, considering the computational limitations, the second series of ETAFs are produced in the linear range of structural analysis. It should be noted that, the required time for the optimization of these AFs is nearly one tenth of the time spent for the optimization of the original ETAFs. While, the average error of the second series of ETAFs is about 30 percent less than the average error of the first series of ETAFs.

Moreover, a procedure for extending the total duration of ETAFs without compromising accuracy and time efficiency was presented. The convergence and level of accuracy to be expected from generating the ETAFs was discussed by applying the generated AFs to SDOF systems as well. It can be concluded that the proposed procedures can be applied successfully to generate usable intensifying AFs which should be applied in response history analysis of structures utilizing the ET methodology.

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NOMENCLATURE

a_g	Acceleration function
B	Building response factor
dt	Time step of time history analysis
ET	Endurance Time method
$F(a_g)$	Objective function of optimization
g	Gravitational acceleration
I	Building importance factor
l	Number of discrete times
$M(a_g)$	Objective matrix of optimization

$N(a_g)$	Compressed objective matrix of optimization
m	Number of period points
n	Number of acceleration points
p	Integer parameter to determine time intervals
R	Response reduction factor
S_a	Spectral acceleration
S_{aC}	Codified acceleration response
S_{aT}	Target acceleration response
$S^a_{i,j}$	Maximum acceleration response for period T_i until time t_j
S_u	Spectral displacement
S_{uT}	Target displacement response
$S^u_{i,j}$	Maximum displacement response for period T_i until time t_j
T	Free vibration period
t	Time
t_{Target}	Target time
$u(t)$	Displacement response time history
$\ddot{u}_g(t)$	Ground acceleration time history
α	Weighting parameter in objective function of optimization
ω_n	Natural circular frequency
ξ	Damping ratio

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