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AN ANT COLONY SYSTEM ALGORITHM FOR THE TIME DEPENDENT NETWORK DESIGN PROBLEM

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ABSTRACT

Network design problem is one of the most complicated and yet challenging problems in transportation planning. The Bi-level, non-convex and integer nature of network design problem has made it into one of the most complicated optimization problems. Inclusion of time dimension to the classical network design problem could add to this complexity. In this paper an Ant Colony System (ACS) has been proposed to solve the Time Dependent Network Design Problem (T-NDP). The proposed algorithm has been used to solve different networks: A small size, a medium size and a large scale network. The results show that the proposed model has superior performance compared to the previous method proposed for solving the T-NDP.

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KEY WORDS: Time Dependent Network Design, Ant Colony System, Mashhad, Semnan

1. INTRODUCTION

The Network Design Problem (NDP) deals with the capacity expansion planning in transportation networks. This problem is known as one of the most challenging problems in mathematics and transportation engineering due to the non-convexity of the objective functions and the NP-Hard nature of the problem. Due to the differences in the objectives, traffic assignment procedures, decision variable types and finally the solution algorithms, variety of NDPs has been proposed previously in the literature. The NDP is traditionally posed in three categories based on the type of decision variable involved; Continuous Network Design (CNDP), Discrete Network Design (DNDP) and Mix Network Design

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(MNDP). In CNDP, the design variables related to the capacity expansions are continuous variables, which determine the amount of capacity expansion proposed for certain links in the network [1]. DNDP deals with discrete (usually binary) decision variables, which determine whether to perform (the value of the decision variable is equal to one) or not to perform (equal to zero) a capacity expansion project [2]. DNDP is theoretically a special case of CNDP which a link could be either expanded to it maximum capacity or could be left with its initial capacity [3] but practically DNDP is harder to solve due to the integrality of the decision variables [4]. When the decision variable is a mixture of both types of variables, NDP is categorized as MNDP [5].

The NDP problem is usually formulated as a bi-level optimization problem, which the lower level problem (User problem) is the so called User Equilibrium (UE) traffic assignment problem and the upper level problem is the planner's problem. Users selfishly seek to minimize their individual travel time, while the planner is interested in minimizing the total system travel time in the network. This structure resembles the Stakelburg, also known as the leader-follower game, in which the planner is the leader and the users are the followers of the game [6]. In this game upon the decision of the leader (planner in the upper level), the followers (users in the lower level) minimize their travel time. This game could also be formulated as a single level Mathematical programming with equilibrium constraints (MPEC) [7].

Different traffic assignment procedures have been previously used in NDP such as Deterministic User Equilibrium problem (DUE) [1, 5], Stochastic User Equilibrium (SUE) [8, 9] and Dynamic Traffic Assignment (DTA) [10, 11]. Demand elasticity is another issue that has been addressed previously by many researchers in order to make NDP more realistic [12-14]. Meanwhile, maximization of the Reserve Capacity (RC) [15], Consumer Surplus (CS) [16], Social Welfare (SW) [17] together with the minimization of total cost and travel time of the network have been considered as the objective function of many previous research on NDPs.

In terms of the solution algorithms, different deterministic, heuristic and meta-heuristic algorithms has been used to solve NDP. Due to the Non-Convexity and NP-Hard nature of the NDP many Meta heuristics such as Simulated Annealing [18-20], Genetic Algorithm [21, 22], Tabu Search [23], Particle Swarm Optimization [24], and Ant Colony Optimization [25, 26] have been recently used to solve NDP. There have been many efforts for deterministic solution of NDP with methods such as Decomposition and Branch and Bound, but they have failed to solve large scale problems in reasonable amount of time [27].

In this paper following the works of Lo and Szeto [28] which have formulated the Time Dependent Network Design Problem (T-NDP) an efficient solution algorithm will be proposed in order to solve large scale T-NDPs more efficiently. The rest of the paper is organized as follows; T-NDP is formulated in Section two. Solution algorithm based on ACS is presented in Section three. Section four belongs to case study, results discussion and sensitivity analysis and finally paper concluded in Section 5.

2. MODEL FORMULATION

In this paper, T-NDP has been formulated as a bi-level optimization problem. As mentioned previously, the lower level problem is the UE traffic assignment problem and the upper level problem is the design problem. In order to evaluate the proposed solution algorithm the result obtained will be compared with the Generalized Reduced Gradient (GRG) [29] method used originally by Lo and Sezeto [28]. It is worth mentioning that Lo and Szeto have used a single level MPEC formulation for T-NDP, while a bi-level optimization approach has been used in this paper.

Generally NDP could be formulated as follows:
In the above equations:

$$\begin{aligned} &(ULP)\, Max\ F\left(X, f\left(u, Q\right)\right) \\ &\qquad s.t.: H\left(X, f\left(u, Q\right)\right) \le 0 \\ &(LLP)\, Min f\left(u, Q\right) \\ &\qquad s.t.: h\left(u, Q\right) \le 0 \end{aligned} \tag{1}$$

where ULP is the upper level problem and LLP is the lower level problem. *F* is the vector of objective functions and *H* is the constraint set of the ULP. In LLP, *X* is the vector of design variables, *U* is the equilibrium flow on the network and *Q* is the demand set. Based on this general formulation of bi-level T-NDP, the following formulation has been proposed in this paper:

ULP:

$$\underset{f,y}{Max}\ CS = \varphi n \sum_t \sum_{rs} \left[\int_0^{q_t^{rs}} D_t^{rs^{-1}}(v)\, dv - u_t^{*rs} q_t^{rs} \right] \tag{2}$$

Subject to:

$$q_t^{rs} = D_t^{rs}\left(u_t^{*rs}, \tilde{q}_t^{rs}\right), \quad \forall r, s, t \tag{3}$$

$$q_t^{rs} = \frac{u_t^{*rs}}{m^{rs}} + \tilde{q}_t^{rs} \tag{4}$$

$$\tilde{q}_t^{rs} = \tilde{q}_{t-1}^{rs}\left(1 + h^{rs}\right) \tag{5}$$

$$c_a^0 + \sum_{i=1}^{t} y_{a,i} \le c_{a,Max} \qquad \forall a, t \tag{6}$$

$$y_{a,t} \geq 0 \tag{7}$$

$$IC_t = \sum_a g_{a,t}\left(y_{a,t}\right) \tag{8}$$

$$g_{a,t}\left(y_{a,t}\right) = k\, t_a^0\, y_{a,t} \tag{9}$$

$$TIC \leq TB \tag{10}$$

$$TB = \sum_t T_t \tag{11}$$

$$IC_t \leq T_t + \sum_{i=1}^{t-1}\left(T_i - IC_i\right) \tag{12}$$

$$IC_t \leq TB - \sum_{i=1}^{t-1} IC_i \tag{13}$$

LLP:

$$Min \sum_a \int_0^{V_{at}} t_{at}(w)\,dw - \sum_{rs} \int_0^{q_t^{rs}} D_{rs}^{-1}(w)\,dw \tag{14}$$

Subject to:

$$V_{at} = \sum_{rs} \sum_p f_{pt}^{rs} \delta_a^p \tag{15}$$

$$u_{pt}^{rs} = \sum_a t_{at} \delta_a^p \tag{16}$$

$$\sum_p f_{pt}^{rs} = q_t^{rs} \tag{17}$$

$$f_{pt}^{rs} \geq 0 \tag{18}$$

$$q_t^{rs} \geq 0 \tag{19}$$

$$t_{at} = t_{at}^{0}\left(1+0.15\left(\frac{V_{at}}{c_{at}^{0}+\sum_{i=1}^{t} y_{at}}\right)^{4}\right) \tag{20}$$

$D_t^{rs^{-1}}(.)$: Is the inverse demand function of the Origin Destination (OD) pair *r* and *s* in time period *t*.

n : Factor that converts hourly network flow to annual network flow.

φ : Network users travel time monetary value.

$u_{p,t}^{rs}, u_t^{*rs}$: The travel time between *r* and s though path *p* and shortest path travel time of the OD pair, respectively.

t_{at} : The travel time of link *a* in time period *t*.

V_{at} : The traffic flow on link *a* in time period *t*.

$\delta_{a,p,t}^{rs}$: A binary variable indicating whether path *p* between *r* and *s* passes though link *a* (the value of the variable will be equal to *1*) or not (value of zero) in time period *t*.

q_t^{rs} : The demand between OD pair *r* and *s* in time period *t*.

f_{pt}^{rs} : The path flow between origin destination pair *r* and *s* though path *p* in time period *t*.

t_a^0 : The free flow travel time of link *a*.

c_a^0 : The initial capacity of link *a*.

$y_{a,t}$: The amount of capacity increase on link *a* in time period *t*.

$c_{a,Max}$: The maximum possible capacity increase on link *a*.

$\tilde{q}_t^{rs}$: The potential demand of OD pair *r* and *s* in time period *t*.

h^{rs} : The demand growth factor between OD pair *r* and *s* in years of design period.

m^{rs} : The parameter of the demand model.

IC_t : Infrastructure building cost at time period *t*.

$g_{a,t}(.)$: The link capacity expansion cost function.

TIC : Total infrastructure cost.

TB : Total budget.

T_t : The budget in time period *t*.

This is a bi-level optimization problem, where ULP is to maximize the consumer surplus and LLP is an elastic demand user equilibrium traffic assignment problem.

The first constraint in the upper level problem is a definitive constraint, which computes the demand in each time period (Eq. (3)). The second constraint is a simple growth model that increases the base demand of the network (Eqs. (4) and (5)). Eqs. (6) and (7) define the maximum allowable capacity expansion on each link. Eq. (8) computes the total expansion cost in each time period. The expansion cost for each link is defined by Eq. (9). The total construction cost is limited to the total budget and the total budget in the summation of budgets in each time period; this is given by Eqs. (10) and (11). The last constraint in ULP is the budget accumulation constraint (Eqs. (12) and (13)).

As LLP assigns the traffic flow in each time period to the road network, the first two definitive constraints define the link flows as a function of path flows between each OD pair and the rout travel time, respectively (Eqs. (15) and (16)). Constraint (17) assures that all the demand is assigned to the active routes between each OD pair and Eq. (20) is the link travel time function.

3. SOLUTION ALGORITHM

Lo and Szeto [28] have used reduced gradient method to solve T-NDP. This paper uses Ant Colony System (ACS) to solve T-NDP. This meta-heuristic algorithms was first developed by Dorigo [30] as Ant System (AS) algorithm. After some modifications in the rules related to the movement, local and global updating of pheromone, Ant Colony system (ACS) was developed [31]. Interested reader are referred to [32] and [33] for more information on the details of ACS, as these have not been presented in this paper for the sake of brievity.

In this paper T-NDP is converted into a Traveler Salesman (TS) problem. The problem is a multi layer network, with each layer representing each time period (one year in this paper) and each node representing a project. As each project might be selected in any time period, the nodes are replicated in each layer. Thus, the selection probability of project j with the predecessor project i by ant k in time period t is equal to $P_{ij}^{k}(t)$. This selection probability depends on the amount of pheromone $\tau_{ij}(t)$, which represent the amount of utility associated with the link in the hypothetical network, and the inverse cost of project selection, denoted by $\eta_{ij}(t)$. In order to compute the utility of each project in the first iteration of the algorithm, the effect of each individual project has been evaluated by assuming that only that single project has been selected. This utility is used in order to compute the initial Pheromone on the network. The initial inverse cost is also considered as the cost of implementing any single project. In ACS's project selection routine, a random number q, is drawn. If the random number q is larger than a pre-specified q_0 then the node with the highest $\left\{\left[\tau_{ij}(t)\right]\times\left[\eta_{ij}(t)\right]^{\beta}\right\}$ will be selected. In this function β is a constant that shows the relative importance of the amount of pheromone and the inverse cost of the project. If q was smaller than q_0 the nodes will be selected by a probability as in Eq. (21):

$$P_{ij}^{k}(t)=\frac{\left[\tau_{ij}(t)\right]\times\left[\eta_{ij}(t)\right]^{\beta}}{\sum_{\ell\in J_i^k}\left[\tau_{i\ell}(t)\right]\times\left[\eta_{i\ell}(t)\right]^{\beta}} \tag{21}$$

At the end of each iteration, the amount of pheromone on the path traversed by the ant with the best objective function will be updated. This will be done using Eqs. (22) and (23):

$$\tau_{ij}(t) = (1-\rho) \times \tau_{ij}(t-1) + \rho \times \Delta\tau_{ij} \tag{22}$$

$$\Delta\tau_{ij} = \begin{cases} \left(L_{gb}\right)^{-1} & if\ (i,j) \in Global\ Best\ Tour \\ 0 & Otherwise \end{cases} \tag{23}$$

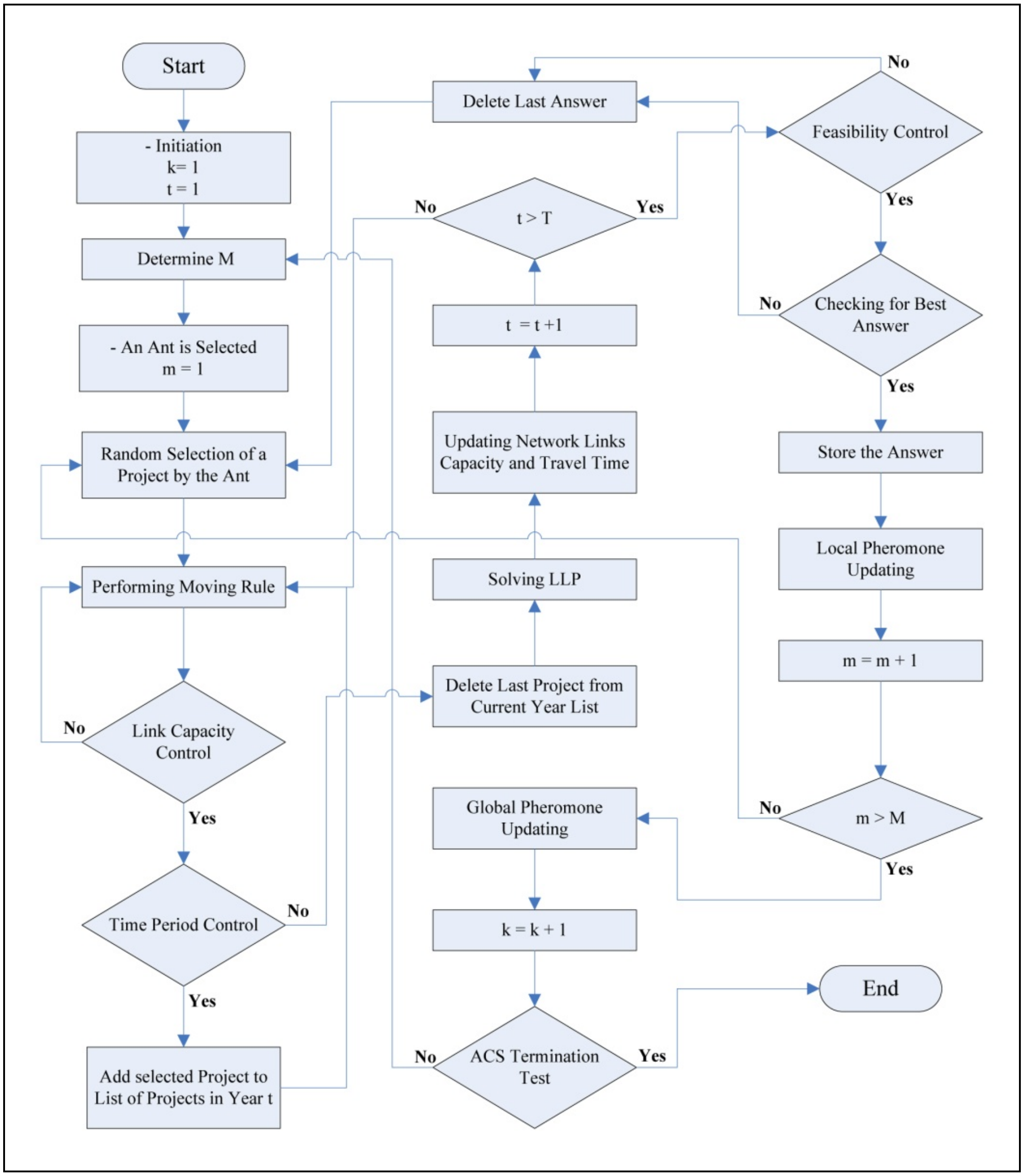


Figure 1. Overall solution algorithm

In these equations ρ is the evaporation parameter, which takes a value between zero and one. L_{gb} is the best solution of ACS so far. The local updating role dynamically changes the utilities of the links. In T-NDP, as the demand and traffic conditions change in each year, the utilities of the links change accordingly.

Three controls have been added to the algorithm in order to check for the constraints: the expansion capacity of links, time period constraint and feasibility. The capacity expansion constraint corresponds to Eqs. (6) and (7). If the constraint is met, the algorithm continues its normal operation, otherwise the selected project is not acceptable and will be omitted from the list of selected projects. If none of the constraints are restricting, several projects could be selected in each time period. The second controller checks for some conditions and if the number of projects in each time period exceeds the allowable number of projects in that time period it will be omitted from the list of projects. Eqs. (8), (9), (12) and (13) will be checked in this control. Finally, the third control checks for constraints (10) and (11), which are the total budget in the time horizon of the problem. The proposed T-NDP algorithm has been coded and implemented in the Visual Basic programming language. Figure 1 summarizes the solution algorithm in the form of a flowchart. In this flowchart *M* refers to the number of ants in ACS, *T* is a refrence to the length of design period; *m* is the current number of ants, *t* is the current year and *k* is the iteration counter.

4. CASE STUDY

In order to check for the quality of results and computational performance of the algorithm, some example networks are used here. The network first consists of 5 origin destination pairs, 9 nodes and 12 links. The network of Semnan City has been also used as a medium, realistic size network, which consists of 54 zones, 79 nodes and 182 links. Finally, a large scale network, Mashhad, with 724 nodes, 253 zones and 1154 links has been tested in this paper. In all three cases, the networks are assumed to be directed and the time horizon is 5 years. The parameters of the model are given in Table 1 and are identical for all the networks. The budget constraints are also assumed to be 300.000 and 4.000.000 and 5.000.000 units for each network, respectively. Because the large number of OD pairs the travel demand matrixes are not presented. The results are given in Table 2, compared to the one proposed by Lo and Szeto [28]. Figure 2 shows the comparative results of the model graphically.

As could be seen in Table 2 and Figure 2, as the size of the test networks increase, the proposed algorithm shows better results compared to GRG. In the small size test network, both the proposed algorithm and GRG have converged to the same results; this is while the proposed algorithm has shown a 34.91% improvement in the run time. As the size of the network grows, for instance in Mashhad network, the improvement in the consumer surplus in 26.5% and total travel time is 12.5%, while the run time has been reduces by more than 77%. This shows the superior performance of the proposed algorithm compared to GRG.

Table 1: Model parameters

$h^{rs}\quad \forall r,s$	$m^{rs}\quad \forall r,s$	n	k	φ
0.08	-1	8760	200	3000

Table 2: Results of model application

	Total Network Consumer Surplus		Design Year Average Network Travel Time		Solution Runtime	
Network	ACS	GRG	ACS (Sec)	GRG (Sec)	ACS (Sec)	GRG (Sec)
Test	1.13×10^{11}	1.13×10^{11}	221	221	1182	1816
Semnan	6.67×10^{14}	5.92×10^{14}	1240	1417	9082	23123
Mashhad	3.51×10^{19}	2.58×10^{19}	2376	2907	27477	120242

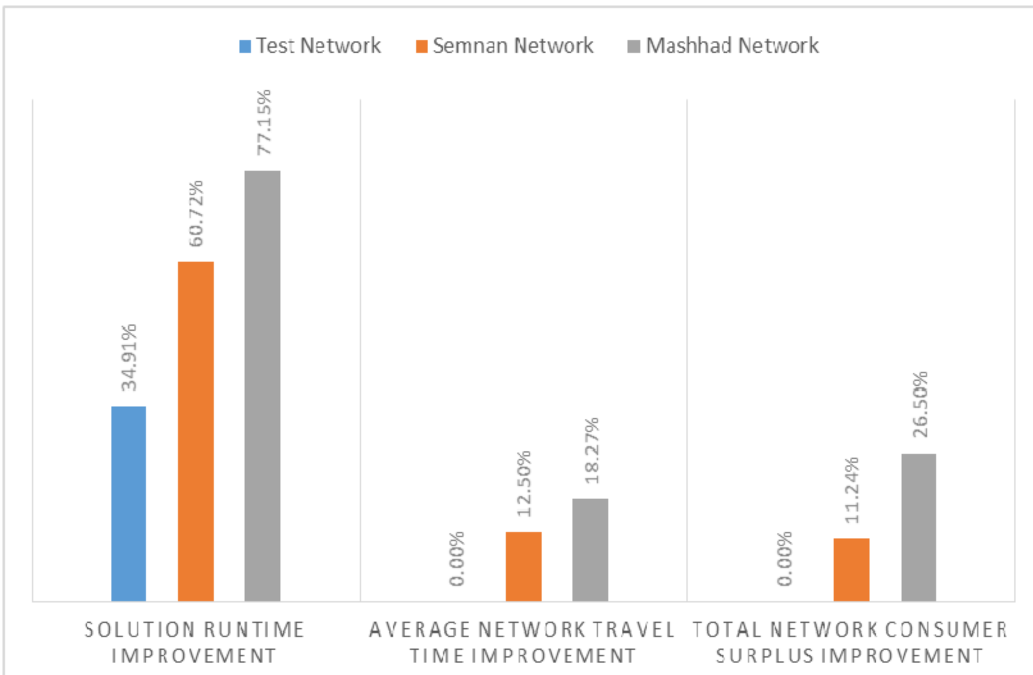
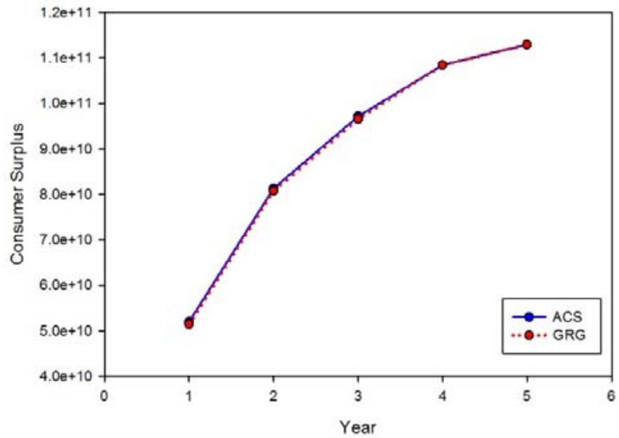


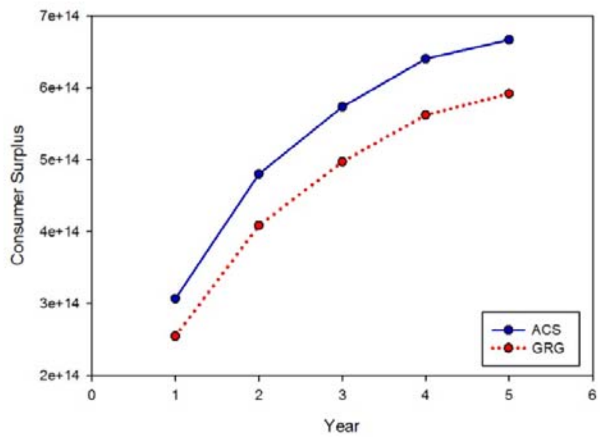
Figure 2. Percentage of the improvement of the proposed algorithm compared to GRG

Figures 3a and 3c show the annual improvement of the proposed algorithm and GRG. Similarly, the proposed algorithm shows superior results in larger networks. This higher performance has been maintained in all time periods in the larger networks.

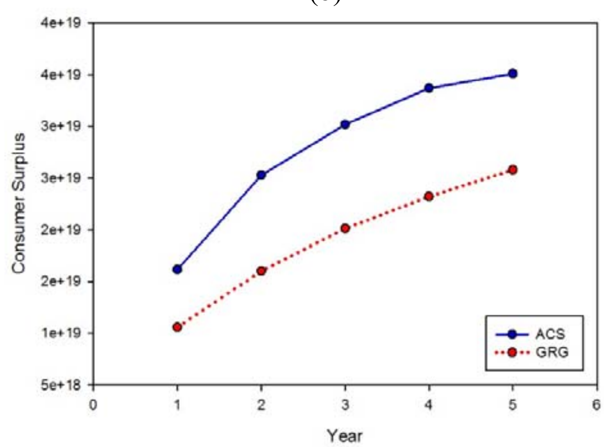
Sensitivity analysis of the parameter of ACS has been also performed in this paper. Number of Ants, q_0, β and ρ have been selected for sensitivity analysis. Figures 4a and 4d show the results of these analyses. The vertical axis shows the changes in the objective function, while the parameters value change according to the horizontal axis.



(a)



(b)



(c)

Figure 3. Comparison of the consumer surplus for the proposed ACS and GRG in the: (a) test network; (b) Semnan; (c) Mashhad

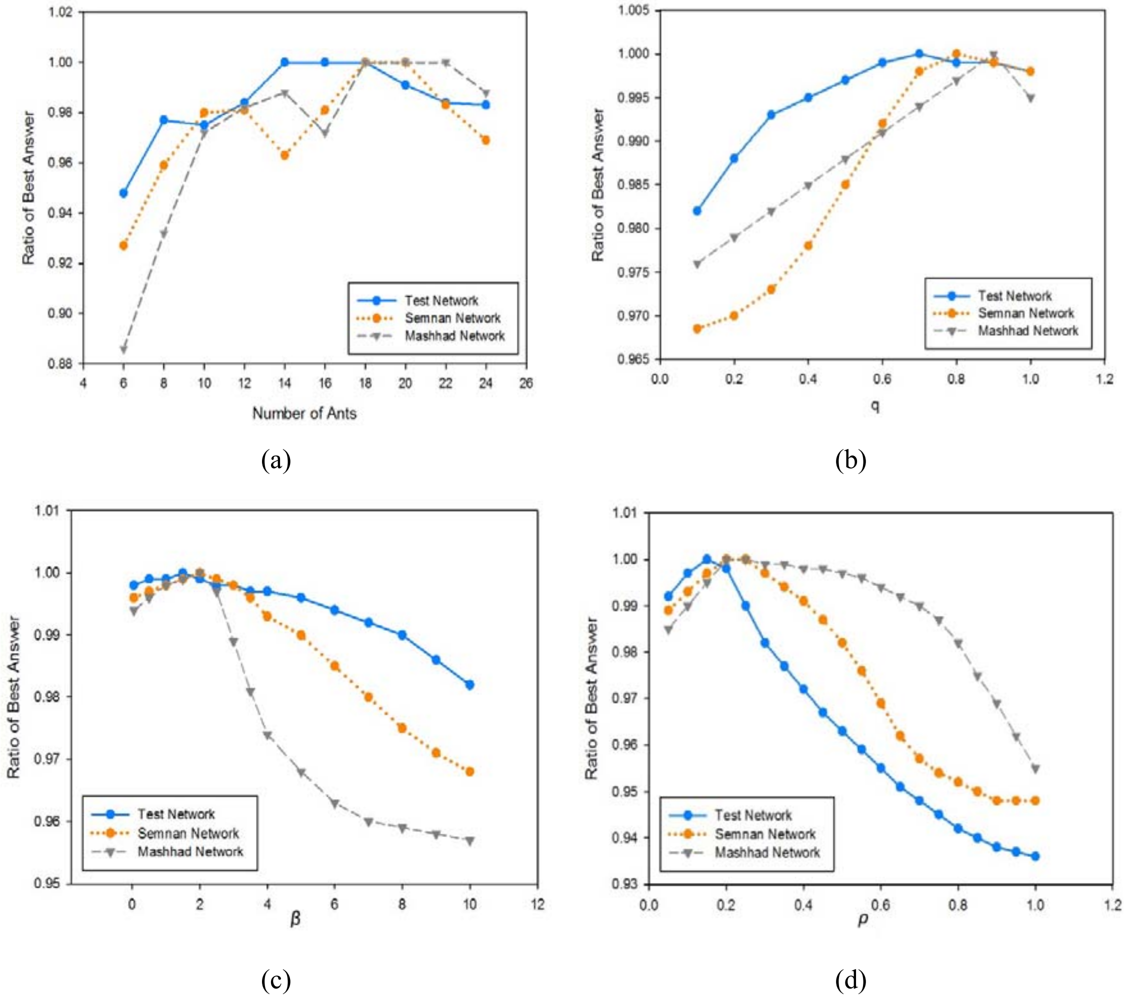


Figure 4. Sensitivity analysis of the objective function with respect to: (a) number of ants; (b) q_0, (c) β; (d) ρ

Figure 4a shows that as the size of the network increases the objective function becomes more sensitive to the number of ants. Meanwhile, in all three networks, increasing the number of ants improves the objective function up to a specific value. This specific value increases as the size of the network increases. Figure 4b shows the sensitivity analysis of the parameter q_0. Although the optimal value of this parameter is similar for all three networks, the pattern of changes differ in them.

As shown in Figure 4c, the network size affects the sensitivity of the objective function with respect to parameter β. Larger networks are more sensitive to changes in β. On the contrary, the sensitivity of the objective function with respect to evaporation rate (ρ), is higher in the smaller networks as shown ih Figure 4d. Summary of sensitivity analysis of ACS parameters are outlined in Table 3.

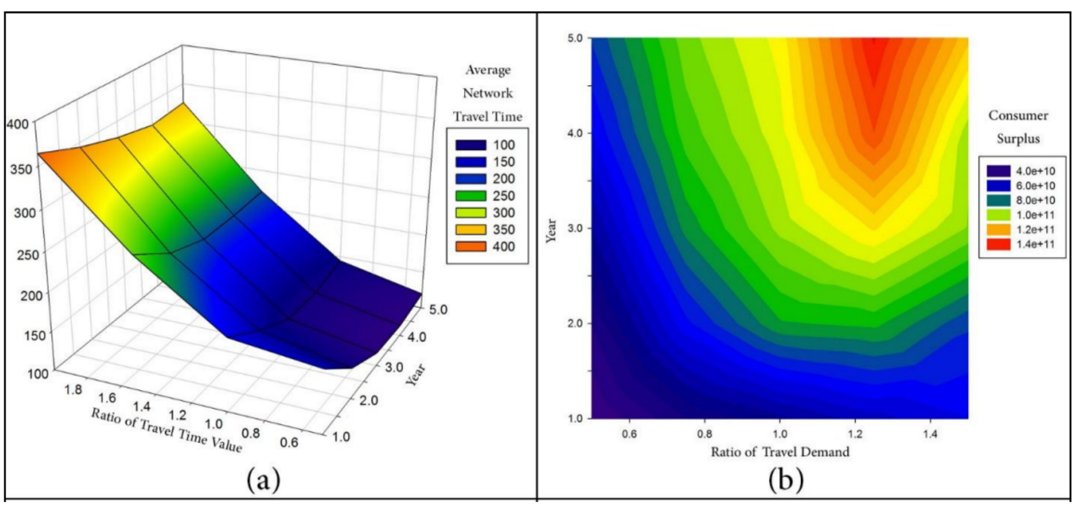
Table 3: Summary of Ant Colony Parameters Sensitivity Analysis

	Number of Ants		*q*		*β*		*ρ*	
Network	Best Value	IPAR*	Best Value	IPAR	Best Value	IPAR	Best Value	IPAR
Test	14-16-18	5.2%	0.7	1.8%	1.5	1.8%	0.15	6.4%
Semnan	18-20	7.3%	0.8	3.15%	2	3.2%	0.2-0.25	5.2%
Mashhad	18-20-22	11.4%	0.9	2.4%	2	4.3%	0.2-0.25	4.5%

*IRAR: Improvement Percentage in Analysis Range

In overall, based on Figure 3 and Table 3 it could be concluded that the number of ants is the most effective parameter in T-TNDP.

In T-NDP, the parameters of users travel time monetary value and travel demand in base year are constants which are presupposed and both could have significant impacts on results. According to best value of ACS parameters that were determined for each of three networks in pervious section, a sensitivity analysis of above mentioned parameters has been performed. The results are given in six segment of (a) to (f) in Figure 10. In graphs, both parameters appear in form of a ratio of presupposed values. This figure shows the paired graphs (a,b), (c,d) and (e,f) which are respectively related to test network, Semnan and Mashhad. By comparing graphs a, c and e it is observed that the increase in travel time value leads to improvement in average network travel time. Comparison of these graphs shows that for any travel time value, networks become improved in design period; with the increase in ratio of travel time value the improvement slope becomes slower. In addition, based on network size, when network size become larger, travel time value became more important and slope of the graph on travel time value axe become steeper. As a result of these two findings, higher ratios of travel time value lead to smaller improvement in larger networks.



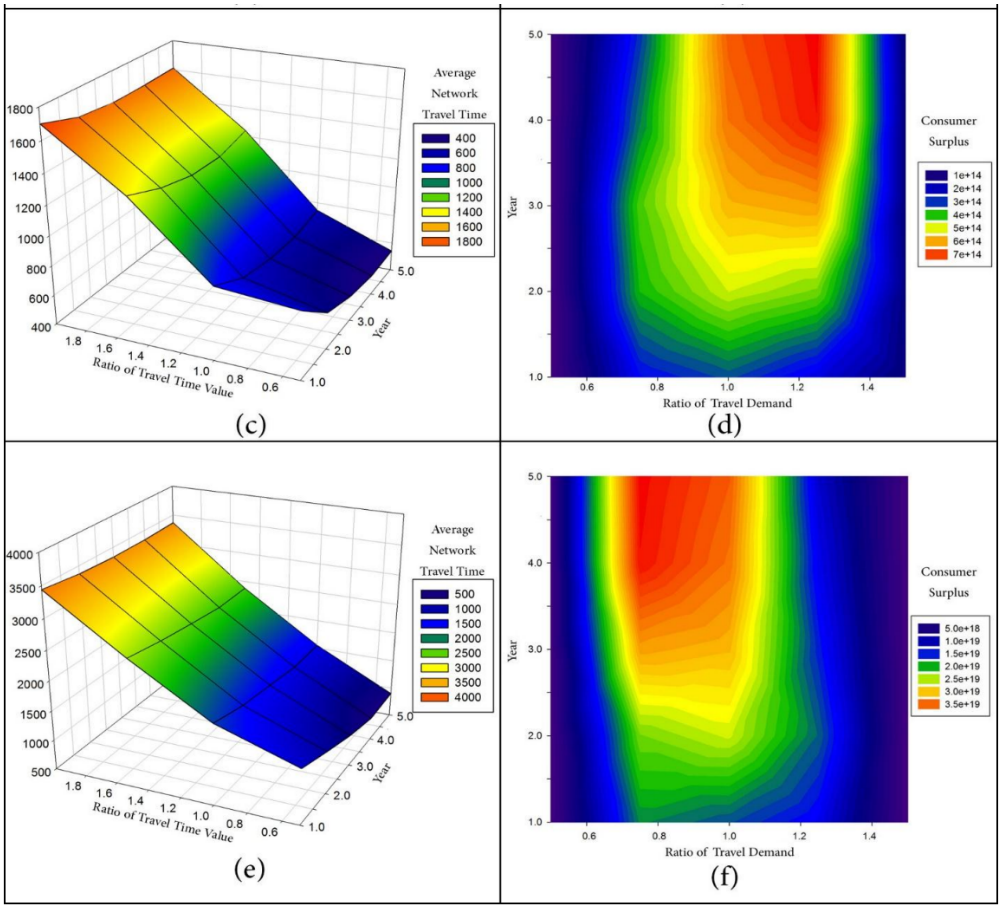


Figure (10) sensitivity analysis of travel time monetary value and travel demand

The comparison of graphs b, d and f shows that in each of network, in a specific range of travel demand ratio to presupposed value, the network design is more beneficial and consumer surplus index grows during design period. Nevertheless, there are clearly two points worth to mention: first the increase in network size results in a reduction in the range of travel demand ratio which leads to better answers. This range is illustrated with warm colors in graphs. The second point concerns the fact that each increass in the network size leads to a further reduction in travel demand ratio that eventuate more consumer surplus. In the other words, becasue there has been a constant amount of budget dedicated to all ratios of travel demand, an increase in travel demand will lead to a less efficient model for larger networks.

6. CONCLUSIONS

In this paper ACS has been applied to T-NDP. The results have shown that the proposed algorithm is both more accurate and faster than the previous methods proposed to solve T-

NDP, namely the GRG algorithm. As the network design becomes more important in larger networks, and as the proposed algorithm shows better performance with any increase in the size of the network, it could be more appropriate for large scale networks. For large scale networks the proposed algorithm has been shown to be approximately four times faster than the traditional GRG. Meanwhile, as T-NDP is a non-Convex problem, deterministic optimization algorithm usually do not converge to the global optimal solution, as could be seen in the case studies. Two sensitivity analyses have been performed for ACS and T-NDP parameters. The first analysis shows that network size has a direct correlation with the number of ants. As network size becomes larger more ants could be more effective to gain better answer. In contrary, evaporation rate and β have inverse effect and as network size become smaller, less value of these parameters lead to better answer of T-TNDP. Based on the second analysis, it is shown that travel time value, network size and average network travel time, have positive correlation with each other. On the other hand, the correlation among travel demand, network size and social benefits of network design, which is calculated in form of consumer surplus, is negative.

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