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ROBUST RESOURCE-CONSTRAINED PROJECT SCHEDULING WITH UNCERTAIN-BUT-BOUNDED ACTIVITY DURATIONS AND CASH FLOWS
I. A NEW SAMPLING-BASED HYBRID PRIMARY-SECONDARY CRITERIA APPROACH

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ABSTRACT

This paper, we presents a new primary-secondary-criteria scheduling model for resource-constrained project scheduling problem (RCPSP) with uncertain activity durations (UD) and cash flows (UC). The RCPSP-UD-UC approach producing a "robust" resource-feasible schedule immunized against uncertainties in the activity durations and which is on the sampling-based scenarios may be evaluated from a cost-oriented point of view. In the presented approach, it is assumed that each activity-duration and each cash flow value is an uncertain-but-bounded parameter, which is characterized by its optimistic and pessimistic estimations. The evaluation of a given robust schedule is based on the investigation of variability of the makespan as a primary and the net present value (NPV) as secondary criterion on the set of randomly generated scenarios given by a sampling-on-sampling-like process. Theoretically, the robust schedule-searching algorithm is formulated as a mixed integer linear programming problem, which is combined with a cost-oriented sampling-based approximation phase. In order to illustrate the essence of the proposed approach we present detailed computational results for a larger and very challenging project instance. A problem specific fast and efficient harmony search algorithm for large uncertain problems will be presented in a forthcoming paper.

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KEY WORDS: project scheduling; scheduling under uncertainty; robustness

1. INTRODUCTION

Traditionally, project schedule uncertainty has been addressed by considering the uncertainty related to activity duration. In general, there are two approaches to dealing with uncertainty in a scheduling environment (Herroelen and Leus [1], and Van de Vonder et al [2]) proactive and reactive scheduling. *Proactive scheduling* constructs a predictive schedule that accounts for statistical knowledge of uncertainty. The consideration of uncertainty information is used to make the predictive schedule more *robust*, i.e., insensitive to disruptions. *Reactive scheduling* involves revising or re-optimizing a schedule when an unexpected event occurs. At one extreme, reactive scheduling may not be based on a predictive schedule at all: allocation and scheduling decisions take place dynamically in order to account for disruptions as they occur. A less extreme approach is to reschedule when schedule breakage occurs, either by completely regenerating a new schedule or by repairing an existing predictive schedule to take into account the current state of the system.

According to the author's opinion, from managerial point of view the "rescheduling of rescheduling" like reactive process, as a problem solving conception, is far from the reality. To avoid the unavoidable combinatorial explosion of scenario-oriented approaches, we have to rethink everything from the beginning. In other words, we have to go back to the proactive schedule, and have to immunize against the possible disruptions. In this paper, we present a new idea about the robustness combining with a cost-oriented uncertainty investigation. The result of the new approach is a makespan minimal robust proactive schedule, which is immune against the uncertainties in the activity durations and which can be evaluated from a cost-oriented point of view on the set of the uncertain-but-bounded duration and cost parameters using a sampling-based approximation.

2. PROBLEM FORMULATION

In this section, we present a new approach for resource-constrained projects with uncertain activity durations and cash flow values (RCPSP-UD-UC). The approach can be used in the project-planning phase to seek answers to the various types of "what if …" questions, therefore, without loss of generality, in this paper, we assume that the resource requirements of the activities, the resource availabilities and the cash-flow-oriented interest rate are known crisp values according to a worst-case (pessimistic) scenario. We have to mentioned, that in the real-word project scheduling problems, the "optimal" performance obtained using conventional deterministic methods can be dramatically degraded in the presence of sources of uncertainty. In this paper, we assume that the only sources of uncertainty are the variability of the activity durations and the cash flow values. The approach produces "robust" schedules, which are immunized against uncertainties in the activity durations. The primary optimality criterion is defined as a linear combination (weighted sum) of the optimistic and pessimistic resource-feasible makespan. In the presented approach, each activity-duration is uncertain-but-bounded parameter, which can be generated from a uniform distribution. Naturally, this simple description of the future can be replaces by a more sophisticated probabilistic or possibilistic imagination. According to our experiences, the presented optimization model is not so sensitive (practically invariant) to the real meaning of the uncertain-but-bounded parameters, which probably may be the result of the "robust nature" of the Central Limit Theorem (CLT).

Theoretically, the optimal robust schedule searching process can formulated as a multi-objective mixed integer linear programming problem (MOMILP), which generate the Pareto-front as a result. In this paper, we replaced the MOMILP with a MILP by scalarization, defining the primary optimality criterion as a sum of the optimistic and pessimistic makespan. Naturally, the weights of the linear combination have a very important meaning from managerial point of view, because these are able to express the personal preferences of a project manager according to his/her risk taking (avoiding) habit. The resulting MILP can be solved directly in the case of small-scale projects within reasonable time. The proposed model is based on the so-called “forbidden set” concept. The output of the model is the set of the optimal conflict repairing relations. Therefore, after inserting the conflict-repairing relations, we get a robust schedule, which is invariant to the uncertain activity durations and the activity movements bounded by the slacks.

We have to mention it, that the result of the MILP formulation is only one makespan minimal robust schedule, which not necessarily will be cost-effective in the defined uncertain environment. The reason is simple: According to the uncertain cash flow values, we have to investigate the uncertain cost-oriented secondary criterion (namely, the variability of NPV values) for all possible uncertain activity duration combinations in the best-worst project duration range. Naturally, the combinatorial explosion (the huge number of alternative scenarios with variable cash-flow values) prevents the exact investigation. Therefore, only one way remains to manage the problem, which means a sampling-based approximation of the cost-oriented schedule characteristics on a sampling-based approximation of the duration-oriented schedule characteristics. When we generate only one makespan-minimal robust schedule, which is invariant to the uncertain activity durations and generate a random scenario set which is large enough to approximate the worst-best NPV values then we know nothing about the competitive alternative solutions. Therefore, we have to generate alternative schedules using different resource-conflict repairing mechanisms; and in the approximated best-worst range, we have to approximate the worst-best NPV values to select an acceptable (not necessarily optimal) solution. In other words, without an appropriate heuristic, which is able to generate alternative robust solutions within reasonable time, we are unable to manage the problem.

In order to model uncertain activity durations in projects, we consider the following resource constrained project-scheduling problem: A single project consists of N real activities. Each activity-duration D_i, $i \in \{1,2,...,N\}$ is a discrete (positive) random variable:

$$D_i \in \{A_i, A_i+1,...,B_i\} \tag{1}$$

where A_i and B_i are the optimistic and pessimistic estimations of D_i, respectively.

The activities are interrelated by precedence constraints: Precedence constraints - as known from traditional CPM-analysis - force an activity not to be started before all its predecessors are finished. These are given by network relations $i \rightarrow j$, where $i \rightarrow j$ means that activity j cannot start before activity i is completed. Furthermore, activity $i = 0 \left(i = N+1\right)$ is defined to be the unique dummy source (sink). Let IP_i, $i \in \{1,...,N+1\}$ denote the set of immediate predecessors for activity i and let NR be the set of the network relations.

Let R denote the number of renewable resources required for carrying out the project. Each resource $r \in \{1,...,R\}$ has a constant per period availability R_r. In order to be processed, each real activity $i \in \{1,2,..,N\}$ requires $R_{ir} \geq 0$ units of resource $r \in \{1,\ldots,R\}$ over its duration.

Let $PS = \left\{ i \rightarrow j \mid i \neq j,\ i \in \{1,...,N\},\ j \in \{1,...,N+1\} \right\}$ denote the set of predecessor-successor relations. A schedule is network-feasible if satisfies the predecessor-successor relations:

$$S_i + D_i \leq S_j \text{, if } i \rightarrow j \in PS \tag{2}$$

Let $\Re$ denote the set of network-feasible schedules. For a network feasible schedule $S \subset \Re$, let $A_t = \left\{ i \mid S_i \leq t < S_i + D_i \right\}, t \in \{1,...,T\}$ denote the set of active (working) activities in period t and let

$$U_{tr} = \sum_{i \in A_t} r_{ir}, t \in \{1,...,T\},\ r \in \{1,...,R\} \tag{3}$$

be the amount of resource r used in period t.

A network-feasible schedule $S \subset \Re$ is resource-feasible if satisfies the resource constraints:

$$U_{tr} \leq R_r,\ t \in \{1,...,T\},\ r \in \{1,...,R\} \tag{4}$$

Let $\overline{\Re} \subseteq \Re$ denote the set of resource-feasible schedules.

As we mentioned, the MILP formulation is based on the forbidden (resource constraint violating) set concept. A forbidden activity set is identified such that: (1) all activities in the set may be executed concurrently, (2) the usage of some resource by these activities exceeds the resource availability, and (3) the set does not contain another forbidden set as a proper subset. (see for example, Bell and Park [3]). A resource conflict can be repaired explicitly by inserting a network feasible precedence relation between two forbidden set members, which will guarantee that not all members of the forbidden set can be executed concurrently. We note, that an inserted explicit conflict repairing relation (as its side effect) may be able to repair one or more other conflicts implicitly, at the same time. Let F denote the number of forbidden sets. Let RR_f denote the set of explicit repairing relations for forbidden set F_f, $f \in \{1,2,...,F\}$. Let $RR = \left\{ \bigcup_f RR_f \,|\, f \in \{1,2,...,F\} \right\}$ denote the set of all the possible repairing relations. In the forbidden set oriented model (see Alvarez-Valdés and Tamarit [5]); a resource-feasible schedule is represented by the set of the inserted conflict repairing relations IR. According to the implicit resource constraint handling, in this model the resource-feasibility is not affected by the feasible activity shifts (movements). In the time oriented model (see Pritsker, Waters, and Wolfe [5]); a resource-feasible schedule is represented by the activity starting times. In this model, according to the explicit resource constraint handling, an activity movement may be able to destroy the resource-feasibility.

Let A and B denote the optimistic and pessimistic makespan of a resource-feasible schedule set, respectively. Let $\{A^*, B^*\}$ denote the set of project's makespan in the optimal resource-feasible schedule set. Let $\overline{T}$ denote an upper bound of the optimal project's makespan in the pessimistic case ($B^* \le \overline{T}$). Let $\overline{T} = \sum_{i=1}^{N} B_i$, which is an "extremely weak" upper bound on the project's makespan B^*, and fix the position of the unique dummy sink in period $\overline{T}+1$. Naturally, this "weak" upper bound can be replaced by any "stronger" one. In our notation, the time periods are labeled by consecutive $t \in \{0,1,...,\overline{T}+1\}$ integers. Note the convention of starting an activity at the beginning of a time period and finishing it at the end of it. According to the applied convention, time period one is the first working period.

Let S_i, $\underline{S}_i \le S_i \le \overline{S}_i$ denote the start time of activity i for $i \in \{1,...,N\}$, where $\underline{S}_i$ $\left(\overline{S}_i\right)$ denotes the earliest (latest) starting time of activity i. Because preemption is not allowed, the ordered set $\boldsymbol{S} = \{S_1,..., S_N\}$ defines a schedule for the project. Naturally, the latest starting times are varying in the function of $\overline{T}$.

Let $D = \{D_1,...,D_N\}$, where $D_i \in [A_i, B_i]$, D_i is integral, for $i \in \{1,2,...,N\}$, the ordered set of the activity durations. By definition, a resource-feasible schedule set remains resource-feasible: (1) for each $D = \{D_1,...,D_N\}$ combination of the feasible activity durations: $D_i \in [A_i, B_i]$, D_i is integral, $i \in \{1,2,...,N\}$; (2) for each $S = \{S_1, S_2,..., S_N\}$ combination of the feasible activity starting times (activity movements): $S_i \in \left[\underline{S}(D)_i, \overline{S}(D)_i\right]$, S_i is integral and $D = \{D_i \,|\, i \in \{1,2,...,N\}\}$.

3. MODEL DESCRIPTION

In this section, we describe a robust scheduling model for the resource-constrained project-scheduling problem with uncertain-but-bounded activity durations. In the presented mixed integer linear programming (MILP) model the total number of zero-one variables is $|RR|$, and the formulation is based on the well-known "big-M" constraints.

Let SA_i, SB_i denote the starting time of activity i, $i \in \{0, 1, 2, \ldots, N+1\}$ in the optimistic and pessimistic resource constrained schedules, respectively. By definition, in the optimistic and pessimistic schedules the activity durations are A_i and B_i for each $i \in \{1, 2, \ldots, N\}$, respectively.

The model gives a robust "makespan minimal" resource constrained schedule, which is not effected by the uncertain activity durations. Here "makespan minimal" means a schedule for which a given linear combination of the optimistic and pessimistic resource constrained makespans is minimal. Naturally, the optimal solution will be a function of the weighting coefficients.

Defining the decision variables:

$$Y_{ij} = \begin{cases} 1 & \text{if } i \to j \text{ inserted} \\ 0 & \text{otherwise} \end{cases}, \text{where } i \to j \in RR, \tag{5}$$

the following MILP model arises:

$$WA * SA_{N+1} + WB * SB_{N+1} \to \boldsymbol{min}, \tag{6}$$

subject to

$$\sum_{i \to j \in RR_f} Y_{ij} \geq 1, \; f \in \{1, \ldots, F\} \tag{7}$$

$$SA_i + A_i \leq SA_j + \left(\overline{SA}_i - \underline{SA}_j + A_i \right) * \left(1 - Y_{ij} \right), \; i \to j \in RR \tag{8}$$

$$SB_i + B_i \leq SB_j + \left(\overline{SB}_i - \underline{SB}_j + B_i \right) * \left(1 - Y_{ij} \right), \; i \to j \in RR \tag{9}$$

$$SA_i + A_i \leq SA_j, \; i \to j \in NR, \tag{10}$$

$$SB_i + B_i \leq SB_j, \; i \to j \in NR, \tag{11}$$

$$SB_{N+1} \leq \overline{T} + 1, \tag{12}$$

$$Y_{ij} \in \{0, 1\}, \; i \to j \in RR. \tag{13}$$

The objective function (6) minimizes the linear combination of the optimistic, most likely, and pessimistic resource constrained makespans. Constraint set (7) assures the resource feasibility (we have to repair each resource conflict explicitly or implicitly, therefore from each conflict repairing set we have to choose at least one element). Constraint sets (8-9) take into consideration the precedence relations between activities in the function of the inserted repairing relations. Constraint sets (10-11) take into consideration the original precedence (network) relations between activities. Finally, constraint (12-13) specifies an upper bound for the pessimistic project makespan.

Naturally, the optimal solution is a function of the $\{WA, WB\}$ coefficients. According to the model construction in the optimal schedule every possible activity movement is resource feasible, and schedule is "robust" because it is invariant to the activity durations in the $[A_i, B_i]$ interval. In other words, a non-critical activity movement (a non-critical delay) or a longer (but possible) activity duration is unable to destroy the resource feasibility of the schedule.

The presented MILP model is a modified and simplified version of the original forbidden set oriented model developed by Alvarez-Valdés and Tamarit [4]. The reason of the modification is simple: in our model the activity durations are uncertain-but-bounded variables. The possibility of the simplification follows from a simple lemma (see Figure 1).

LEMMA: The optimal solution $Y^* = \left\{ Y_{ij} \middle| Y_{ij} = 1, i \to j \in RR \right\}$ is acyclic.

PROOF: In order to prove the lemma, suppose that there is at least one cycle in the optimal solution (as is shown by Figure 4). In this case, there is an arc (path) from i to j ($i \to \cdots \to j$) and an other arc (path) from j to i ($j \to \cdots \to i$). According to the inserted repairing relations, $S_i + D_i \le S_j$ and $S_j + D_j \le S_i$. From the premise it follows, that $S_i + D_i \le S_j \le S_j + D_j \le S_i$. Therefore, D_i will be zero. This contradicts that $D_i > 0$. ♦

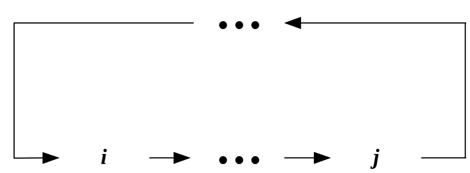

Figure 1. Demonstration of the acyclic nature of the conflict-repairing process

According to the above lemma, we can omit from the original model the following explicit constraints served to eliminate cycles:

$$0 \le Y_{ij} + Y_{ji} \le 1, \; i \to j \in RR, \; j \to i \in RR \qquad (14)$$

$$Y_{ik} \ge Y_{ij} + Y_{jk} - 1, \; i \to k \in RR, \; i \to j \in RR, \; j \to k \in RR \qquad (15)$$

Naturally, the original explicit cycle eliminating constraint set may be useful in the relaxed linear programming (LP) problem solving process (when we applied it as a core element of a hybrid heuristic), in spite of the fact that, according to the former experiences Csébfalvi [6], Csébfalvi and Láng [7], Csébfalvi and Csébfalvi [8], Csébfalvi and Szendrői

[9], its constraining power is not so high.

The core element of the sampling-based cost-oriented schedule evaluation is a MILP problem, in which we try to maximize the NPV fixing the activity durations and the cash-flow values according to the generated random numbers. Generally, the solution of a MILP problem is a costly operation.

When the MILP problem is a core element of a simulation process, the total time requirement of the MILP problem solutions may be a critical factor of the simulation, which may degrade the quality of the sample-based approximation.

Fortunatelly, according to the applied implicit resource-constraint handling, the constraint set of the MILP contains only precedence constraints, which consist of the original predecessor-successor relations and the inserted resource-conflict repairing relations.

Replacing the standard precedence constraints:

$$S_i + D_i \le S_j ,\ i \to j \in NR \cup RR , \tag{16}$$

with a totally unimodular (TU) formulation, the resource-constrain-free net present value problem (NPVP) can be solved in polynomial time as a LP problem (see Pritsker et al. [5]):

$$\boldsymbol{max}\left[NPV = \sum_{i=1}^{N} \sum_{t \in T_i} C_{it} * X_{it} \right] = NPV^* , \tag{17}$$

$$\sum_{p=T_i}^{\overline{X}_i} X_{ip} + \sum_{q=\underline{X}_p}^{T_i + D_i - 1} X_{jq} \le 1 ,\ T_i \in \left\{ \underline{X}_i , \underline{X}_i + 1, \ldots, \overline{X}_i \right\} ,\ i \to j \in PS \cup RR , \tag{18}$$

$$X_{N+1} = \overline{T} + 1 , \tag{19}$$

$$\sum_{t \in T_i} X_{it} = 1 ,\ i \in \left\{ 1,2,\ldots,N \right\} , \tag{20}$$

$$C_{it} = C_i * e^{-\alpha (t + D_i - 1)} ,\ i \in \left\{ 1,2,\ldots,N \right\} ,\ t \in T_i , \tag{21}$$

$$X_{it} \in \left\{ 0,1 \right\} ,\ t \in T_i ,\ i \in \left\{ 1,2,\ldots,N \right\} . \tag{22}$$

Objective (17) maximizes the discounted value of all cash flows that occur during the life of the project. Note that early schedules do not necessarily maximize the NPV of cash flows. Constraints (18) represent the "strong" precedence relations. In constraint (19) the resource-constrained project's makespan T can be replaced by its estimated upper bound. Constraints (20) ensure that each activity i, $i \in \left\{ 1,2,\ldots,N \right\}$ has exactly one starting time within its time window $T_i = \left\{ \underline{X}_i , \underline{X}_i + 1, \ldots, \overline{X}_i \right\}$ where $\underline{X}_i$ ($\overline{X}_i$) is the early (late) starting time for activity i according to the precedence constraints and the latest project completion time $\overline{T}$. Constraint set

(21) describes for each activity the change of the cash flow in the function of the completion time. The binary decision variable set (22) specifies the possible starting times for each activity. Using a fast interior-point-solver [11-13] the modified LP problem can be solved nearly 100 times faster then with a traditional simplex solver.

4. COMPUTATIONAL EXPERIMENTS

The algorithm of the proposed model has been programmed in Compaq Visual Fortran® Version 6.5. The algorithm, as a DLL, was built into the *ProMan* system (Visual Basic® Version 6.0) developed by Ghobadian and Csébfalvi [12], . To solve the MILP problem a fast state-of-the-art solver, namely the CPLEX 12.0 in AIMMS 3.10 for Windows environment was used. The solver, as an AIMMS COM object, was integrated into *ProMan*. The computational results were obtained by running *ProMan* on a 1.8 GHz Pentium IV IBM PC with 256 MB of memory under Microsoft Windows XP® operation system. At the running of the resource-constrained project borrowed from Golenko-Ginzburg and Gonik [13] we changed the default optimality tolerance parameters (Relative Gap =0.01 % and Absolute Gap = 5 period) and the Time Limit parameter (10 hours). In the presented example, the $\{WA,WB\}=\{1,1\}$ weight set was used.

In this section, as a motivating example, we consider a larger resource constrained project with 36 real activities presented by Golenko-Ginzburg and Gonik [13]. In contrast to the instances of the well-known and popular PSPLIB (Kolisch and Schpreher [14]), this instance already includes information of random activity durations, that is, for each activity *i* the optimistic and pessimistic duration time $[A_i, B_i]$, $i \in \{1,2,\ldots,36\}$ is given.

In this problem, there is only one resource type and 50 units are available from this type in each period. In this study, we assumed that each activity duration is an "uncertain-but-bounded" parameter without any possibilistic or probabilistic interpretation.

The instance contains $F=3730$ forbidden sets, which means that the problem is challenging one from methodological point of view. The unfeasible early start schedule of the project in activity-on-node representation mode with theoretically correct resource-profile visualization Csébfalvi [15] is presented in figures 2 and 3. In these figures the random part of each activity-duration is represented by a light gray bar. The predecessor-successor relations as represented by lines. The unconstrained optimistic (pessimistic) makespan is $173(265)$ time units, respectively. The initial data of the project are given in tables 1 and 2. This instance is a really hard RCPSP for the applied state-of-the-art CPLEX 12.0 solver. For each real activity, according to the usual managerial assumptions, the randomly generated uncertain-but-bounded cash flow values are presented in tables 3 and 4.

Using the CPLEX 12.0 solver with the mentioned setting, the solving process was terminated prematurely as a result of reaching the extremely large 10 hours time limit. Therefore, the given final solution $\{A^*, B^*\}=\{340,500\}$ is only a good one. The possibilistic range of the makespan $[\underline{A}, \overline{B}]=[395\ ,\ 465]$ and the net present value $[\underline{NPV}, \overline{NPV}]=[1579\ ,\ 2111\ ]$ were estimated by simulation.

The pseudo-code of the cash flow generation process is the following (***INT2000*** random number generator calls ***RAND2000*** (Microsoft Q86523) random number generator to generate integer random numbers from a given interval):

```
For A = 1 To RealActivities
  Do
    CashFlow_M(A) = INT2000(-500, 1000)
  Loop While Abs(CashFlow_M(A)) < 100
  If CashFlow(A) > 0 Then
    CashFlow_A(A) = CInt(CashFlow_M(A) - 0.15 * CashFlow_M(A))
    CashFlow_B(A) = CInt(CashFlow_M(A) + 0.05 * CashFlow_M(A))
  Else
    CashFlow_A(A) = CInt(CashFlow_M(A) + 0.15 * CashFlow_M(A))
    CashFlow_B(A) = CInt(CashFlow_M(A) - 0.05 * CashFlow_M(A))
  End If
Next A
```

Table 1: The initial data of the Golenko-Ginzburg and Gonik project

a	A_a	B_a	R_{ia}	IP_a
0	0	0	0	
1	16	60	16	{0}
2	15	70	15	{0}
3	18	35	18	{0}
4	19	45	19	{0}
5	10	33	10	{0}
6	18	15	18	{1}
7	24	50	24	{1}
8	25	18	25	{6}
9	16	24	16	{6}
10	19	38	19	{2}
11	20	22	20	{2}
12	18	32	18	{3}
13	15	45	15	{4}
14	16	78	16	{5}
15	17	45	17	{14}
16	19	35	19	{14}
17	21	60	21	{10, 13}
18	24	50	24	{10, 13}

Table 2: The initial data of the Golenko-Ginzburg and Gonik project

a	A_a	B_a	R_{1a}	IP_a

19	13	42	13	{17}
20	16	30	16	{15, 18}
21	12	21	12	{15, 18}
22	14	20	14	{15, 18}
23	16	42	16	{16}
24	15	40	15	{12}
25	13	28	13	{12}
26	14	35	14	{8, 11}
27	18	24	18	{7, 9}
28	22	22	22	{7, 9}
29	10	18	10	{26, 27}
30	18	38	18	{24, 28}
31	16	55	16	{24, 28}
32	17	30	17	{25}
33	19	37	19	{20, 23}
34	20	38	20	{21, 32}
35	15	55	15	{19, 22}
36	24	22	24	{29, 30, 34}
37	0	0	0	{31, 33, 35, 36}

Table 3: The generated uncertain-but-bounded cash flow values

a	***CF_A***$_a$	***CF_M***$_a$	***CF_B***$_a$
1	482	567	595
2	335	394	414
3	93	109	114
4	416	489	513
5	268	315	331
6	275	323	339
7	-363	-316	-300
8	258	304	319
9	299	352	370
10	354	416	437
11	150	176	185
12	92	108	113
13	734	863	906
14	-378	-329	-313
15	-499	-434	-412
16	694	816	857
17	802	943	990
18	-125	-109	-104

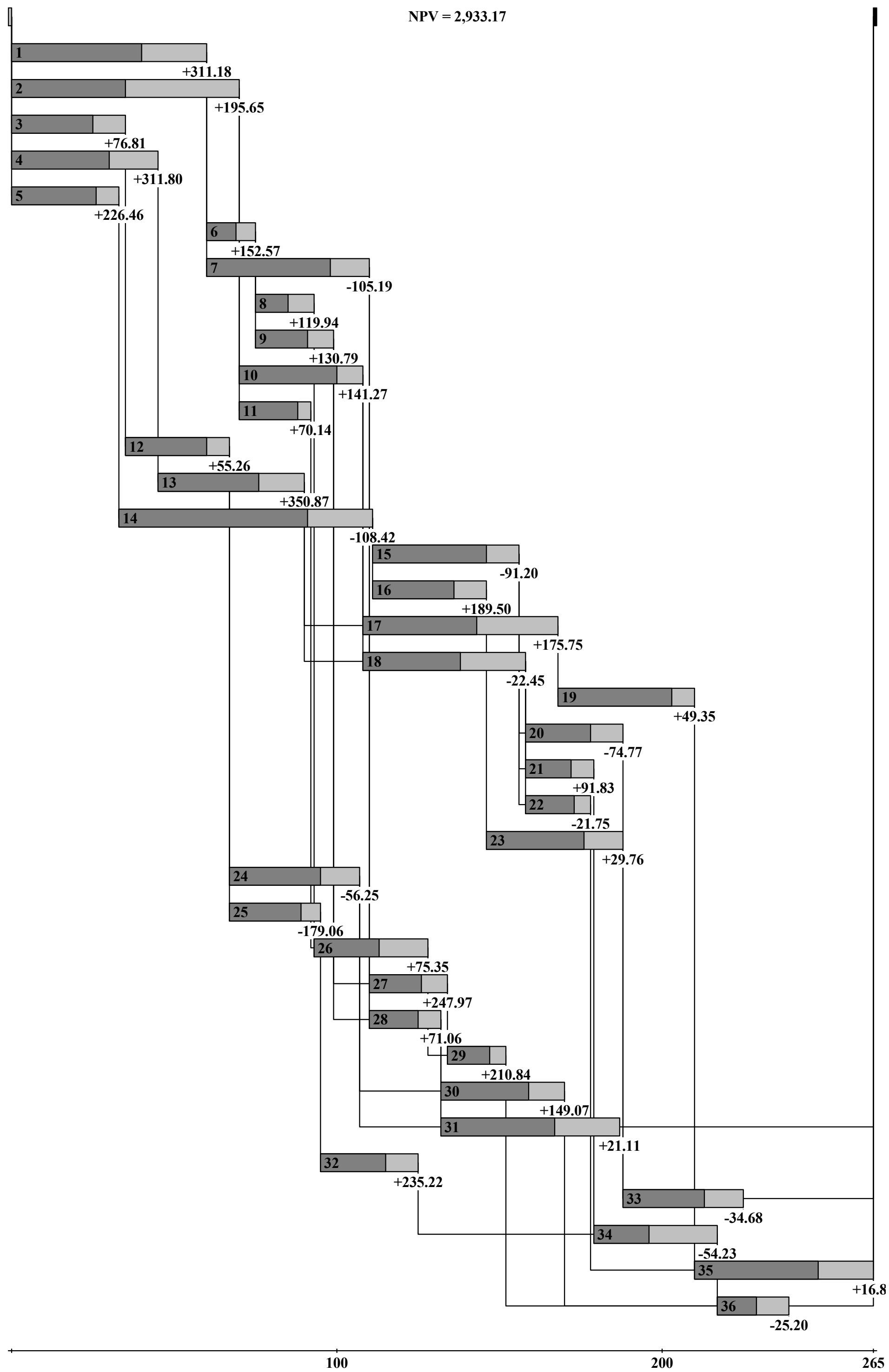

Figure 2. Cash flow oriented early start project visualization with the nominal cash flow values.

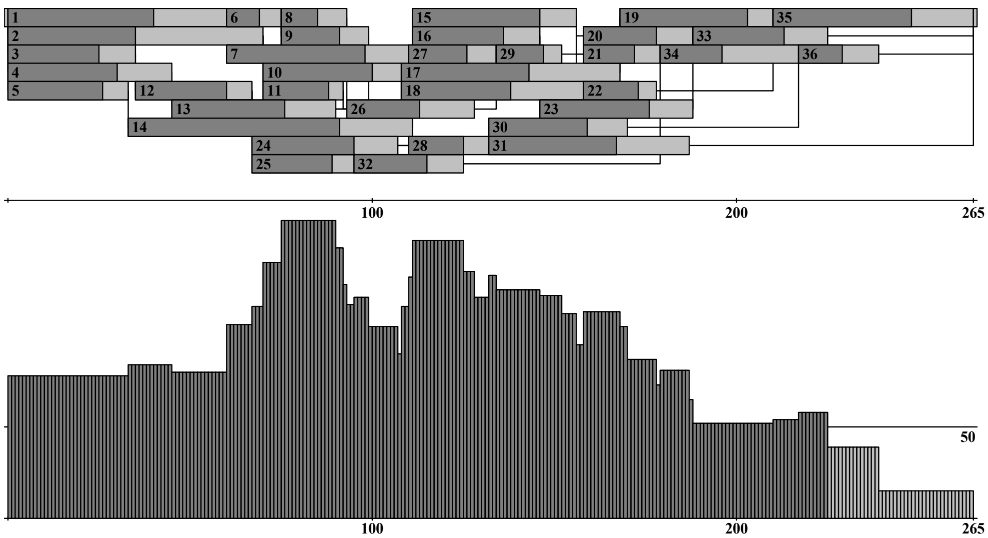

Figure 3. Resource-usage oriented theoretically correct early start project visualization

Table 4: The generated uncertain-but-bounded cash flow values

a	***CF_A_a***	***CF_M_a***	***CF_B_a***
19	343	403	423
20	-563	-490	-465
21	468	550	578
22	-148	-129	-123
23	166	195	205
24	-189	-164	-156
25	-532	-463	-440
26	230	271	285
27	805	947	994
28	226	266	279
29	819	964	1012
30	694	816	857
31	116	137	144
32	698	821	862
33	-378	-329	-313
34	-546	-475	-451
35	202	238	250
36	-316	-275	-261

The total number of the generated random schedules was 1000 using the uniform random number generator to generate the durations and the cash flow values. We have to note again, that in our case the simulation is not a costly operation, because using a fast interior point solver and a totally unimodular formulation, the optimistic and pessimistic NPV optimization problem can be solve within a fraction of a second.

6. CONCLUSONS

In this paper, we presented new primary-secondary-criteria scheduling model for resource-constrained project scheduling problem (RCPSP) with uncertain activity durations and cash flows (RCPSP-UD-UC) approach producing a “robust” resource-feasible schedule, which is immune against uncertainties in the activity durations and which is on the sampling-based scenarios may be evaluated from duration and cost oriented point of view. In the presented approach, it is assumed that each activity duration and each cash flow value is an uncertain-but-bounded parameter without any probabilistic or possibilistic interpretation and characterized by an optimistic and pessimistic estimations. The evaluation of a given robust schedule is based on the investigation of variability of the makespan as a primary and the net present value (NPV) as secondary criterion on the set of randomly generated scenarios given by a sampling-on-sampling-like process. Theoretically, the robust schedule-searching algorithm was formulated as a mixed integer linear programming problem, which is combined with duration and cost oriented sampling-based approximation phase. The model is invariant to the “real meaning” of the duration and cash flow estimations, therefore in the sampling-phase the pure "uncertain-but-bounded" approach can be replaced by possibilistic (membership function oriented) or probabilistic (density function oriented) or a mixed approach. In order to illustrate the essence of the proposed approach we presented detailed computational results for a larger and very challenging project instance borrowed from Golenko-Ginzburg and Gonik [14] and discussed by several authors in the literature. A problem specific fast and efficient harmony search algorithm for large uncertain problems, namely the RCPSP-UD-UC version of the Sounds of Silence (SoS) metaheuristic is a new member of the SoS family, which was originally developed by Csébfalvi et al. [16-17], more over it was extended for a wide range of the RCPSP [18-23], that will be presented in a forthcoming paper.

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