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Improved group-theoretical method for eigenvalue problems of special symmetric structures, using graph theory

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This paper is dedicated to Professor Franz Ziegler of TU-Wien on the occasion of his 70th birthday.

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ABSTRACT

Group-theoretical methods for decomposition of eigenvalue problems of skeletal structures with symmetry employ the symmetry group of the structures and block-diagonalize their matrices. In some special cases, such decompositions can further be continued. This particularly happens when submatrices resulted from the decomposition process, correspond to substructures with new symmetrical properties which are not among the properties of the original structure. Thus, a group-theoretical method is not able to recognize such additional symmetry from the original problem. In this paper, an algorithm is presented based upon a combination of group-theoretical ideas and graph-methods. This algorithm identifies the cases where the structure has the potential of being further decomposed, and also finds the symmetry group, and subsequently the transformation which can further decompose the system. It is also possible to find out when the block-diagonalization is complete and no further decomposition is possible. This is particularly useful for large eigenvalue problems such as calculation of the buckling load or natural frequencies of vibrating systems with special symmetries.

1. Introduction

Symmetry has been widely studied in science and engineering [1–5]. Large eigenvalue problems arise in many scientific and engineering problems [6–8]. While the basic mathematical ideas are independent of the size of matrices, the numerical determination of eigenvalues and eigenvectors requires additional considerations as the dimensions and the sparsity of matrices increase. Special methods are needed for efficient solution of such problems.

Methods are developed for decomposing the graph models of structures in order to calculate the eigenvalues of matrices with special patterns [9,10]. The eigenvectors corresponding to such patterns are studied in [11]. The application of these methods is extended to the vibration of mass-spring systems [12], and free vibration of frames [13].

Group theory has been known as the "mathematical language of the symmetry", and group representation theory is the vehicle for exploring symmetry in linear problems [14]. Methods based on group theory, have been implemented thoroughly in different fields of science such as physics and chemistry [14–16]. In structural mechanics, also such methods have been established for diverse problems such as dynamic analysis, bifurcation problems, force methods of structural analysis, and finite element methods [4,5,17–24]. However, it can be possible to process the results of such techniques, using methods of graph theory in order to explore if there is the possibility of further decomposition for a given problem which has been block-diagonalized by the means of group-theoretical methods. In this paper, after a brief review on the conventional methods available for decomposition of a symmetrical system by the means of group theory, physical interpretation of subproblems resulted by such techniques are studied using the graph model of the system in order to find the possibility of further decomposition.

2. Symmetry groups and representation theory; basic concepts

2.1. Symmetry and symmetry operations

The symmetry of a body is described by introducing the set of all those transformations which preserve the distance between all pairs of points of the body and maps the body into coincidence with itself [15]; either the result of the transformation is equivalent or it is identical with the primary arrangement. Each of these

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